Propagation of Uncertainty in a Maritime Risk Assessment Model utilizing Bayesian Simulation and Expert Judgment Aggregation Techniques

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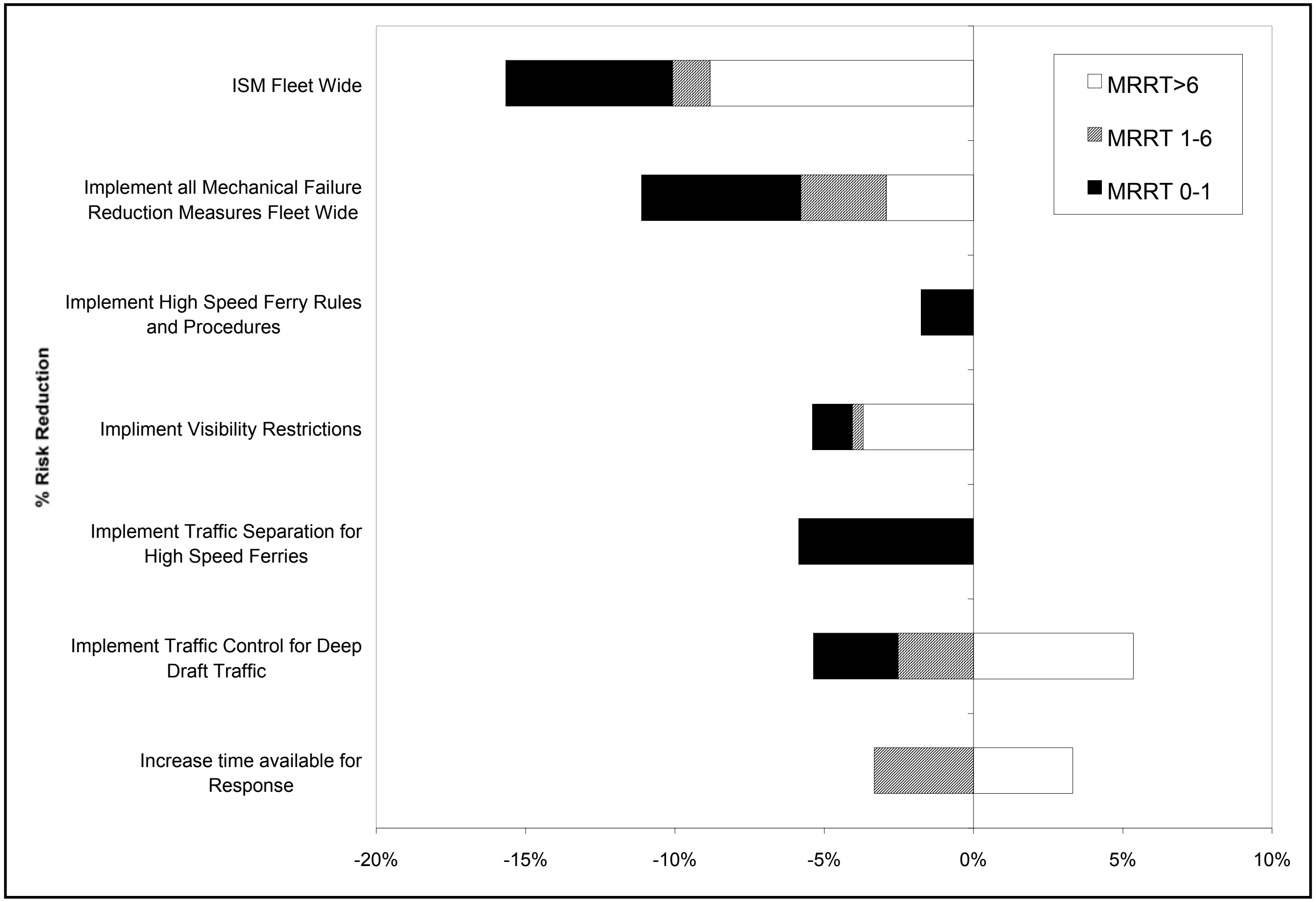
1. ORGANIZATION

- Briefly review Bayesian Simulation of San Francisco Bay Exposure Analysis.

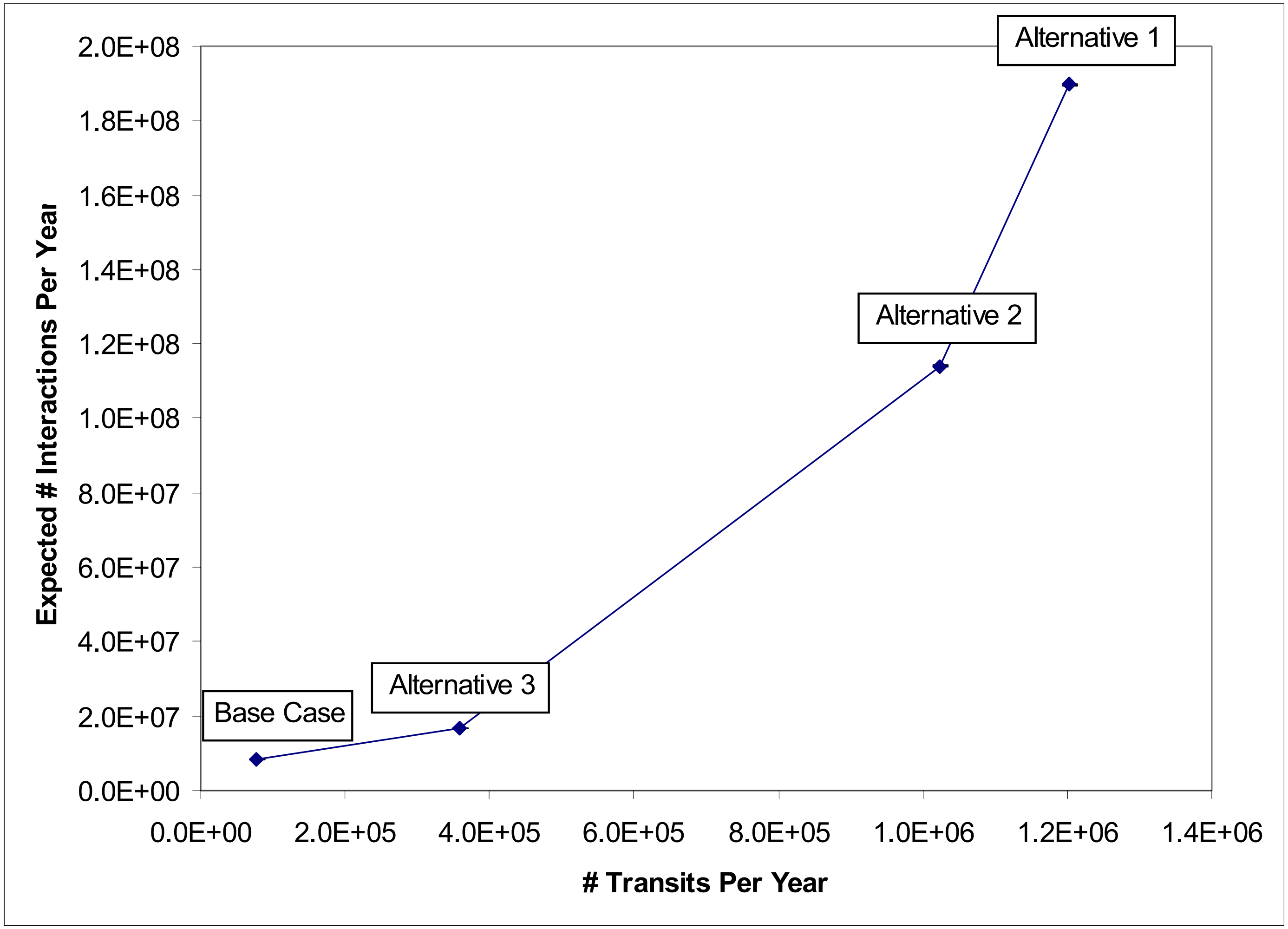
- Discuss in detail **Bayesian methodology** for assessing **relative accident probabilities** and their uncertainty using **paired comparisons** to elicit expert judgments. Approach is illustrated using expert judgment data elicited for **The Washington State Ferry Risk Assessment** in 1999.

