A pre-designed protection scheme for WDM ring networks

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ABSTRACT
We consider a packet-switched network overlaid on a wavelength-routing optical network. A pre-designed path protection scheme to restore traffic upon a single link or node failure in bidirectional rings is presented. An approach to restore affected traffic using a pre-designed logical topology that is embedded after failure on the surviving topology is discussed. We present several logical topology designs with the goal of minimizing the maximum logical distance between any two nodes. The designs are rigorously analyzed and bounds (which are tight in some cases) are presented.

Keywords: Wavelength routing, survivability, pre-designed protection, multi-hop networks, rings, logical topology.

1. INTRODUCTION
As a result of recent progress in the field of optical fibers and lightwave communications technology, tremendous optical bandwidth has become available for the use of wide-area and metropolitan area networks. With a few orders of magnitude separating the current optical bandwidth and peak electronic clock speeds, wavelength-division multiplexed (WDM) networks have emerged as strong candidates for the construction of next generation optical networks.

Wavelength routing, achieved by wavelength-aware routing nodes, is an approach that is used to route light signals according to their carrier wavelengths and points of origination. WDM wavelength routing networks enable the formation of lightpaths which are envisioned to be the transport links for a variety of networks that may be overlaid on the physical optical infrastructure. Lightpaths are all-optical circuit-switched paths which are formed by choosing one wavelength on each link from the source to the destination of the lightpath and concatenating them. When the routing nodes are incapable of wavelength conversion, the wavelengths on a lightpath’s route must all be the same.

We consider a multi-hop WDM wavelength routing ring network with a packet router attached to every wavelength router/wavelength add-drop multiplexer. Lightpaths are established between certain packet router pairs, and traffic between two end-nodes flows over possibly multiple lightpaths and is processed electronically at intermediate nodes. The optical network infrastructure, namely, the wavelength routers and fibers interconnecting them, is referred to as the physical topology. The logical topology (LT) refers to the set of lightpaths that are established over the physical topology. Each lightpath is thus a logical link. In this context, we may say that user packet traffic flows from source to destination using possibly many links in the LT.

In this paper, we consider the restoration of traffic upon occurrence of a single link or node failure. The two conventional approaches to protecting the network from service disruptions are line protection and path protection. Line protection refers to a technique in which the failed link is backed up by an alternative route between the end-nodes of the link, whereas in path protection, all the affected paths are switched to their respective backup protection paths. In this work, we present a protection scheme for WDM ring networks that can be used to restore traffic between all end-node pairs that use an affected lightpath under normal operation. In other words, we propose a scheme to reroute end-to-end traffic over new logical paths that do not use any of the affected logical links (lightpaths). Although this idea is not new, the work in the literature mainly take one of two forms: minimize the spare resources (fibers, wavelengths, etc.) required to restore all affected connections, or maximize the number of connections restored for a given amount of spare resources. A main feature of our proposal is the partial restoration of traffic between all affected node pairs for a given amount of protection resources. Such an approach could be used to gracefully
degrade the quality of service of all affected connections while the failed link or node is repaired. In contrast, previous approaches (in which protection resources are constrained) would restore some traffic on some connections and drop the other connections altogether.

Our proposed scheme is a pre-computed, failure-dependent restoration scheme. When a link or node failure is detected, we embed a pre-computed set of lightpaths (that depends on the failed component, in general) that do not use the failed component. In this work, we do not assume that the set of affected node-pairs is known. Instead, we assume that any of the node-pairs can be affected, and focus on providing all-to-all connectivity between the surviving end-nodes on the logical topology using the protection resources exclusively. Note, however, that those node-pairs that were not affected may continue to use their normal working paths and wavelengths.

We illustrate our approach with the help of an example below. Starting with an $N$-node bidirectional ring, the surviving topology upon a single link failure is a bidirectional line topology with $N$ nodes, and that upon a single node failure is a line topology with $N - 1$ nodes. The node-pair $(u,v)$ in Figure 1 uses three lightpaths, one of which uses the failed link (shown by an “X”). Traffic from $u$ to $v$ will be restored (partially, in general) using a set of lightpaths that are established (using wavelengths reserved for protection) after the link failure is detected.

We assume in this paper that the constraint on the protection resources is a constraint only on the number of protection wavelengths per link. With this background, the problem now is to design a logical topology that provides all-to-all connectivity between nodes on the surviving line topology with a limited number of wavelengths.

