Optimal Wavelength Assignment Algorithms for Permutation Traffic in Multi-Fiber WDM Ring Networks*

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Abstract

Permutation traffic occurs in a number of networking applications. In this paper, the problem of wavelength assignment for permutation traffic in multi-fiber WDM rings with and without wavelength conversion is considered. We focus on a special class of permutation traffic and analyze the bounds on the number of wavelengths required to establish the connections. Lower bounds and optimal algorithms are presented for all the cases. The results indicate that a small number of fibers is sufficient to provide most of the benefits that wavelength conversion provides for this class of permutation traffic.

Keywords: Wavelength-routing, WDM, wavelength assignment, permutation, multi-fiber rings.

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*This work was supported in part by DARPA (co-funded by NSA) Grant #N66001-00-1-8949, and by NSF Grants ANI-9973098 and ANI-9973111. An earlier version of this paper appeared as [1].

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1 Introduction

Wavelength routing, in conjunction with Wavelength Division Multiplexing (WDM), appears to be the most promising mechanism for information transport in metropolitan and wide-area core networks [2]. WDM wavelength routing networks enable the formation of lightpaths, which constitute the optical layer. Lightpaths are all-optical circuit-switched paths which are formed by choosing one channel (wavelength on a fiber) on each link from the source to the destination of the lightpath and concatenating them. Different lightpaths may carry different types of client-layer traffic, e.g., ATM, IP, analog signals, etc. The channels that can be chosen for a lightpath depend on the capability of the nodes to perform wavelength conversion, i.e., switching an incoming channel at one wavelength to an outgoing channel at a different wavelength. If the nodes are incapable of wavelength conversion, then all channels of a lightpath have to be on a single wavelength. Wavelength conversion, by allowing more flexibility in assigning channels to a lightpath, is likely to improve channel utilization. There are a number of studies in the literature on the performance benefits due to wavelength conversion [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14]. However, the performance advantage of wavelength conversion is offset by the high cost of all-optical wavelength converters.

Channel utilization can be increased even without wavelength conversion in multi-fiber networks if switching between channels at the same wavelength is enabled [15, 16]. For example, space switches may be used to switch an incoming channel to an outgoing channel at the same wavelength, but on an arbitrary fiber. In this case, lightpaths may be formed by concatenating channels on any one of the many fibers on each link along the lightpath’s route, but on a single wavelength. Note that channels at different wavelengths may not be used for a lightpath if wavelength conversion is not available.

Our focus in this paper is the off-line embedding of a special class of permutations in multi-fiber all-optical wavelength-routing WDM rings. Specifically, we solve the wavelength assignment problem and analyze the number of wavelengths required to establish the connections in a given permutation. Routing permutation traffic has long been used as a measure of a network’s ability to establish a set of pre-determined connections (for example, see [17]). More recently, it has been used as a measure of the performance advantage of wavelength conversion [18] in optical networks.

The off-line wavelength assignment problem in general topologies is known to be NP-hard, and remains NP-hard in single-fiber rings without wavelength conversion when the traffic pattern
is arbitrary [19]. Wavelength assignment in single-fiber rings for arbitrary connection patterns corresponds to the coloring of an arbitrary circular-arc graph [20], and an algorithm which requires no more than twice the optimal number of wavelengths was presented in [21]. When the connection pattern is a permutation, better bounds can be obtained for single-fiber rings [18].

In [18], two types of multiplexing schemes, namely link and path multiplexing (LM and PM), were compared through the amount of resources required to embed permutations in various rearrangeable non-blocking network topologies including rings. LM and PM are essentially equivalent to networks with and without wavelength conversion. Our work here is different from that in [18] in the following respect. Firstly, only single-fiber networks with and without wavelength conversion were considered in [18] whereas we consider multi-fiber networks. It is well known that multi-fiber networks are equivalent to networks with limited wavelength conversion with regard to wavelength assignment [22]. Secondly, the bounds on the number of wavelengths for bidirectional rings in [18] were for the worst-case permutation traffic. Hence, wavelength assignment for many permutations can possibly be done using many fewer wavelengths than the bound given in [18]. Here, we provide an optimal wavelength assignment algorithm for any given permutation (of a special class, to be described later).