- Propagate simulation uncertainty and expert judgment uncertainty in **a proof of concept case study** using WSF expert judgment and SF bay exposure simulation

2. WHY ADDRESS UNCERTAINTY?



Example of WSF Risk Assessment Results



Example of SF Bay Exposure Assessment Results

One problem with the representations in figures above is **the apparent finality** of the results. The decision-maker is led to believe that the results are **definitive and are in no way uncertain.** The **National Research Council** performed a peer review of the Prince William Sound Risk Assessment (the father and grand father risk assessment of the Washington State Ferry Risk Assessment and San Francisco Bay Exposure Assessment, respectively) and concluded that the **underlying methodology shows "promise"** to serve as a systematic approach for making risk management decisions for marine systems, **but uncertainty in results needs to be addressed.**

"Risk management ... should answer whether evidence is sufficient to prove specific risks and benefits"
(A. Elmer, President, SeaRiver Maritime, Inc. in National Research Council, 2000).

"Since the truth is, we always have uncertainty, we say that speaking in probability curves is telling the truth".
(see, e.g., Kaplan, 1997, p. 412)

1. BAYESIAN SIMULATION IN A NUTSHELL

Bayesian simulation differs from classical simulation analysis in that **probability distributions** are used to represent the uncertainty about model parameters rather than point estimates and confidence intervals. Such treatment is applied to **both random inputs to the model and the outputs from the model.**

Bayesian San Francisco Bay Exposure Analysis:

1. Bayesian Poisson process models of traffic arrivals were created for all 5,277 arrivals processes (excluding the Ferries since these run on a tight schedule).

2. For each arrival process a **Gamma Prior Distribution** was postulated for the arrival rate λ of the **exponential distributed inter arrival time**.

3. For each arrival process a **Gamma Posterior distribution** for the arrival rate λ was determined by a **conjugate Bayesian analysis** utilizing the inter arrival data for that process.

4. In the simulation, for each arrival one **first samples an arrival rate λ** from its Gamma distribution and **next sample the inter arrival time from the exponential distribution** utilizing the sampled arrival rate λ, etc.

HENCE, **BAYESIAN SIMULATION TECHNIQUES**
ADDRESS BOTH:

THE RANDOMNESS WITHIN
THE SYSTEM UNDER CONSIDERATION
(I.E. ALEATORY UNCERTAINTY)

AND

LACK OF KNOWLEDGE ABOUT THE SYSTEM
(I.E. EPISTEMIC UNCERTAINTY)

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5. As our output data is in the form of **a count** (i.e. number of interactions per grid cell or total number of interaction), this number of vessel interactions can be naturally modeled using **a Poisson distribution with rate $\boldsymbol{\lambda}$**, with a conjugate prior gamma distribution with **shape $\boldsymbol{\alpha}$ and scale $\boldsymbol{\gamma}$.**

6. For s replications the posterior distribution of the number of vessel interactions is again a Poisson distribution with rate λ, with a conjugate posterior gamma distribution with **shape $\boldsymbol{\alpha + \sum_{j=1}^{s} n_j}$ and scale $\boldsymbol{\gamma + s}$**, where n_j is the number of counts observed in the j-th replication and s is the total number of (independent) replications of the simulation.

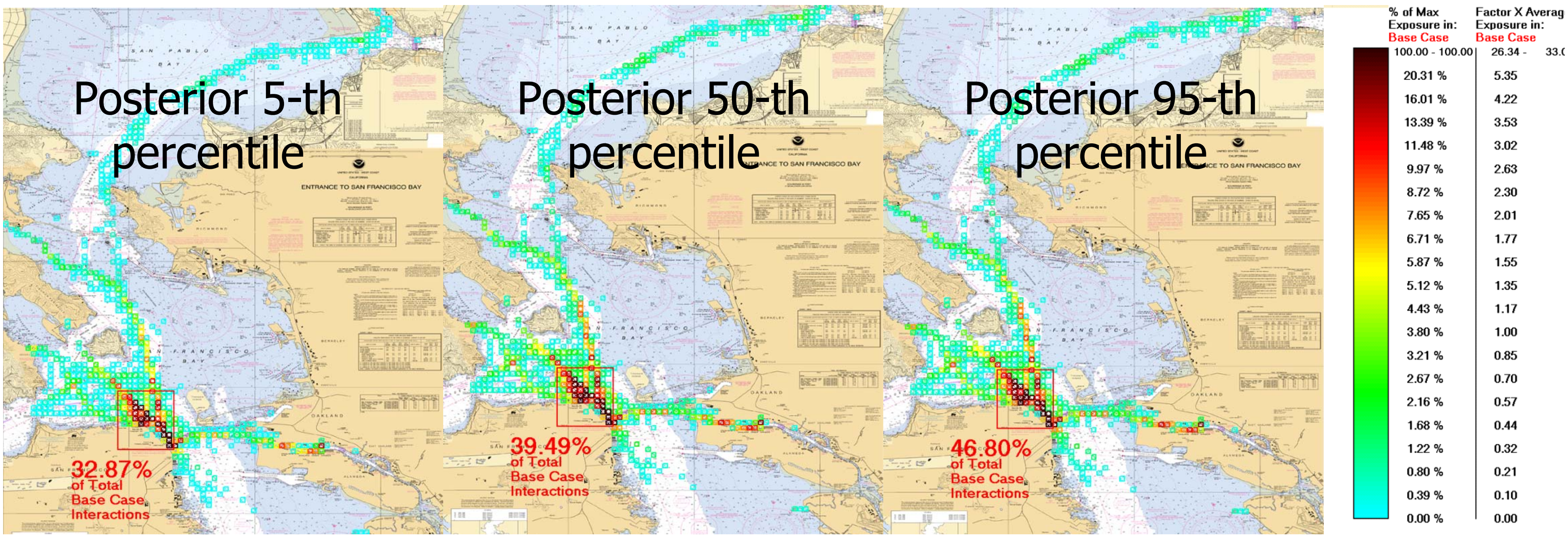
7. Our predictive distribution for the number of interactions (per grid cell) is then a Poisson Gamma distribution.

$$Pr(X = x) = \left(\frac{\gamma + s}{\gamma + s + 1}\right)^{\alpha + \sum_{j=1}^{s} n_j} \frac{\Gamma(\alpha + \sum_{j=1}^{s} n_j + x)}{\Gamma(x)\Gamma(\alpha + \sum_{j=1}^{s} n_j)} \frac{1}{x(\gamma + s + 1)^x}, x = 0, 1, 2, \ldots$$