![Figure 1. Logical path rerouting after link failure in a bidirectional ring network.](image)

We now briefly review some of the related literature on logical topology design. The problem of deciding whether a given set of lightpaths can be established using a given number of wavelengths on an arbitrary physical topology has been shown to be NP-complete, and heuristics for maximizing the number of established lightpaths for a given number of wavelengths were also given. Another paper assumes that average traffic demand between node pairs is given, and presents heuristics to maximize the single-hop traffic for a given number of wavelengths. In another work, the number of wavelengths in establishing a given set of lightpaths is attempted to be minimized. In a slightly different formulation, the number of transceivers rather than the number of wavelengths is assumed to be limited, and the problem of designing the LT such that the load on the maximally loaded logical link is minimized is considered. A similar problem is also studied which considers two objective functions – minimizing the maximum load over all logical links, and minimizing the average packet delay. The problem formulation is extended to multiple fibers per link. The focus of some papers is the optimal reconfiguration of the LT in response to changes to traffic. All of the papers (with an exception) consider an arbitrary mesh topology as the physical topology. The general approach in most of the literature is to pose the problem as either a mixed-integer program (linear or non-linear), obtain bounds, and propose heuristics to solve the problem. While this is inevitable because of the complexity of the problem in a mesh topology, we shall see that the simplicity of the line network enables us to present a rigorous analysis of the LTs designed.
The rest of the paper is organized as follows. The network model and the problem statement are presented in Section 2. As we shall see in the next section, our design objective in this paper is the minimization of the maximum number of optical hops between any two nodes. We propose several LT designs and analyze them in Sect. 3. Finally, our conclusions and directions for future work in Sect. 4 complete the paper.

2. NETWORK MODEL AND PROBLEM STATEMENT

The physical topology of the surviving network (the line) is specified by a directed graph $G = (V, E)$, where the set of nodes $V = \{1, 2, \ldots, N\}$ corresponds to optical wavelength routers and the set of edges $E$ correspond to point-to-point single-fiber links in the network. The physical distance between nodes $i$ and $j$, denoted by $P(i, j)$, is the number of edges in the shortest path from $i$ to $j$ in $G$. Each wavelength router is connected to a packet router that electronically processes any packets arriving at the node. We assume that each fiber can carry $F$ wavelengths, $\lambda_1, \lambda_2, \ldots, \lambda_F$, and thus, indirectly limit the amount of optical terminating equipment. Wavelength conversion is assumed to be unavailable.

The logical topology $G' = (V, E')$ is a directed graph consisting of the set of nodes corresponding to the packet routers$^1$. A directed edge $(u, v) \in E'$ iff a lightpath from $u$ to $v$ is established. The terms lightpath and logical links are used interchangeably throughout the paper. The logical distance between two nodes $u$ and $v$, denoted $d(u, v)$, is defined to be the number of edges in $E'$ in a shortest path from $u$ to $v$ in $G'$. $R[u, v]$ denotes the shortest logical path from $u$ to $v$ that is chosen to route packets from $u$ to $v$. The diameter of the logical topology $D = \max_{u,v} d(u, v)$ is the distance between the farthest nodes in $G'$. Henceforth, unless otherwise specified, diameter and distance refer to those terms in the logical topology.

Our general objective in designing the LT is to minimize packet delay. Packet delay can be broken down into (a) transmission delay on each logical link, (b) propagation delay on each logical link, (c) processing delay at each router, and (d) queuing delay at each router. In an optical network with high-capacity wavelengths, it is reasonable to assume that transmission and queuing delays$^2$ are negligible in comparison with propagation and electronic processing delays. We ignore propagation delays and traffic intensities in this paper, and assume that packet delay is solely affected by the number of optical hops between nodes. In this work, our goal is to design an LT to be embedded on top of the surviving line network using $F$ wavelengths that are reserved for protection. The specific objective function that we choose to minimize is the maximum logical distance between any two nodes (the diameter). Note that this function corresponds to minimizing the worst-case delay between any two nodes. Other objective functions such as average delay and throughput may also be appropriate.

In the next section, we derive non-trivial lower bounds for the diameter where possible, and present LT design algorithms and upper bounds for the diameter.

3. LOGICAL TOPOLOGY DESIGN ALGORITHMS AND BOUNDS

Suppose the nodes of the surviving line network are numbered $1, 2, \ldots, N$ from left to right. Since a unique physical route exists between any given pair of nodes, logical link $(i, j)$ in this topology consists of physical links $\{(i, i+1), (i+1, i+2), \ldots , (j-1, j)\}$. Thus any logical link $(i, j)$ such that $j > i$ uses physical link $(i, i+1)$. Similarly, any logical link $(i, j)$ such that $j < i$ uses physical link $(i, i-1)$. Therefore, for any given node $i$, the number of logical links to nodes $j > i$ is at most $F$, and the number of logical links to nodes $j < i$ is also not more than $F$.

We start by considering the case of a single wavelength. As we shall see, the single-wavelength case is more amenable to analysis, and almost optimal designs can be achieved. Further, it provides some insight into designing LTs for the multi-wavelength case.

3.1. Single Wavelength

From the above discussion, the maximum in-degree (or out-degree) of any node $v$ in the LT is 2 (except nodes 1 and $N$ which can have an in-degree (out-degree) of at most one). Further, if $(i, j) \in E(G')$ and $(i, k) \in E(G')$, then $j < i < k$ or $j > i > k$. Thus, the LT looks like a concatenation of unidirectional rings attached at connecting nodes as shown in Figure 2. A straightforward LT design that establishes one lightpath in each direction per physical link uses $N-1$ rings of size 2, and gives a diameter of $N-1$ and average distance of approximately $N/3$.

$^1$For example, the number of receivers at a node is limited to $F\Delta$ where $\Delta$ is the in-degree of the node in $G$.