The rest of the paper is organized as follows. The network model is presented in Section 2, which also contains some definitions and relevant notation. Lower bounds for the required number of wavelengths, and optimal algorithms for embedding permutations are presented in Section 3. Some numerical results and discussion are presented in Section 4, and the paper is concluded in Section 5.

2 Network Model and Notation

We consider unidirectional and bidirectional ring networks. The network consists of $N$ nodes numbered $0, 1, \ldots, N-1$ (which may be imagined to be arranged in clockwise order), and is denoted by a graph $G = (V, E)$. Each directed link $e \in E$ consists of $d$ fibers, numbered $0, 1, \ldots, d-1$, where all fibers are oriented in the direction of the link. Each fiber is assumed to carry $W$ wavelengths, numbered $0, 1, \ldots, W-1$. A channel in the network is identified by its link, fiber, and wavelength – the channel on link $e \in E$ on wavelength $i$ and fiber $j$ is denoted by $\lambda^j_i(e)$. In unidirectional rings, we assume that links exist in the clockwise direction only, i.e., from node $i$ to $(i+1) \mod N$. We
assume the existence of $W$ $d \times d$ space switches, one per wavelength plane, at an arbitrary node, say node 0, that allow an optical signal on any of the $d$ input fibers to be switched to any of the $d$ output fibers, on the same wavelength.

A permutation traffic is defined as a set of connections $\{\{i, \pi(i)\} : i = 0, 1, \ldots, N - 1\}$ where $\pi$ is a permutation function of the numbers $0, 1, \ldots, N - 1$. We say that such a permutation traffic is induced by permutation $\pi$. In this paper, we restrict ourselves to permutation traffic induced by permutations in which $\pi(i) = (i + x) \mod N$, $i = 0, 1, \ldots, N - 1$ for an arbitrary integer $x$, $1 \leq x < N$. The induced traffic is denoted by $\Pi_N(x)$.

In describing our algorithm and proving its optimality, we use some terminology and results presented in a previous paper [23]. For completeness, we present the relevant terminology here.

A multicycle of routes (MCR) is a sequence of routes (connections)\(^1\) $(r_0, r_1, \ldots, r_{l-1})$ for some $l$ such that $d_i = s_{(i+1) \mod l}$ for $i = 0, \ldots, l - 1$, where $s_i$ and $d_i$ are the source and destination nodes of route $r_i$. In other words, an MCR is a sequence of routes that starts at a node, goes clockwise around the ring one or more times, and ends at the same node. The multiplicity of an MCR is the number of times the ring is circumvented by the routes in the MCR. For example, in Figure 1, $\{r_0, r_1, r_2, r_3, r_4, r_5, r_6, r_7, r_8, r_9\}$ is an MCR of multiplicity 3 and $\{r_0, r_1, r_2, r_3, r_4, r_8, r_9\}$ is an MCR of multiplicity 2, both starting at node 0. There is no MCR of multiplicity 1 starting at node 0 in this example.

![Figure 1: Illustration of a multi-cycle of routes (MCR).](image)

The load of a traffic is defined as the maximum number of connections that cross any directed link for a given routing algorithm. The traffic is said to be uniform if the number of connections crossing any link is equal to the load. For example, in a unidirectional ring (which has a unique route between any two nodes), $\Pi_N(x)$ is uniform with load $x$.

\(^1\)The terms “route”, “connection”, and “lightpath” are used interchangeably throughout the paper.
3 Optimal Wavelength Assignment Algorithms

In this section, we present optimal wavelength assignment algorithms for embedding $\Pi_N(x)$ for arbitrary values of $x$ in both uni- and bidirectional rings when there is no wavelength conversion. We also present the number of wavelengths required to embed $\Pi_N(x)$ when wavelength conversion is available at a single node. Note that when wavelength conversion is available at a single node, wavelength assignment is known to be trivial (for example, see [10]). We consider unidirectional rings first.

3.1 Unidirectional Rings

We first start by presenting the number of wavelengths necessary and sufficient to embed $\Pi_N(x)$ when there is a wavelength converter node.

**Theorem 3.1** The number of wavelengths $W$ necessary and sufficient for embedding $\Pi_N(x)$ in a unidirectional ring network with wavelength conversion is $\lceil \frac{N}{d} \rceil$.