See, e.g., Bernardo and Smith (1994), *Bayesian Theory,* Wiley

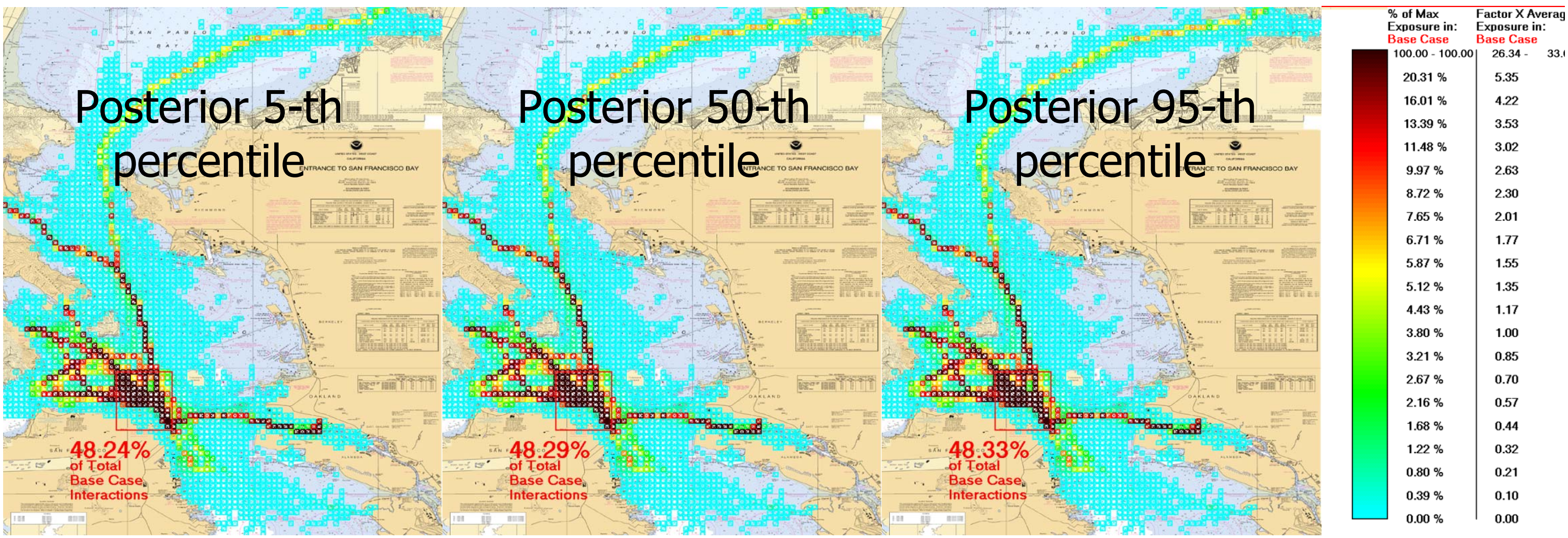
3. SF BAY EXPOSURE ASSESSMENT RESULTS WITH UNCERTAINTY

After 1 day of simulation

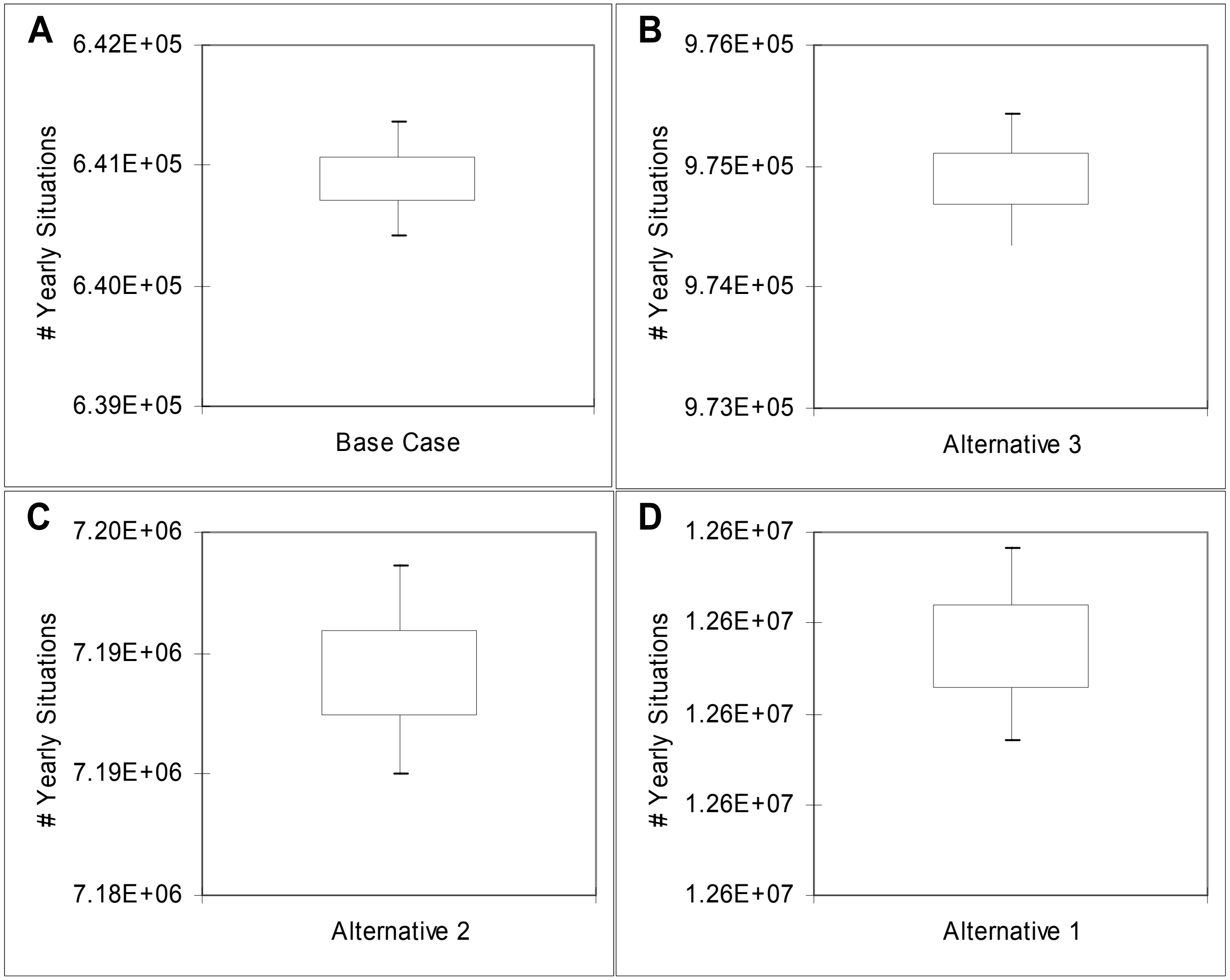


3. SF BAY EXPOSURE ASSESSMENT RESULTS WITH UNCERTAINTY

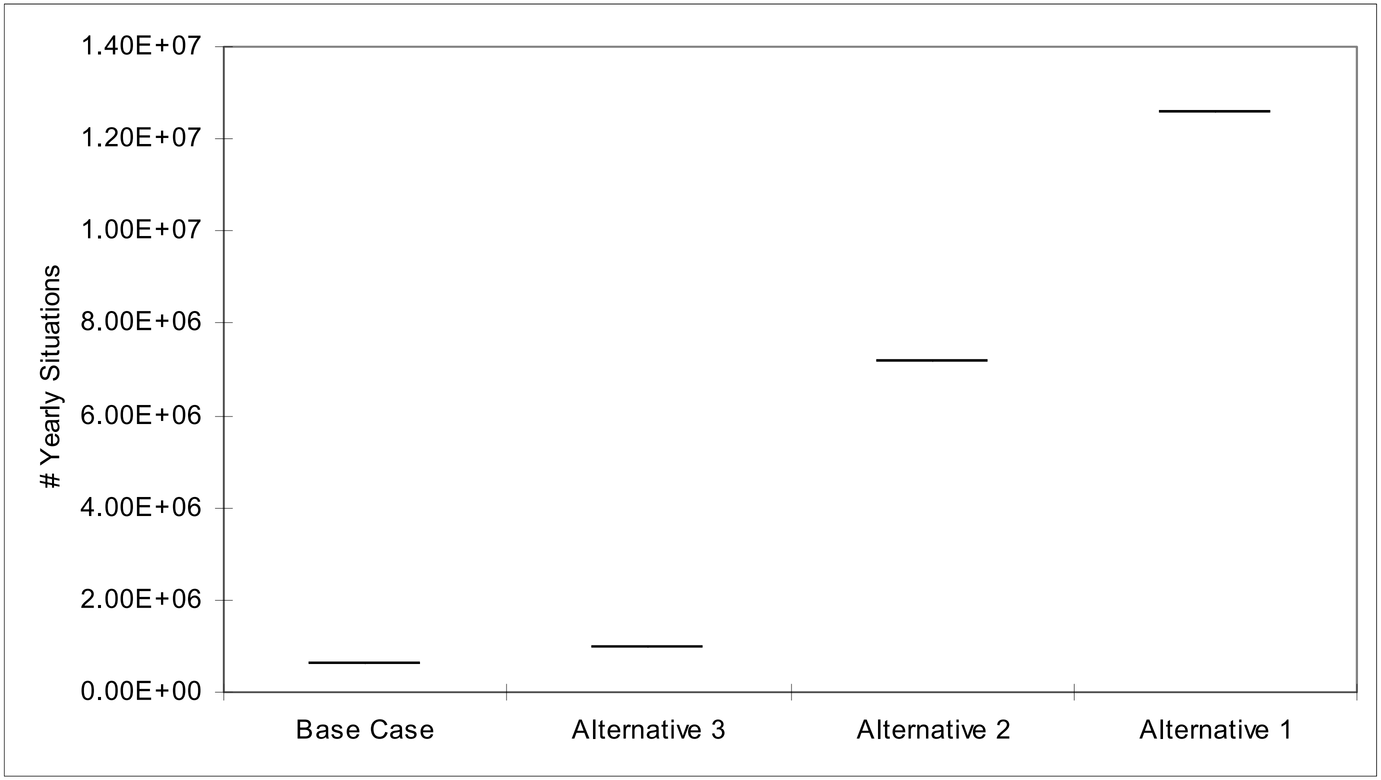
After 50 years of Simulation



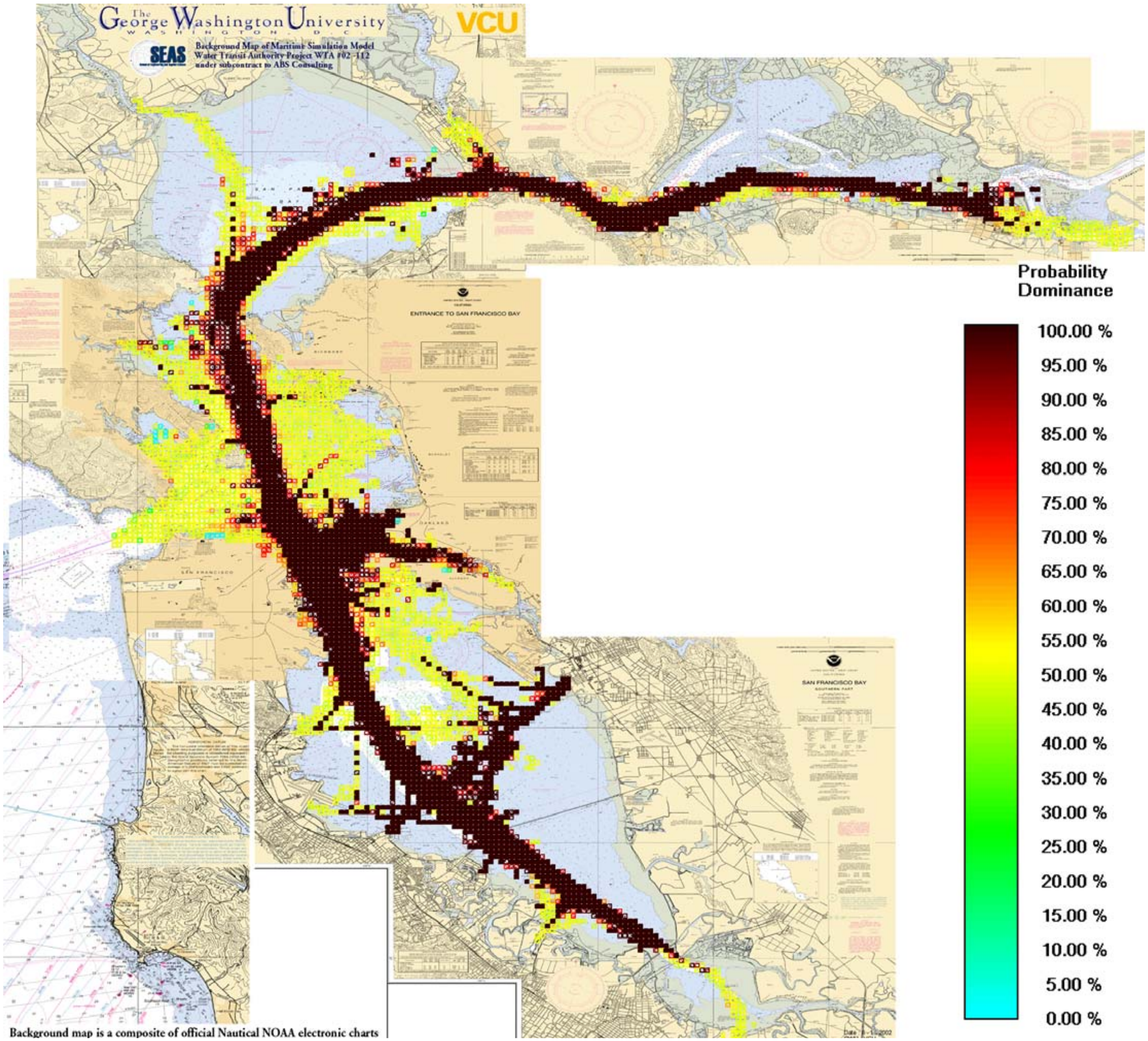
4. AGGREGATE SF BAY EXPOSURE ASSESSMENT RESULTS WITH UNCERTAINTY



5. AGGREGATE SF BAY EXPOSURE ASSESSMENT COMPARISON RESULTS WITH UNCERTAINTY



BOX PLOTS A-D OF PREVIOUS SLIDE IN A SINGLE GRAPH


The George Washington University
WASHINGTON DC
VCU
SEAS
Background Map of Maritime Simulation Model
Water Transit Authority Project WTA #02-112
under subcontract to ABS Consulting
ENTRANCE TO SAN FRANCISCO BAY
SAN FRANCISCO BAY
Probability Dominance
100.00 %
95.00 %
90.00 %
85.00 %
80.00 %
75.00 %
70.00 %
65.00 %
60.00 %
55.00 %
50.00 %
45.00 %
40.00 %
35.00 %
30.00 %
25.00 %
20.00 %
15.00 %
10.00 %
5.00 %
0.00 %
Background map is a composite of official Nautical NOAA electronic charts