$^2$We abuse notation slightly and use $V$ to denote the set of packet routers as well.

$^3$Queuing delays may become significant for large packet sizes and very high lightpath loadings, for example, $> 90\%$. 
Suppose a design has $m$ logical rings. In general, the logical rings $R_1, R_2, \ldots, R_m$ can be of different sizes. Suppose that the number of nodes in ring $R_i$ is $r_i$. Since there are $m - 1$ connecting nodes that are part of two logical rings, we have that $\sum_{1 \leq i \leq m} r_i = N + m - 1$. Let the number of links in ring $R_i$ from lower (higher) numbered nodes to higher (lower) numbered nodes in ring $R_i$ be given by $\alpha_i$ ($\beta_i$). Then, $\alpha_i + \beta_i = r_i$. In the figure for $R_i$, $\alpha_i = 3$, $\beta_i = 4$ and $r_i = 7$. We now present a simple but non-trivial lower bound on the LT diameter. Let $T(i,j) \triangleq \sum_{k=1}^{j} r_k$.

3.1.1. Lower Bounds

**Theorem 3.1 (A Lower Bound).** $D \geq \lceil N/2 \rceil$.

**Proof.** Recalling that $d(i,j)$ is the logical distance between nodes $i$ and $j$, we have $d(1,N) = \sum_{1 \leq i \leq m} \alpha_i$ and $d(N,1) = \sum_{1 \leq i \leq m} \beta_i$. Since $d(1,N) + d(N,1) = \sum_{1 \leq i \leq m} (\alpha_i + \beta_i) = N + m - 1$, and since $m \geq 1$, we must have $D \geq \max \{d(1,N), d(N,1)\} \geq \lceil N/2 \rceil$. $\Box$

It is possible to improve this bound significantly as we show next.

**Theorem 3.2 (An Improved Lower Bound).** Let $f(m) \triangleq \frac{1}{2} \left[ \frac{2^m (N + m - 1)}{2^m - 1} \right] - 2$ for even integers $m$, and let $f(m) \triangleq \frac{1}{2} \left[ \frac{3\cdot2^{m-1} \cdot (N + m - 1)}{3\cdot2^{m-1} - 1} \right] - 2$ for odd integers $m \geq 3$. Also, let $f(1) \triangleq N - 1$. Then, $D \geq \lceil \{ \min_{1 \leq m \leq N - 1} f(m) \} \rceil$.

**Proof.** Suppose the LT consists of $m$ rings, and let $D(m)$ be the diameter of the LT. Let $Q(i,j)$ be defined to be the maximum distance in the LT from a node in $R_i$ to a node in $R_j$, i.e., $Q(i,j) \triangleq \max_{u \in R_i, v \in R_j} d(u,v)$. Then, for $i < j$, $Q(i,j) = \sum_{1 \leq k \leq j} \alpha_k + \beta_j + \beta_j - 2$ and $Q(j,i) = \sum_{1 \leq k \leq j} \beta_k + \alpha_i + \alpha_i - 2$.

Let $S \triangleq 2D(m) - N - m + 5$, and suppose for the moment we can show that for $j > i > 0$ and $i + j = m + 1$, $r_i + r_j \leq 2^{i-1} \cdot S$ for $2 \leq m \leq N - 1$, and $r_{m+1} \leq S \cdot (2^{|m+1| - 1})$ for all odd $m$ s.t. $3 \leq m \leq N - 1$. Then, for even $m$, we have:

$$N + m - 1 = \sum_{i=1}^{m} r_i \leq \sum_{i=1}^{m} 2^{i-1} \cdot S = (2^m - 1)S.$$ 

Substituting for $S$, and rearranging terms, we get

$$D(m) \geq \frac{1}{2} \left[ \frac{2^m (N + m - 1)}{2^m - 1} \right] - 2 = f(m).$$

For odd $m \geq 3$, we have

$$N + m - 1 = \sum_{i=1}^{m} r_i = \sum_{i=1}^{m} (r_i + r_{m+1-i}) + r_{m+1} \leq \sum_{i=1}^{m} 2^{i-1} \cdot S + (2^{|m+1|} - 1)S = (3 \cdot 2^{|m|} - 1 - 1)S.$$
Substituting for \( S \) and rearranging terms, we get
\[
D(m) \geq \frac{1}{2} \left[ \frac{(3 \cdot 2[\frac{m}{2}]^{-1}) (N + m - 1)}{3 \cdot 2[\frac{m}{2}]^{-1} - 1} \right] - 2 = f(m).
\]

Finally, \( D(1) \) is obviously equal to \( N - 1 \) which is also \( f(1) \). Noting that \( D \) is an integer and the range of \( m \) is from 1 to \( N - 1 \), we get
\[
D \geq \min_{1 \leq m \leq N-1} [D(m)] \geq \min_{1 \leq m \leq N-1} [f(m)].
\]