**Proof:** Note that the routing is fixed in a unidirectional ring. There are $N$ lightpaths each of which cross exactly $x$ links. Hence, the load of the traffic is $x$ (note that the traffic is uniform). Since each wavelength has $d$ channels and channels can be maximally used with wavelength conversion, the result follows. \qed

We now consider networks with no wavelength conversion (but with fiber switching).

**Theorem 3.2** The number of wavelengths necessary for embedding $\Pi_N(x)$ in a unidirectional ring without wavelength conversion is

$$W \geq \lceil \frac{N}{dN/x} \rceil.$$ 

**Proof:** Note that the number of channels on any wavelength is $dN$, and since each route has length $x$, the number of routes covered by a single wavelength is at most $[dN/x]$. Suppose there exists an embedding for $\Pi_N(x)$ using $W$ wavelengths. Since there are $N$ routes in $\Pi_N(x)$, the maximum number of routes covered by $W$ wavelengths is at most $W[dN/x]$, and this number has to be at least $|\Pi_N(x)| = N$. Therefore, $W \geq N/[dN/x]$. Since $W$ is an integer, $W \geq \lceil N/[dN/x] \rceil$. \qed

We next present several properties of MCRs of a permutation traffic that will be used in developing the optimal wavelength assignment algorithm. As shown earlier [23], any uniform traffic can
be partitioned into a set of MCRs. Let $M_0, M_1, \ldots, M_{k-1}$ be an MCR partition of $\Pi_N(x)$, and let $m_0, m_1, \ldots, m_{k-1}$ be their corresponding multiplicities.

**Lemma 3.1** For any $x$, $m_0 = m_1 = \ldots = m_{k-1}$.

**Proof:** Since any connection from the permutation traffic exists in exactly one MCR, and each connection’s route has $x$ hops, the multiplicities of the MCRs must be equal.

Let $m$ be the (common) multiplicity of the MCRs, and $\text{LCM}(x, y)$ denote the least common multiple of two integers $x$ and $y$.

**Lemma 3.2** $m = \frac{\text{LCM}(N, x)}{N}$.

**Proof:** From the definition of the multiplicity of an MCR, $m = \xi x/N$ where $\xi$ is the least integer such that $\xi x \mod N = 0$. In other words, $m = \frac{\text{LCM}(N, x)}{N}$.

**Lemma 3.3** The number of MCRs in $\Pi_N(x)$ is $k = \frac{x}{m}$.

**Proof:** The number of connections in each MCR is $mN/x$ because the sum of the hop-lengths of the routes in an MCR is $mN$ and $x$ is the hop-length of each connection in $\Pi_N(x)$. Since there are $N$ connections in any $\Pi_N(x)$ and each connection is in a single MCR, the number of MCRs is $x/m$.

### 3.1.1 The Wavelength Assignment Algorithm

We now proceed to describe our algorithm which we call Permutation Wavelength Assignment Algorithm for Unidirectional Rings (PWAA-UR). The proof of optimality of PWAA-UR depends on partitioning $\Pi_N(x)$ in a specific way. Let MCR $M_j$ be defined\(^2\) as $M_j = \{r_{j+b} \mid 0 \leq b \leq \frac{mN}{x} - 1\}$ for $0 \leq j \leq k - 1$, where $r_i$ denotes the route from $i$ to $i + x$. This is clearly possible if $k = 1$. A little bit of thought would confirm its possibility for other values of $k$ as well. Note that the starting node of $M_j$ is $j$ for each $0 \leq j \leq k - 1$. For example, Figure 2 shows MCRs $M_0, M_1$, and $M_2$ when $N = 21$ and $x = 9$. In any MCR partition of $\Pi_{21}(9)$, $m = 3$, and $k = 3$. We now proceed to describe our optimal wavelength assignment algorithm taking $M_0, \cdots, M_{k-1}$ as an input. Figure 3, which illustrates a wavelength assignment for $\Pi_{21}(9)$, may be used to follow the description of the algorithm. Recall that fiber switching is possible only at node 0.

\(^2\)Throughout the paper, operations on node indices are understood to be modulo $N$.  