6. CONCLUSION OF BAYESIAN SIMULATION OF SF BAY MTS IN A NUTSHELL

The uncertainty in the random arrival patterns of **non-ferry traffic** does not seem to affect in this particular study the conclusions regarding the different expansion scenarios (Base Case, Alternative 3, Alternative 2 and Alternative 1) from **an Exposure Perspective.** **(which could be explained by the fact that interactions are predominantly FERRY to FERRY interactions which run on a tight schedule)**.

For more detailed information see:

Merrick, J. R. W., J. R. van Dorp and V. Dinesh (2003). Assessing Uncertainty in Simulation Based Maritime Risk Assessments. Submitted to *Risk Analysis*.

Merrick, J. R. W., V. Dinesh, A. Singh, J. R. van Dorp and T. Mazzuchi (2003). Propagation of Uncertainty in a Simulation-Based Maritime Risk Assessment Model Utilizing Bayesian Simulation Techniques. *Winter Simulation Conference* 2003 Proceedings.

7. ACCIDENT PROBABILITY ASSESSMENT USING BAYESIAN PAIRED COMPARISON ELICITATION

- An important class of elicitation techniques consists of the psychological scaling models that use the concept of paired comparisons. Origins can be traced back to **Thurstone's (1927) and Bradley (1953)).**

- Another popular paired comparison elicitation technique is called **the Analytical Hierarchy Process (AHP)** developed by **Saaty (1977, 1980).** The AHP Process is primarily used for the construction of value functions $V(\underline{X})$ involving multiple contributing factors $\underline{X} = (X_1, X_2, \ldots, X_p)$ (see, e.g. Foreman and Selly (2002)).

- The **popularity of the paired comparison method** can perhaps be contributed to the observation that experts are **more comfortable** making comparisons rather than directly assessing a quantity of interest.

- To the best of our knowledge, **Pulkkinen (1993, 1994)** was first to introduce a **Bayesian paired comparison aggregation method** for the elements of a multivariate random vector $\underline{\beta} = (\beta_1, \beta_2, \ldots, \beta_p)$ by multiple experts. Pulkkinen's (1993, 1994) exposition is mainly theoretical and limited to a discussion of mathematical properties.

- Similar to the AHP process, we are interested in the functional relationship between **contributing factors** $\underline{X} = (X_1, X_2, \ldots, X_p)$ and an accident probability $Pr(Accident|\, Incident, \underline{X})$ defined by

$$Pr(Accident|Incident, \underline{X}) = P_0 Exp\left(\underline{\beta}^T \underline{X}\right). \qquad (1)$$

- $\underline{X} = (X_1, X_2, \ldots, X_p)$ describes **a system state** during which an incident (e.g. a mechanical failure) occurred.

- The accident probability model (1) resembles the well-known **proportional hazards model** originally proposed by **Cox (1972)** and builds on the assumption that accident risk behaves **exponentially** rather than linearly with changes in covariate values.

- Our goal is to establish the uncertainty distribution of the accident probability $Pr(Accident|Incident, \underline{X})$ in entirety rather than a point estimate.