It now remains for us to show that \( r_i + r_j \leq 2^{i-1}S \), when \( i + j = m + 1 \), \( j > i > 0 \), \( 2 \leq m \leq N - 1 \). We do this by induction on \( i \), for each value of \( m \). First, suppose \( m \) is even. Then, since \( D(m) \) is the diameter, we must have
\[
Q(1,m) + Q(m,1) = \sum_{1 \leq k \leq m} \alpha_k + \beta_1 + \beta_m - 2 + \sum_{1 \leq k \leq m} \beta_k + \alpha_1 + \alpha_m - 2 = N + m - 1 + r_1 + r_m - 4 \leq 2D(m).
\]
This implies that \( r_1 + r_m \leq 2D(m) - N - m + 5 = S \). Now, suppose \( r_x + r_y \leq 2^{x-1}S \) for all \( 1 < x < i \) and \( j+1 < y < m \) s.t. \( x + y = m + 1 \), and consider \( r_i + r_j \). Then,
\[
Q(i,j) + Q(j,i) = \sum_{i=k}^j \alpha_k + \beta_i + \beta_j - 2 + \sum_{i=k}^j \beta_k + \alpha_i + \alpha_j - 2
= T(1,m) + r_i + r_j - 4 - T(1,j-1) - T(j+1,m)
= N + m - 1 + r_i + r_j - 4 - \sum_{k=1}^{i-1} 2^{k-1}S
= N + m - 5 + r_i + r_j + (2^{i-1} - 1)S.
\]

Since \( Q(i,j) + Q(j,i) \leq 2D(m) \), we have \( r_i + r_j \leq 2D(m) - N - m + 5 + (2^{i-1} - 1)S = 2^{i-1}S \). This completes the proof for even values of \( m \). For odd \( m \geq 3 \), we still have to show that \( r_{[\frac{m}{2}]} = (2^\frac{m}{2} - 1)^{S} \). Now,
\[
Q \left( \left[ \frac{m}{2} \right], \left[ \frac{m}{2} \right] \right) + Q \left( \left[ \frac{m}{2} \right], \left[ \frac{m}{2} \right] + 1 \right) = r_{[\frac{m}{2}]} + r_{[\frac{m}{2}]} - 2 + r_{[\frac{m}{2}]} + r_{[\frac{m}{2}]} - 2
= r_{[\frac{m}{2}]} + r_{[\frac{m}{2}]} - 4 + T(1,m) - T \left( 1, \left[ \frac{m}{2} \right] - 1 \right) - T \left( \left[ \frac{m}{2} \right] + 1, m \right).
\]

Similarly, it can be shown that
\[
Q \left( \left[ \frac{m}{2} \right], \left[ \frac{m}{2} \right] + 1 \right) + Q \left( \left[ \frac{m}{2} \right] + 1, \left[ \frac{m}{2} \right] \right) = r_{[\frac{m}{2}]} + r_{[\frac{m}{2}]} - 4 + T(1,m) - T \left( 1, \left[ \frac{m}{2} \right] \right) - T \left( \left[ \frac{m}{2} \right] + 2, m \right).
\]

Since \( Q(i,j) + Q(j,i) \leq 2D(m) \), we have
\[
Q \left( \left[ \frac{m}{2} \right], \left[ \frac{m}{2} \right] \right) + Q \left( \left[ \frac{m}{2} \right], \left[ \frac{m}{2} \right] + 1 \right) + Q \left( \left[ \frac{m}{2} \right] + 1, \left[ \frac{m}{2} \right] \right) + Q \left( \left[ \frac{m}{2} \right] + 1, \left[ \frac{m}{2} \right] + 1 \right)
\leq 2D(m).
\]

Substituting for \( T(1,m), T \left( \left[ \frac{m}{2} \right] - 1 \right), \) and \( T \left( \left[ \frac{m}{2} \right] + 2, m \right), \) we have \( 2r_{[\frac{m}{2}]} - 8 + 2T(1,m) - 2T \left( \left[ \frac{m}{2} \right] - 1 \right) - 2T \left( \left[ \frac{m}{2} \right] + 2, m \right) \)
\leq 4D(m). Hence, \( r_{[\frac{m}{2}]} = (2^\frac{m}{2} - 1)^{S} \). This completes the proof. \( \Box \)

Unfortunately, a closed form expression for the above lower bound seems to be hard to compute, and the lower bound has to be numerically computed.
3.1.2. An LT Design

We now present an algorithm for designing the LT, and show its optimality in certain cases. Consider an \( N \)-node linear topology. Let us define \( h(k) \triangleq 2^{k+2} - 2k - 3 \) for any integer \( k \geq 0 \). Note that \( h(k) \) is an increasing function of \( k \). Let \( k^* \) be the largest integer such that \( h(k^*) \leq N \), and let \( w = N - h(k^*) \). If \( w = 0 \), set \( x = 0 \), otherwise, set \( x = w + 1 \).