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Figure 2: The MCR partition of $P_{21}(9)$ used by the algorithm.
Figure 3: A wavelength assignment for $\Pi_{21}(9)$ in a 2-fiber unidirectional ring network.
In our algorithm, channels are assigned to routes following the fixed order of MCRs $M_0, M_1, \ldots, M_{k-1}$, and routes within each $M_j$ are also considered in the same sequence used to form $M_j$. The channel assignment is done in such a way that channels on wavelength $i$ are utilized as much as possible before channels on wavelength $i + 1$ are considered, for $0 \leq i < W - 1$. Note that the number of channels on each wavelength $i$ is $dN$. The network model assumes that a single node (node 0) is equipped with $W$ $d \times d$ switches, and this allows switching from $\lambda_i^j$ to $\lambda_i^{j'}$ at node 0 for any $i, j$, and $j'$.

Within each wavelength $i$, we first assign channels on fiber 0, then switch to fiber 1 at node 0, then to fiber 2 at node 0 after circumnavigating the ring once, and so on. A final switching from fiber $d - 1$ to fiber 0 is done if necessary.

For example, Figure 3 shows an assignment of channels to routes in $M_0$, $M_1$, and $M_2$ (see Figure 2). In this example, $d = 2$, and $W = 6$ wavelengths are used. Note that some channels on each wavelength may not be utilisable because of the switching limitations (which would not exist, for example, if wavelength conversion were available). Let a gap be defined as a (maximal) set of unused same-wavelength channels on contiguous links.

In general, there are two gaps, denoted by $g_i^0$ and $g_i^1$ on any wavelength $i$ in our wavelength assignment. Consider the routes that are assigned an arbitrary wavelength $i$. In general, wavelength $i$ has some routes from some MCR $M_a$, all routes from some MCRs, say, $M_{a+1}, \ldots, M_{\beta-1}$, and finally some routes from MCR $M_\beta$. Let $r_a$ be the first route in $M_a$ that is assigned wavelength $i$, and let $r_\beta$ be the last route in $M_\beta$ that is assigned wavelength $i$.

The first gap $g_i^0$ is the set of unused channels between the starting (also ending) nodes of $M_{\beta-1}$ and $M_\beta$. In other words, when all routes from an MCR are assigned wavelength $i$, and only some routes from the next MCR can be assigned wavelength $i$, then, the set of unused channels between the starting nodes of these two MCRs forms the first gap. Since the MCRs are formed such that the starting node of $M_j$ is $j$, we note that $g_i^0 = 0$ or 1 for any wavelength $i$. Note that a gap of 0 occurs only when wavelength $i$ can be assigned to routes in a single MCR. The second gap $g_i^1$ is the set of unused channels between the ending node of $r_\beta$ and the starting node of $r_a$.

Let $R_i$ denote the set of routes assigned to wavelength $i$ using our algorithm. Figure 3 then shows that $R_0 = \{r_0, r_9, r_8, r_6\}$, $R_1 = \{r_{15}, r_3, r_{12}, r_1\}$, $R_2 = \{r_{10}, r_{19}, r_7, r_{16}\}$, $R_3 = \{r_4, r_{13}, r_2, r_{11}\}$, $R_4 = \{r_{20}, r_8, r_{17}, r_5\}$, and $R_5 = \{r_{14}\}$. The two gaps for each wavelength are shown by dotted and
dashed lines in the figure.

3.1.2 Optimality of the Algorithm

We are now ready to show the optimality of PWAA-UR.

**Theorem 3.3** The algorithm PWAA-UR can be used to embed $\Pi_N(x)$ using $W$ wavelengths, where

$$W = \left\lfloor \frac{N}{dN/x} \right\rfloor.$$

**Proof:** Suppose for now that $g_i^0 + g_i^1 < x$ for each $0 \leq i < W - 1$. (This hypothesis is shown to be true in Lemma 3.4.) We then observe that each wavelength $i$ for $0 \leq i < W - 1$ covers $\lfloor dN/x \rfloor$ routes. We also note that wavelength $W - 1$ covers at least one route and let $z$ denote the number of routes covered by wavelength $W - 1$. The total number of routes covered by $W$ wavelengths is then $(W - 1)\lfloor dN/x \rfloor + z$, and this must be equal to $N$, since $\Pi_N(x)$ has $N$ routes. Therefore,

$$W = \frac{N - z + \lfloor dN/x \rfloor}{\lfloor dN/x \rfloor}.$$

Since $z \geq 1$, the above equation gives

$$W \leq \frac{N - 1 + \lfloor dN/x \rfloor}{\lfloor dN/x \rfloor} < \frac{N}{\lfloor dN/x \rfloor} + 1,$$

which implies that $W \leq \left\lfloor \frac{N}{\lfloor dN/x \rfloor} \right\rfloor$ since $W$ is an integer. This result, together with Theorem 3.2, implies that $W = \left\lfloor \frac{N}{\lfloor dN/x \rfloor} \right\rfloor$. \hfill \Box