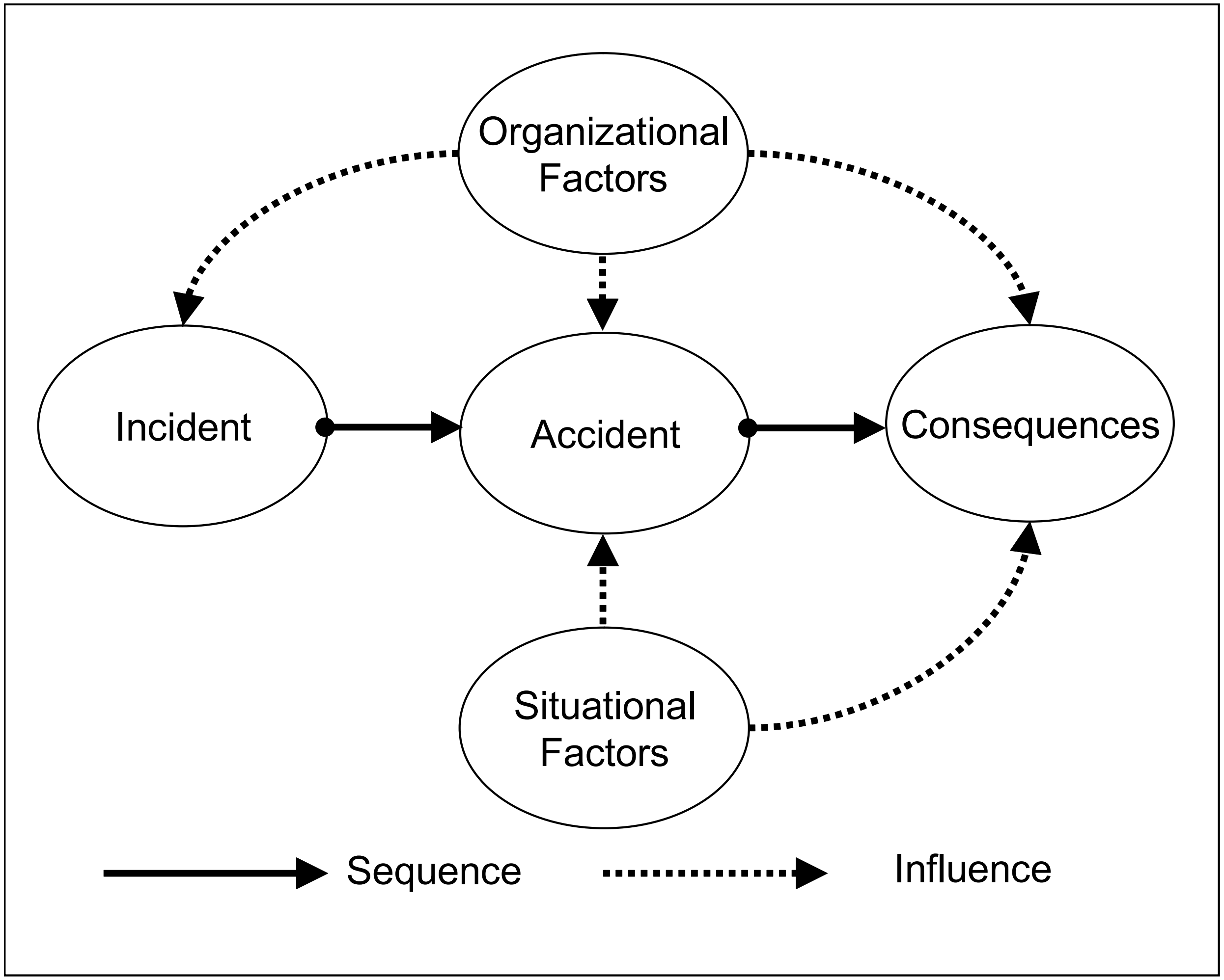
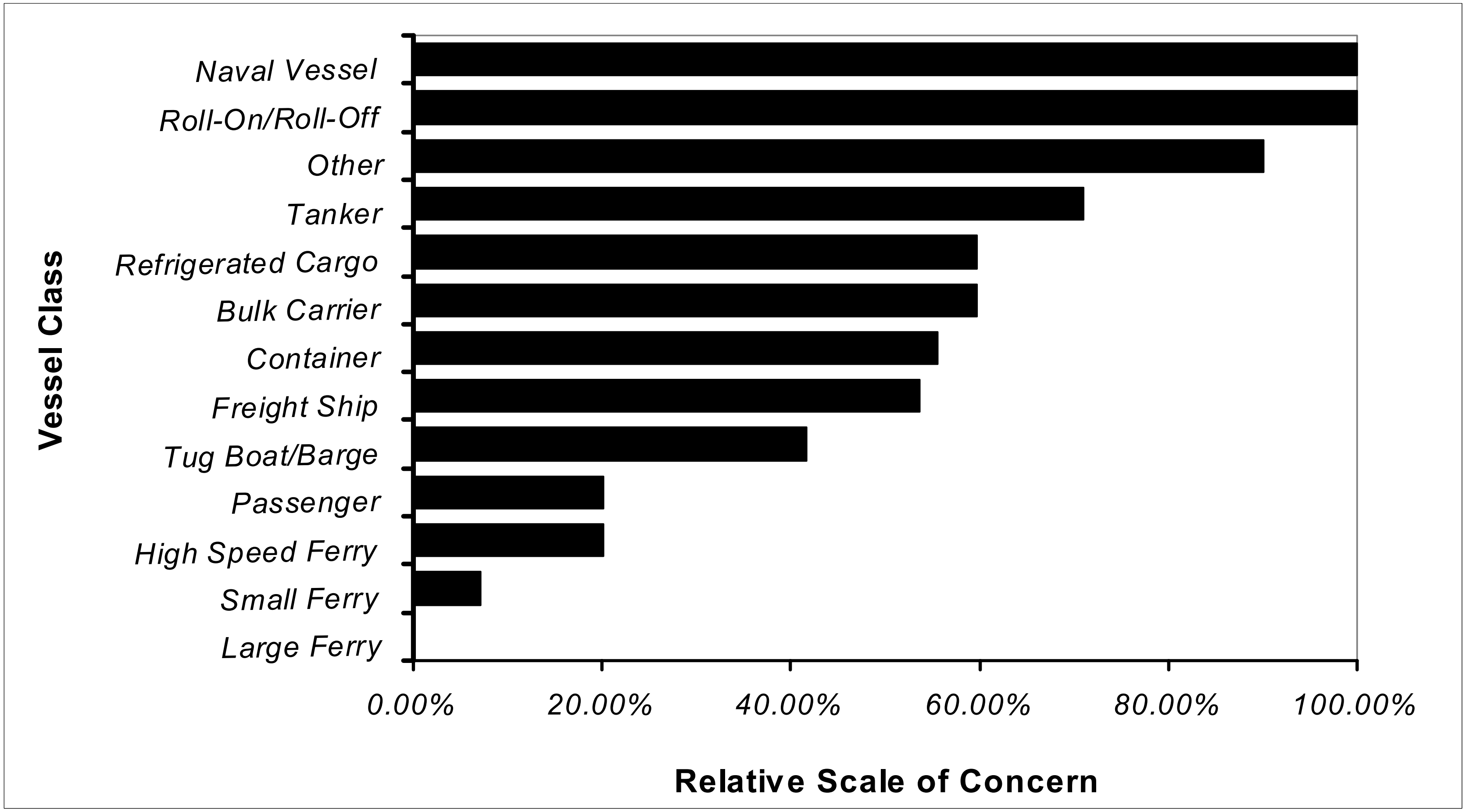


Table 1. Description of 10 contributing factors to $Pr(Accident \mid Incident, \underline{X})$ in WSF Risk Assessment

	Designation	*Description*	*Discretization*
X_1	FR_FC	Ferry route-class combination	26
X_2	TT_1	1st interacting vessel type	13
X_3	TS_1	Scenario of 1st interaction	4
X_4	TP_1	Proximity of 1st interaction	*Binary*
X_5	TT_2	2nd interacting vessel type	5
X_6	TS_2	Scenario of 2nd interaction	4
X_7	TP_2	Proximity of 2nd interaction	*Binary*
X_8	VIS	Visibility	*Binary*
X_9	WD	Wind direction	*Binary*
X_{10}	WS	Wind speed	*Continuous*

- $\underline{X} \in [0,1]^p$, $\underline{\beta} \in \mathbb{R}^p$ and $P_0 \in (0,1)$. The covariate X_i, $i = 1, \ldots, p$ are **normalized** so that $X_i = 1$ describes the **"worst" case scenario** and $X_i = 0$ describes the **"best" case scenario.**



Constructed Covariate Scale for Interacting Vessels

Question: 32 48

Situation 1	Attribute	Situation 2
Super	**Ferry Class**	-
SEA-BAI	**Ferry Route**	-
Naval Vessel	**1st Interacting Vessel**	-
Crossing the bow	**Traffic Scenario 1st Vessel**	-
1 to 5 miles	**Traffic Proximity 1st Vessel**	-
Deep Draft	**2nd Interacting Vessel**	-
Crossing the bow	**Traffic Scenario 2nd Vessel**	-
1 to 5 miles	**Traffic Proximity 2nd Vessel**	-
more than 0.5 mile	**Visibility**	less than 0.5 mile
Along Ferry	**Wind Direction**	-
40 knots	**Wind Speed**	-
	9 8 7 6 5 4 3 2 1 2 3 4 5 6 7 8 9	
Situation 1 is worse	**<===================X===================>**	**Situation 2 is worse**

An example question appearing in one of the questionnaires used in the WSF risk assessment

$$P(\underline{X}^1, \underline{X}^2 \,|\, \underline{\beta}\,) = Exp\{\underline{\beta}^T \left(\underline{X}^1 - \underline{X}^2\right)\} \in [0, \infty]. \tag{2}$$

$$Log\{P(\underline{X}^1, \underline{X}^2 \,|\, \underline{\beta}\,)\} = \underline{\beta}^T \left(\underline{X}^1 - \underline{X}^2\right) \in (-\infty, \infty) \tag{3}$$

8. THE LIKELIHOOD OF A SINGLE EXPERT'S RESPONSE

$$Y_j = \text{Experts response to ratio } \frac{Pr(Accident|Incident,\ \underline{X}_j^1)}{Pr(Accident|Incident,\ \underline{X}_j^2)},$$

$$Z_j = Log\,Y_j, j = 1, \ldots, n.$$

The response of the expert to such a question is uncertain and will assumed to be **normal distributed** such that

$$(Z_j \mid \mu_j, r) \sim N(\mu_j, r),\ r = 1/\sigma^2 \tag{4}$$

$$\mu_j = q_j^T \underline{\beta}, \underline{q}_j = (\underline{X}_j^1 - \underline{X}_j^2) \tag{5}$$

$$f_{Z_j}(z_j) \propto \sqrt{r}\ exp\left\{ -\frac{r}{2}(z_j - \mu_j)^2 \right\}. \tag{6}$$