Our LT design is symmetric about the middle node or link (depending on whether \( N \) is odd or even), and consists of \( m^* \triangleq 2k^* + [x] \) logical rings where \([x] = 0 \) if \( x = 0 \), \([x] = 1 \) if \( 0 < x \leq 2^{k^*+1} + 1 \), and \([x] = 2 \) otherwise. The sizes of the rings are as follows: \( r_1 = r_{m^*} = 2 \), \( r_2 = r_{m^*-1} = 4 \), \ldots, \( r_{k^*} = r_{m^*-k^*} = 2^{k^*} \). If \( 0 < x \leq 2^{k^*+1} + 1 \), \( r_{k^*+1} = x \). Otherwise, \( r_{k^*+1} = y \triangleq \lfloor \frac{x+1}{2} \rfloor \) and \( r_{k^*+2} = z \triangleq \lceil \frac{x+1}{2} \rceil \). For \( 1 \leq i < k^* \), ring \( R_i \) is connected to ring \( R_{i+1} \) at node \( 2^{i+1} - i - 1 \). The connections of rings \( R_i \) for \( m^* - k^* \leq i \leq m^* \) are defined by symmetry. Note that all rings have an even number of nodes except possibly the middle one. For ring \( R_i \), \( 1 \leq i \leq k^* \), we make \( \alpha_i = \beta_i = 2^{i-1} \). \( \alpha \)'s and \( \beta \)'s for the other rings are defined by symmetry. We illustrate the design with an example below. In Figure 3(a), a 30-node linear chain, and the logical rings along with their connecting nodes are shown. The implementation of rings \( R_3 \) and \( R_4 \) on the physical topology are shown in Figure 3(b).

\[ \begin{array}{c}
R_1 \quad R_2 \quad R_3 \quad R_4 \quad R_5 \\
1 \quad 2 \quad 3 \quad 4 \quad 5 \\
6 \quad 7 \quad 8 \quad 9 \quad 10 \\
11 \quad 12 \quad 13 \quad 14 \quad 15 \\
16 \quad 17 \quad 18 \quad 19 \quad 20 \\
21 \quad 22 \quad 23 \quad 24 \quad 25 \\
26 \quad 27 \quad 28 \quad 29 \quad 30
\end{array} \]

\[ \begin{array}{c}
R_1 \quad R_2 \quad R_3 \quad R_4 \\
1 \quad 2 \quad 3 \quad 4 \\
5 \quad 6 \quad 7 \quad 8 \\
9 \quad 10 \quad 11 \quad 12 \\
13 \quad 14 \quad 15 \quad 16 \\
17 \quad 18 \quad 19 \quad 20 \\
21 \quad 22 \quad 23 \quad 24 \\
25 \quad 26 \quad 27 \quad 28 \\
29 \quad 30
\end{array} \]

\[ \begin{array}{c}
R_1 \quad R_2 \quad R_3 \quad R_4 \\
1 \quad 2 \quad 3 \quad 4 \\
5 \quad 6 \quad 7 \quad 8 \\
9 \quad 10 \quad 11 \quad 12 \\
13 \quad 14 \quad 15 \quad 16 \\
17 \quad 18 \quad 19 \quad 20 \\
21 \quad 22 \quad 23 \quad 24 \\
25 \quad 26 \quad 27 \quad 28 \\
29 \quad 30
\end{array} \]

\[ \begin{array}{c}
R_1 \quad R_2 \quad R_3 \quad R_4 \\
1 \quad 2 \quad 3 \quad 4 \\
5 \quad 6 \quad 7 \quad 8 \\
9 \quad 10 \quad 11 \quad 12 \\
13 \quad 14 \quad 15 \quad 16 \\
17 \quad 18 \quad 19 \quad 20 \\
21 \quad 22 \quad 23 \quad 24 \\
25 \quad 26 \quad 27 \quad 28 \\
29 \quad 30
\end{array} \]

\[ \begin{array}{c}
R_1 \quad R_2 \quad R_3 \quad R_4 \\
1 \quad 2 \quad 3 \quad 4 \\
5 \quad 6 \quad 7 \quad 8 \\
9 \quad 10 \quad 11 \quad 12 \\
13 \quad 14 \quad 15 \quad 16 \\
17 \quad 18 \quad 19 \quad 20 \\
21 \quad 22 \quad 23 \quad 24 \\
25 \quad 26 \quad 27 \quad 28 \\
29 \quad 30
\end{array} \]

Figure 3. The LT design for a 30-node linear topology with \( F = 1 \). (a) The set of logical rings, and (b) the implementation of rings \( R_3 \) and \( R_4 \).

3.1.3. Upper Bounds

It can be easily shown that, for the above LT design, the diameter \( D = Q(1, m^*) \) if \( 0 < x \leq 2^{k^*+1} + 1 \), and, otherwise \( D = \max\{Q(1, m^*), Q(1, k^*+2)\} \). Using the definition of \( Q(i, j) \), \( D = \min(2(2^k - 1) + [x/2], \max\{2^k - 1 + [\frac{x}{2}], y - 1, 2(2^k - 1) + [\frac{x}{2}], [\frac{x}{2}]\}) \). The first term in the minimum expression is the diameter that occurs in the design when \( 0 < x \leq 2^{k^*+1} + 1 \), and the second term is the diameter otherwise.

This design is, in fact, optimal if \( N = 2^{k^*+2} - 2k - 3 \) or \( N = 3 \cdot 2^k - 2k - 2 \) for some integer \( k > 0 \). The optimality is consequent to a certain property of \( f(m) \), which we prove next.