**Lemma 3.4** $g_i^0 + g_i^1 < x$ for each $i$ in $0 \leq i < W - 1$.

**Proof:** To prove this lemma, we consider three cases.

*Case 1: $d$ is a multiple of $m$.*

It is easy to see in this case that $g_i^0 = g_i^1 = 0$ for all $i$, since all the routes of exactly $d/m$ MCRs may be assigned the same wavelength.

*Case 2: $d < m$.*

In this case, routes assigned to any wavelength $i$ are all from at most two consecutive MCRs $M_j$ and $M_{j+1}$. Suppose they are from only one MCR. Then, $g_i^0 = 0$, and $g_i^1$ must be less than $x$ since otherwise, wavelength $i$ would have accommodated at least one more route.
Now assume that wavelength \( i \) is assigned to at least one route in \( M_j \) and at least one route in \( M_{j+1} \). We then note that \( g_i^0 = 1 \), and claim that \( g_i^1 < x - 1 \). Suppose for contradiction that \( g_i^1 = x - 1 \). Let \( r_a \) and \( r_b \) be the first route in \( M_j \) and the last route in \( M_{j+1} \) both assigned wavelength \( i \). Then, \( g_i^1 = a - (b + x) \). Let the subset of \( M_j \) that are assigned wavelength \( i \) (the first of which is \( r_a \)) be denoted \( P_j(a) \). Note that the sum of the hop-lengths of the routes in \( P_j(a) \) is a multiple of \( x \). Let \( c_j \) denote the number of times the ring is circumvented by following the routes in \( P_j(a) \). Similarly, if \( P_{j+1}(b + x) \) denotes the subset of \( M_{j+1} \) that is not assigned wavelength \( i \) (the first of which is \( r_{b+x} \)), the total hop-length of the routes in \( P_{j+1}(b + x) \) must be a multiple of \( x \). Let \( c_{j+1} \) denote the number of full circumventions by the routes in \( P_{j+1}(b + x) \). Since we assumed that \( g_i^1 = x - 1 \), it is not difficult to see that \( c_j = c_{j+1} \). Figure 4 illustrates this situation. The total number of channels on wavelength \( i \) assigned to routes in \( M_j \cup M_{j+1} \) is then \( d_j + d_{j+1} \), where \( d_j \) (respectively, \( d_{j+1} \)) denotes the number of channels on wavelength \( i \) assigned to routes in \( M_j \) (respectively, \( M_{j+1} \)). From the above, we have \( d_j = j - a + c_j N \) and \( d_{j+1} = mN - [c_{j+1} N + (j + 1) - (b + x)] \). Since \( c_j = c_{j+1} \), \( d_j + d_{j+1} = mN - [a - (b + x)] - 1 = mN - x \), and since \( N > x \), \( d_j + d_{j+1} > (m - 1)N \geq dN \). This is impossible since wavelength \( i \) has only \( dN \) channels.

![Figure 4](image-url)

Figure 4: Illustration of the case when \( d = 2 \) is less than \( m = 3 \).
Case 3: \( d > m, \) and \( d \) is not a multiple of \( m. \)