- Expert answers ***n* paired comparison questions** defined by $\underline{q}_j = (\underline{X}_j^1 - \underline{X}_j^2)$, $j = 1, \ldots, n$, Define Q to be the $p \times n$ matrix and $\mathcal{Z}$ to be the vector with log responses of expert

$$Q = [\underline{q}_1, \ldots, \underline{q}_n],\ \mathcal{Z} = (z_1, \ldots, z_n). \tag{7}$$

- **Likelihood of an expert responding $\mathcal{Z}$ to questionnaire Q**, may be derived from (6) as being proportional to

$$\mathcal{L}(\mathcal{Z} \mid \underline{\beta}, r, Q) \propto r^{\frac{n}{2}} exp\left\{ -\frac{r}{2}(c - 2\,\underline{b}^T \underline{\beta} + \underline{\beta}^{\mathrm{T}} A\, \underline{\beta}) \right\}. \tag{9}$$

where

$$A = \sum_{j=1}^{n} \underline{q}_j \underline{q}_j^T \,;\, \underline{b} = \sum_{j=1}^{n} \underline{q}_j z_j \,;\, c = \sum_{j=1}^{n} z_j^2 \tag{10}$$

If columns of Q span $\mathbb{R}^p$ the matrix A can be shown to be symmetric, positive definite and henceforth invertible.

9. PRIOR DISTRIBUTION

- To allow for a conjugate Bayesian analysis **a multivariate normal/gamma prior** is proposed for the joint distribution of $(\underline{\beta}\,, r)$ similar to the one described in **West and Harrison (1989).**

$$\prod(r\,|\alpha,\nu)=\frac{\frac{\nu}{2}^{\frac{\alpha}{2}}}{\Gamma(\frac{\alpha}{2})}r^{\frac{\alpha}{2}-1}exp(-\frac{r}{2}\nu),\text{ i.e. } Gamma(\frac{\alpha}{2},\frac{\nu}{2}). \qquad (11)$$

$$\prod(\,\underline{\beta}\,|\,r\,)\ \propto r^{\frac{p}{2}}\,exp\left\{-\frac{r}{2}(\underline{\beta}-\underline{m})^{T}\Delta(\underline{\beta}-\underline{m})\right\},\text{ i.e. } MVN(\underline{m},r\Delta).(12)$$

Hence, **the joint prior distribution** on $(\underline{\beta}\,, r)$ follows from (11) and (12) to be

$$\prod(\,\underline{\beta}\,,\,r\,)\propto r^{\frac{\alpha}{2}-1}\,exp(-\frac{r}{2}\nu)\times r^{\frac{p}{2}}exp\left\{-\frac{r}{2}(\underline{\beta}-\underline{m})^{T}\Delta(\underline{\beta}-\underline{m})\right\}.(13)$$

- The marginal distribution of $\underline{\beta}$ may be derived from (14), yielding

$$\prod(\underline{\beta}) \propto \left[1+\frac{1}{\nu}(\underline{\beta}-\underline{m})^T\Delta(\underline{\beta}-\underline{m})\right]^{-\frac{\alpha+p}{2}} \tag{14}$$

and is recognized as a ***p*-dimensional multivariate *t*-distribution** with α degrees of freedom, location vector $\underline{m}$ and precision matrix $\frac{\alpha}{\nu}\Delta$.

- From (14) and (3) follows that the **log-relative probability** $Log\{P(\underline{X}^1,\underline{X}^2\,|\,\underline{\beta})\}$ has a **prior *t*-distribution** with mean and precision

$$\underline{m}^T(\underline{X}^1-\underline{X}^2),\ \frac{\alpha}{\nu}(\underline{X}^1-\underline{X}^2)^T\Delta(\underline{X}^1-\underline{X}^2) \tag{15}$$

9.1. Prior Parameter Specification

- A **prior chi-squared distribution** with α degrees of freedom (equivalent to a gamma distribution $Gamma(\frac{\alpha}{2},\frac{\nu}{2})$ with $\nu=1$) and $E[r|\alpha,\nu{=}1]=\alpha$.

- The prior parameter α will be set equal to **the reciprocal of the variance** of **an expert responding at random** and depends on the scale that is used in the paired comparison questions to collect the expert responses.

$$\alpha = E[r|\alpha, \nu{=}1] = \left\{\frac{2}{17}\sum_{k=2}^{9}\{Log(k)\}^2\right\}^{-1} \approx 0.380341. \qquad (16)$$

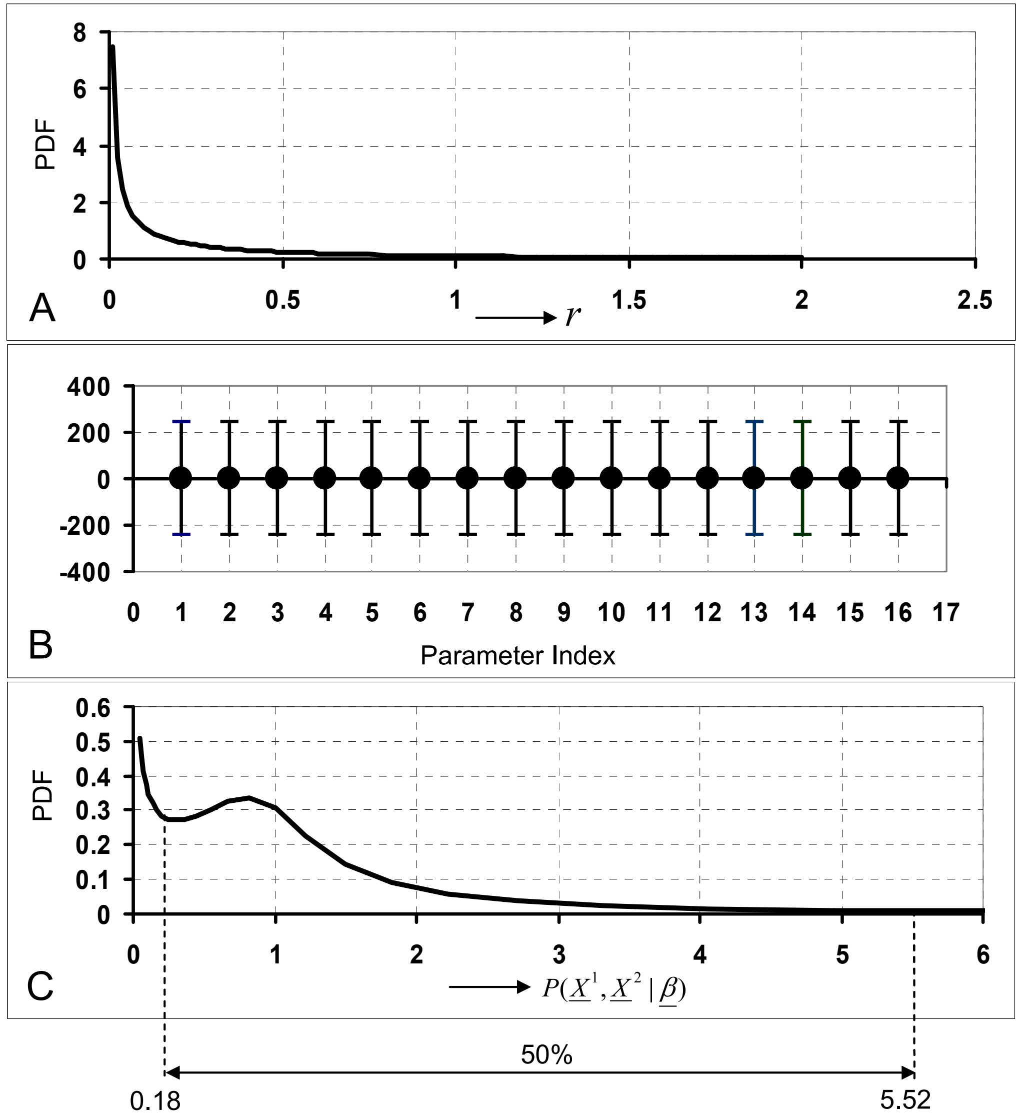
- For distribution of $(\underline{\beta}|r)$ we may select **a location vector** and the **unit precision matrix**