**Theorem 3.3.** There exists an integer \( m' \) such that \( f(1) \geq f(2) \geq \cdots \geq f(m') \leq f(m'+1) \leq \cdots \leq f(N - 1) \).

**Proof.** We will show that there exists an \( m' \) s.t. \( f(i) - f(i+1) \geq 0 \) for \( i < m' \), \( f(i) - f(i+1) \leq 0 \) for \( i \geq m' \), \( i \) even; and \( f(i-1) - f(i) \geq 0 \) for \( i < m' \), \( f(i-1) - f(i) \leq 0 \) for \( i > m' \), \( i \) odd. Note that the proof of both of these inequalities is necessitated by the different definitions of \( f(i) \) for even and odd values of \( i \).

Let \( i \) be even and consider \( f(i) - f(i+1) \). Using the definition of \( f(i) \), we have

\[
f(i) - f(i+1) = \frac{1}{2} \left[ \frac{2^i(N + i - 1)}{2^i - 1} - \frac{3 \cdot 2^{i-1}(N + i)}{3 \cdot 2^{i-1} - 1} \right]
\]

\[
= \frac{N + i + 2 - 3 \cdot 2^i}{2(3 \cdot 2^{i-1} - 1)}
\]
after some algebra. Since the denominator of the above expression is always positive, \( f(i) - f(i+1) \geq 0 \) if \( N+i+2 \geq 3 \cdot 2^k \). Let \( m' - 1 \) be defined to be the largest value of \( i \) for which this latter inequality is true. It can easily be verified that for \( i \leq m' \), \( f(i) - f(i) \geq 0 \). □

Observe that Theorem 3.3 is also useful in efficiently computing the lower bound of \( D \) in Theorem 3.2. We can now show the optimality of our design algorithm in the cases mentioned above.

**Theorem 3.4.** The LT design algorithm is optimal if \( N = 2^{k+2} - 2k - 3 \) or \( N = 3 \cdot 2^k - 2k - 2 \) for some integer \( k > 0 \).

**Proof.** Let \( N = 2^{k+2} - 2k - 3 \). According to the design algorithm, there are \( 2k \) rings, and the size of the \( i \)th ring \( r_i = 2^i \), \( i = 1, 2, \ldots, k \). The other \( k \) rings are defined by symmetry. It is seen that \( D \leq T(1, 2k) = \sum_{1 \leq i \leq k} 2\alpha_i = 2^{k+1} - 2 \). Now, consider the lower bound of Theorem 3.2. It is easily verified that \( f(2k) = f(2k+2) = 2^{k+1} - 2 \). We now claim that \( f(2k) > f(2k+1) < f(2k+2) \) and that \( [f(2k+1)] = f(2k) = f(2k+2) \). Together with Theorem 3.3, this would prove that \( 2^{k+1} - 2 \) is also a lower bound.

From the definition of \( f(m) \) and using \( N = 2^{k+2} - 2k - 3 \),

\[
f(2k+1) = \frac{1}{2} \left[ \frac{3 \cdot 2^{k-1}(2^{k+2} - 2k - 3 + 2k)}{3 \cdot 2^{k-1} - 1} \right] - 2
\]

\[
= \frac{1}{2} \left[ \frac{4 \cdot 2^k(3 \cdot 2^{k-1} - 1) - 2^{k-1}}{3 \cdot 2^{k-1} - 1} \right] - 2
\]

\[
= 2^{k+1} - 2 - \frac{\frac{2^k}{2} - 1}{3 \cdot 2^{k-1} - 1}.
\]

Since \( 0 < \frac{2^{k+2}}{3 \cdot 2^{k-1} - 1} < 1 \) for \( k > 1 \), we have \( f(2k) > f(2k+1) < f(2k+2) \) and \( [f(2k+1)] = f(2k) = f(2k+2) \).

A similar proof is possible for \( N = 3 \cdot 2^k - 2k - 2 \). We omit the algebra, and simply state that the design in this case has \( D = 3 \cdot 2^{k-1} - 2 \), which is also the lower bound. □

Though we have only been able to analytically show optimality for certain special values of \( N \), the design is almost always optimal as can be seen from Figure 4. The bounds for the diameter are plotted against \( N \), the size of the network. As can be seen, the diameter increases linearly with \( N \). More importantly, we have observed that the difference between the bounds is at most 1, and is, in fact, 0 for about 80% of the values of \( N \) between 2 and 1000.

![Figure 4](image)

**Figure 4.** The upper and lower bounds on diameter, \( D \) vs. number of nodes, \( N \), obtained for the linear topology with \( F = 1 \).
3.2. Multiple Wavelengths

In this section we consider the general problem of embedding lightpaths, given a set of \(F (> 1)\) wavelengths, i.e., every physical link can now be a part of \(F\) lightpaths. At present, we do not have lower bounds for this problem. In the following, we present some LT designs that have a significantly smaller diameter than the LT which has \(F\) lightpaths (spanning one physical link each) over each physical link.

3.2.1. LT Designs and Upper Bounds

For simplicity of exposition, let us consider the case of two wavelengths, and then present the general case. From the previous discussion, each physical link can be a part of at most two lightpaths.