Let \( M_i, M_{i+1}, \ldots, M_j, M_{j+1} \) be the consecutive MCRs such that routes assigned wavelength \( i \) are all from these MCRs. Clearly, all routes in \( M_{i+1} \cup \cdots \cup M_j \) are assigned wavelength \( i. \) Let \( r_a \) and \( r_b \) denote the first route in \( M_i \) and the last route in \( M_{j+1} \) that are assigned wavelength \( i. \) Then, \( g_i^1 = a - (b + x). \) Let \( P_l(a) \) denote the subset of routes in \( M_j \) that are assigned wavelength \( i \) (the first of which is \( r_a \)), and let \( P_{j+1}(b + x) \) denote the subset of routes in \( M_{j+1} \) that are not assigned wavelength \( i \) (the first of which is \( r_{b+x} \)). Let the number of ring circumventions by the routes in \( P_l(a) \) (respectively, \( P_{j+1}(b + x) \)) be denoted by \( c_l \) (respectively, \( c_{j+1} \)). Suppose \( g_i^1 = x - 1. \) We then observe that there exists a route in \( P_{j+1}(b + x) \) starting at \( a, \) and the length of the subpath of \( P_{j+1}(b + x) \) starting at \( a \) through the end of \( P_{j+1}(b + x) \) must be equal to the length of \( P_l(a). \) This observation then implies that the end of \( M_{j+1} \) must be \( l + 1 \) and \( c_l = c_{j+1}. \) This in turn implies that routes assigned wavelength \( i \) are all from at most two MCRs \( M_i \) and \( M_{j+1}. \) When \( d_l \) (respectively, \( d_{j+1} \)) denotes the number of channels on wavelength \( i \) assigned to routes in \( M_i \) (respectively, \( M_{j+1} \)), the total number of utilized channels on wavelength \( i \) is \( d_l + d_{j+1}, \) where \( d_l = l - a + c_jN \) and \( d_{j+1} = mN - [c_{j+1}N + (j + 1) - (b + x)]. \) Since \( c_l = c_{j+1} \) and \( l = j, \) \( d_l + d_{j+1} = mN - [(a - (b + x)] - 1. \) Note that \( a - (b + x) = g_i^1 \) and we assumed that \( g_i^1 = x - 1. \) Therefore, \( d_l + d_{j+1} = mN - x. \) Since \( m \leq d - 1, \) the total number of utilized channels on wavelength \( i \) is \( d_l + d_{j+1} \leq (d - 1)N - x < (d - 1)N, \) which is impossible. Figure 5 illustrates the Case 3.

This completes the proof of the lemma.

We now turn to bidirectional rings. We will see that most of the proofs and results for unidirectional rings will apply for this case as well.

### 3.2 Bidirectional Rings

We start by considering a ring network with a single wavelength converter node.

**Theorem 3.4** The number of wavelengths necessary and sufficient to embed \( \Pi_N(x) \) in a bidirectional ring network with wavelength conversion is \( W = \left\lceil \frac{x(N-x)}{Nd} \right\rceil. \)
Figure 5: Illustration of the case when $d > m$ and $d$ is not a multiple of $m$.

**Proof:** The number of wavelengths necessary to embed $\Pi_N(x)$ can be obtained by routing the lightpaths so as to minimize the load of the traffic.\(^3\) Suppose $a$ lightpaths are routed clockwise and $b$ lightpaths are routed counterclockwise. Since there are $N$ lightpaths in any permutation traffic, $a + b = N$. Now, the load due to the lightpaths routed in clockwise direction alone is $\lceil ax/N \rceil$. Similarly, the load due to the lightpaths routed in the counterclockwise direction alone is $\lceil b(N-x)/N \rceil$. It follows that setting $a = N-x$ and $b = x$ minimizes the load due to all lightpaths, and the minimum load thus obtained is $\lceil (N-x)/N \rceil$. Thus, the number of wavelengths necessary, $W$, is given by

$$W \geq \left\lceil \frac{(N-x)/N}{d} \right\rceil = \left\lceil \frac{(N-x)x}{Nd} \right\rceil.$$  

Since channels can be maximally used with wavelength conversion, $\left\lceil \frac{(N-x)x}{Nd} \right\rceil$ wavelengths are also sufficient for embedding $P_i \Pi_N(x)$. \(\square\)

Next, assume wavelength conversion is not available, but inter-fiber switching on the same wavelength is possible at some node.