Consider the following setup of lightpaths: there are \(\sqrt{N}\) lightpaths in either direction on \(\lambda_1\), each spanning \(\sqrt{N}\) physical links; there are \((N - 1)\) lightpaths in either direction on \(\lambda_2\), each spanning a single physical link. We observe that in this case, the diameter is bounded by \(2\sqrt{N}\) (a maximum of \(\sqrt{N}/2\) hops on \(\lambda_2\) to get to a node where a lightpath on \(\lambda_1\) begins, a maximum of \(\sqrt{N}\) hops on \(\lambda_1\), and then at most \(\sqrt{N}/2\) hops on \(\lambda_2\) to get to the destination node). We show below that we can do significantly better, and, in fact, even decrease the order.

Before presenting our new design, let us consider a related problem, the solution of which, is used in our LT design. In a line network of \(N\) nodes, we wish to set up lightpaths using only one wavelength such that: the maximum of the distance from any interior node, say \(i\), to one of the two end-nodes, and the distance from an end-node (whichever is closer) to \(i\) is minimum. Let us denote this distance by \(\xi(i)\). Also, let us define \(\eta(N, 1)\) to be the maximum value of \(\xi(i)\) over the entire range of \(i\). More formally:

\[
\xi(i) = \max \{\min \{d(1, i), d(N, i)\}, \min \{d(i, 1), d(i, N)\}\},
\eta(N, 1) = \max_{1 \leq i \leq N} \xi(i)
\]

Note that in the above equations, 1 and \(N\) are the two end-nodes of the linear topology. The second argument in the definition of \(\eta(N, 1)\) is the number of available wavelengths, which we are currently considering to be one. Now the question we ask is: what is the minimum value of \(\eta(N, 1)\)?

Notice that since we are only allowed to use one wavelength, the setup is once again a combination of logical rings as in Section 3.1. However our objective function in this case is quite different and so the bounds presented before do not apply as such.

We next prove the following results for \(\eta(N, F)\).

**Lemma 3.5.** \(\eta(N, 1) \leq 2\sqrt{N}\).

**Proof.** Consider the setup shown in figure 5. The logical topology consists of \(\sqrt{N}\) rings, and \(\alpha_i = 1\) and \(\beta_i = \sqrt{N}\), \(1 \leq i \leq \sqrt{N}\). The distance from node 1 to any node \(i\) is at most \(2\sqrt{N} - 1\). Also, the distance from any interior node to node \(N\) is at most \(2\sqrt{N} - 1\). Thus, \(\eta(N, 1) \leq 2\sqrt{N} - 1\). \(\Box\)

![Figure 5](image)

**Figure 5.** A setup to minimize \(\eta(N, 1)\).

It is possible to improve the upper bound slightly to \(3\sqrt{N}\), and provide a corresponding lower bound of \(3\sqrt{N} - 2\), but we do not present it here due to its complexity. Next, we present a generalization of the setup for minimizing \(\eta(N, F)\) for \(F > 1\).

Before presenting an analysis, we develop some notation for use in this section. As we shall see, our LT design has the following structure: the number of physical links in each lightpath using a given wavelength in a given direction is the same. Let \(x_i\) be the number of physical links in each lightpath using \(\lambda_i\) from left to right, and let \(X_i\) be the set of such lightpaths. Similarly, let \(y_i\) be the number of physical links in each lightpath using \(\lambda_i\) from right to left, and let \(Y_i\) be the set of such lightpaths. Let \(E'_i = X_i \cup Y_i\) be defined to be the set of lightpaths (logical links) using \(\lambda_i\). Note that \(E' = \cup_{i = 1}^{F} E'_i\).
Furthermore, let $V_i$ denote the set of nodes at which $\lambda_i$ is terminated. In all our designs, if a lightpath using $\lambda_i$ is terminated at a node, there is also a lightpath starting from that node using $\lambda_i$.

**Lemma 3.6.** $\eta(N, F) \leq 2F \cdot \lceil N^{1/2F} \rceil$.

**Proof.** We present a constructive proof by showing a setup of lightpaths that guarantees the required value of $\eta$. Since the number of wavelengths $F$ is greater than one, the values of $\alpha$ and $\beta$ are no longer sufficient to explain the structure of the lightpath setup.

Given an arbitrary value of $N$, let $\hat{N} \triangleq \lceil [N^{1/2F}] \rceil^{2F}$. Observe that $\eta(N, F)$ is an increasing function of $N$, but the choice of $\hat{N} (\geq N)$ makes the design easier to understand. We use the set of wavelengths in a hierarchical manner. Lightpaths on $\lambda_F$ span $\lfloor N^{1/(2F-1)} \rfloor$ physical links going from left to right, and span $\lceil N^{1/(2F-1)} \rceil$ physical links going from right to left, i.e., $x_F = N^{\lfloor 1/(2F-1) \rfloor}$, $y_F = N^{\lceil 1/(2F-1) \rceil}$. $Y_F$ is the set of lightpaths using $\lambda_F$ and going from right to left. The set of nodes $V_F^{(1)}$ at which the lightpaths in $Y_F$ are terminated leads to the definition of a segment: a sub-network of the linear topology between two consecutive nodes in $V_F^{(1)}$. A recursive structure using the remaining wavelengths $\lambda_1, \lambda_2 \ldots \lambda_{F-1}$ is replicated within each segment (of length $N^{1/(2F-1)}$) defined by $\lambda_F$. The sizes of logical links using each wavelength are defined as follows: $x_i = N^{\lfloor 1/(2F-1) \rfloor}$, and $y_i = N^{\lceil 1/(2F-1) \rceil} 1 \leq i \leq F$.

It can be easily verified that in this setup of lightpaths, $\eta(\hat{N}, F) \leq 2F \cdot \hat{N}^{1/F}$, from which we obtain $\eta(N, F) \leq 2F \cdot \lceil N^{1/2F} \rceil$. □

We are now ready to use the solution for obtaining $\eta(N, F)$ in the LT design to minimize the diameter.

**Theorem 3.7 (An upper bound).** $D(N, F) \leq (4F - 3) \cdot \lceil N^{1/(2F-1)} \rceil$.

**Proof.** We prove the result by demonstrating a setup of lightpaths that guarantees the required value for the diameter. Let $\hat{N} \triangleq \lceil [N^{1/(2F-1)}]^{2F-1} \rceil$. The idea behind the setup is that, given $F$ wavelengths, we use $\lambda_F$ to establish lightpaths of length $x_F = y_F = N^{1/(2F-1)}$. Within each segment defined by $\lambda_F$, we use the solution for minimizing $\eta(N^{1/(2F-1)}, F - 1)$. Note that $y_1 = 1$, and all-to-all connectivity is achieved since $\lambda_1$ is terminated at every node in both directions. The diameter $D \leq \eta(N^{1/(2F-1)}, F - 1) + \eta(N^{1/(2F-1)}, F - 1) + \eta(N^{1/(2F-1)}, F - 1) = (4F - 3) \cdot \lceil N^{1/(2F-1)} \rceil$.

The first term is the maximum number of logical hops to reach a node at which $\lambda_F$ originates, the second term is the maximum number of logical links traversed on $\lambda_F$, and the third term is the maximum number of logical links traversed in getting from a node at which $\lambda_F$ is terminated to the destination node. The final logical topology is obtained by coalescing all nodes $i \geq \hat{N}$ into a single node, and terminating at node $\hat{N}$ all lightpaths ending or starting at $i > \hat{N}$. The entire setup is outlined in figure 6. □

![Figure 6. A setup to minimize diameter](image)

We observe that, for any given $N$, the bound increases after a threshold value of $F$, whereas $D(N, F)$ is clearly a non-increasing function of $F$. For example, when $F \geq N^2/4$, a diameter of one is possible by establishing lightpaths between every node pair. Similarly, a diameter of 2 is possible when $F \geq N - 2$, by setting lightpaths \{(i, i), (i, N), (N, i), (1, i) : 1 < i < N\}. Implicit in our LT design is the assumption that the number of wavelengths used for restoration is small. However, note that not all $F$ wavelengths need to be used to achieve a certain diameter.

The designs we have presented for specific values of $N$ and $F$ can be used to obtain a good design by considering various possibilities for $\hat{N}$ and $\hat{F}$ where $\hat{N}$ and $\hat{F}$ are the number of nodes and wavelengths used to design an LT for achieving a certain diameter. For example, one could choose $\hat{N} \geq N$ and $\hat{F} < F$, and the diameter is the smallest. If $\hat{F} < F$ in such a design, we may simply use the other wavelengths to establish additional lightpaths according to the same structure, i.e., more than lightpath may be established between the same pair of nodes, to increase the traffic carrying capacity.
4. CONCLUSIONS AND FUTURE WORK

In this paper, we presented a pre-designed protection scheme for handling single node or link faults in WDM ring networks with a multi-hop packet overlay network. The scheme involves reserving a limited number of wavelengths for protection use, and embedding a pre-designed logical topology using these wavelengths on the surviving line network. The logical topology must be pre-computed for each possible location and type of fault.

We then presented logical topology designs using a limited number of wavelengths that achieve all-to-all connectivity on the surviving line topology. Our objective was to minimize the electronic processing delay for the worst-case traffic flow. For uniform traffic, this is equivalent to minimizing the diameter of the logical topology, and we presented rigorous analyses for the lower bounds. We also proposed logical topology design algorithms that are optimal in some cases, and in all cases, produce diameters of the same order as the lower bounds.

Our work can be extended in several ways. Firstly, the bounds may be improved, and we believe this to be the case especially for moderately large number of wavelengths. Secondly, the intensity of traffic, which we assumed to be uniform, may be considered in minimizing the worst-case delay. Thirdly, the topologies may be designed to achieve other objectives such as minimizing average delay and maximizing throughput. We are currently working on some of these problems and results will be reported in the future. Our protection scheme is failure-dependent. Future work may involve the design of logical topologies that ensure connectivity irrespective of which component fails. Finally, considering arbitrary mesh topologies can be an ambitious direction for further research, more so if the formulation does not evolve into an MILP formulation.

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