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\(^3\)Recall that the load is defined as the maximum number of connections that cross any directed link.
**Theorem 3.5** The number of wavelengths necessary, \( W \), for embedding \( \Pi_N(x) \) in a bidirectional ring network without wavelength conversion is

\[
W \geq \left\lceil \frac{N}{[dN/x] + [dN/(N-x)]} \right\rceil.
\]

**Proof:** Note that the number of channels on any wavelength is at most \( 2dN \), \( dN \) in the clockwise direction and \( dN \) in the counterclockwise direction. Since each route has length \( x \) in clockwise direction \( (N-x \) in counterclockwise direction), the number of routes covered by a single wavelength is at most \( \left\lfloor \frac{dN}{x} \right\rfloor + \left\lfloor \frac{dN}{N-x} \right\rfloor \). Suppose there exists an embedding for \( \Pi_N(x) \) using \( W \) wavelengths. Since there are \( N \) routes in \( \Pi_N(x) \), the maximum number of routes covered by \( W \) wavelengths is at most \( W\left(\left\lfloor \frac{dN}{x} \right\rfloor + \left\lfloor \frac{dN}{N-x} \right\rfloor \right) \), and this number has to be at least \( |\Pi_N(x)| = N \). Therefore, \( W \geq \lceil N/(\lfloor dN/x \rfloor + \lfloor dN/(N-x) \rfloor) \rceil \). \( \square \)

### 3.2.1 The Wavelength Assignment Algorithm

In our algorithm called as Permutation Wavelength Assignment Algorithm for Bidirectional Rings (PWAA-BR), channels in the clockwise direction are first assigned to as many routes as possible using the algorithm PWAA-UR. For the remaining routes, channels in the counterclockwise direction are assigned following the fixed order as in the the clockwise direction.

**Theorem 3.6** The algorithm PWAA-BR can be used to embed \( \Pi_N(x) \) using \( W \) wavelengths, where

\[
W = \lfloor N/(\lfloor dN/x \rfloor + \lfloor dN/(N-x) \rfloor) \rfloor.
\]

**Proof:** One wavelength can be assigned to \( \lfloor dN/x \rfloor \) routes in the clockwise direction by Lemma 3.4. By similar arguments, one can show that a single wavelength may be assigned to as many as \( \lfloor dN/(N-x) \rfloor \) routes in the counterclockwise direction. We then have

\[
W(\lfloor dN/x \rfloor + \lfloor dN/(N-x) \rfloor) \geq N = |\Pi_N(x)|.
\]

This completes the proof for bidirectional rings.

In the next section, we present some numerical results and compare the wavelength requirements for networks with and without wavelength conversion.
4 Numerical Results

Here, we present some numerical results to obtain some qualitative insight into the usefulness of wavelength conversion. These are obtained using the theoretical results of the previous section. We first plot the number of wavelengths to embed $\Pi_N(x)$ as a function of $x$ for a 50-node unidirectional ring for various values of $d$ with and without wavelength conversion, in Figure 6.

![Figure 6: Wavelength requirements for embedding $\Pi_N(x)$ in a 50-node unidirectional ring network with and without wavelength conversion for various values of $d$.](image)

A similar figure is plotted for a 100-node network in Figure 7.

We observe that the number of wavelengths is the same for networks with and without wavelength conversion for some values of $x$, but they can significantly differ for other values of $x$. For instance, when $x = 18$ in a 50-node single-fiber ring, 7 more wavelengths are required when wavelength conversion is unavailable. However, this difference rapidly decreases as $d$ is increased to even moderate values such as 3 or 4. This is a corroboration of the observation of previous researchers that limited channel switching provides most of the benefits of full channel switching for a variety of traffic models [7, 23].

The trends are similar for bidirectional rings with even less of a difference in the number of wavelengths between rings with and without wavelength conversion. We therefore do not show those results here.
Figure 7: Wavelength requirements for embedding $\Pi_N(x)$ in a 100-node unidirectional ring network with and without wavelength conversion for various values of $d$.

5 Conclusions and Future Work

In this paper, we considered the problem of embedding a special class of permutations in WDM ring networks off-line. In this class, all sources and destinations are separated by an equal number of nodes. An application of this class of permutation traffic is in all-to-all message broadcast. Networks with and without wavelength conversion were considered, and lower bounds on the number of wavelengths required to embed a permutation were presented. Wavelength assignment algorithms for embedding the traffic that are optimal in the required number of wavelengths were also presented.

Future work could consider tightening the bounds on the number of required wavelengths to embed arbitrary permutations. Furthermore, the problem of optimally scheduling all-to-all broadcast message transmissions using the special class of permutations, when the number of wavelengths is limited, appears to be an interesting problem worthy of consideration.

References