$$\underline{m} = (0, \ldots, 0)^T, \Delta = \begin{pmatrix} 1 & & \emptyset \\ & \ddots & \\ \emptyset & & 1 \end{pmatrix}, \qquad (17)$$

as long as the prior distribution on the relative accident probabilities (2) are flat.

- The **pdf of the relative accident probability** in our previous question is a **log-*t* distribution** (see, e.g., McDonald and Butler (1987)) with prior parameters

$$\underline{m}^T(\underline{X}^1 - \underline{X}^2) = 0, \alpha = 0.380341,\ \nu = 1\ ,\ \delta_{ii} = (\underline{X}^1 - \underline{X}^2)^T \Delta (\underline{X}^1 - \underline{X}^2) = 4.$$



Prior on $(\underline{\beta}\,, r)$ and $P(\underline{X}^1, \underline{X}^2 \mid \underline{\beta}\,)$ of previous question

- **The prior median of $P(\underline{X}^1, \underline{X}^2 \mid \underline{\beta})$ equals 1** (indicating indifference in collision likelihood between system states $\underline{X}^1$ and $\underline{X}^2$).

- **A 50% credibility interval** of $P(\underline{X}^1, \underline{X}^2 \mid \underline{\beta})$ in the figure above equals $[0.181, 5.515]$. **A 75% credibility interval** of $P(\underline{X}^1, \underline{X}^2 \mid \underline{\beta})$ equals $[2.012 \cdot 10^{-5},\ 4.971 \cdot 10^{4}]$ (which is quite wide).

Table 2. Interaction Variables associated with the contributing factors in Table 1.

	Name	*Description*	*Discretization*
X_{11}	FR_FC · TT_1	Interaction	13
X_{12}	FR_FC · TS_1	Interaction	13
X_{13}	FR_FC · VIS	Interaction	4
X_{14}	TT_1 · TS_1	Interaction	*Binary*
X_{15}	TT_1 · VIS	Interaction	13
X_{16}	TS_1 · VIS	Interaction	4

10. POSTERIOR ANALYSIS

Applying Bayes theorem utilizing the likelihood (9) , the prior distribution (13) and it follows that the posterior distribution $\prod(\underline{\beta}, r \mid \underline{\mathcal{Z}}, Q)$ is proportional to

$$\prod(\underline{\beta}, r \mid \underline{\mathcal{Z}}, Q) \propto r^{\frac{\alpha+n}{2}-1} exp\left\{-\frac{r}{2}\left(1+c+\underline{m}^T \Delta \underline{m}\right)\right\} \times \tag{18}$$
$$r^{\frac{p}{2}} exp\left\{-\frac{r}{2}\left(-2[\underline{b}+\Delta \underline{m}]^T \underline{\beta} + \underline{\beta}^T [A+\Delta] \underline{\beta}\right)\right\}.$$

Defining Δ^u to be $\Delta^u = A + \Delta$ and implicitly defining $\underline{m}^u$ satisfying

$$\left[\underline{b}+\Delta\, \underline{m}\right]^T \underline{\beta} = \left[\Delta^u \underline{m}^u\right]^T \underline{\beta} \tag{19}$$

for all $\underline{\beta}$, it follows that

$$\underline{b} + \sum \underline{m} = \Delta^u \underline{m}^u \Leftrightarrow \underline{m}^u = \left(\Delta^u\right)^{-1}\left(\underline{b} + \Delta\, \underline{m}\right). \tag{20}$$

Utilizing (20) and $\Delta^u = A + \Delta$ we derive from (18) that

$$\prod(\underline{\beta}, r \mid \mathcal{Z}, Q) \propto r^{\frac{\alpha+n}{2}-1} exp\Big\{-\tfrac{r}{2}\Big(1 + c + \underline{m}^T \Delta \underline{m} - \Big[\underline{m}^u\Big]^T \Delta^u \underline{m}^u\Big)\Big\} \times$$
$$r^{\frac{p}{2}} exp\Big\{-\tfrac{r}{2}\Big[\underline{\beta} - \underline{m}^u\Big]^T \Delta^u \Big[\underline{\beta} - \underline{m}^u\Big]\Big\}. \qquad (21)$$

From (21) it follows that $(\underline{\beta} \mid \mathcal{Z}, Q) \sim MVN(\underline{m}^u, r\Delta^u)$ where

$$\begin{cases} \Delta^u = \sum_{j=1}^{n} \underline{q}_j \underline{q}_j^T + \Delta \\ \underline{m}^u = \Big(\Delta^u\Big)^{-1}\Big(\sum_{j=1}^{n} \underline{q}_j z_j + \Delta\, \underline{m}\Big) \end{cases} \qquad (30)$$

and $(r|\mathcal{Z}, Q) \sim Gamma(\frac{\alpha^u}{2}, \frac{\nu^u}{2})$ with

$$\begin{cases} \alpha^u = \alpha + n \\ \nu^u = \nu + \sum_{j=1}^{n} z_j^2 + \underline{m}^T \Delta \underline{m} - \Big[\underline{m}^u\Big]^T \Delta^u \underline{m}^u \end{cases} \qquad (31)$$

11. EXAMPLE FROM WSF RISK ASSESSMENT

- **8 Experts** were selected amongst WSF captains and WSF first mates who had extensive experience with all 13 different ferry routes over an extended period of time (more than 5 years). **Combination** of the responses of these 8 experts follows naturally by **exploiting the conjugacy of the analysis** in Section 3, 4 and 5 through **sequential updating.**

Table 3. Expert Response to Previous Paired Comparison Question

Expert Index	1	2	3	4	5	6	7	8
Response	5	5	3	9	7	9	3	0.5

- **During the WSF risk assessment in 1998** expert responses were aggregated by taking **geometric means of their responses** and using them in a **classical log linear regression analysis** approach to assess relative accident probabilities given by (2). **Classical point estimates** for the parameters $\beta_j, j = 1, \ldots, 16$ **will be compared** to their **Bayesian counterparts** following our Bayesian aggregation method.

- Expert were instructed to assume that **a navigation equipment failure** had occurred on the **Washington State Ferry** and were next asked to assess **how much more likely a collision is to occur** in Situation 1 (good visibility in previous question) as compared to Situation 2 (bad visibility in previous question) taking into account the value of all the contributing factors. **Total of 60 Questions.** The questions were **randomized** in order and were **distributed evenly over the 10 contributing factors** in Table 1 (i.e. 6 questions per changing contributing factor).

11.1. The elements A, $\underline{b}$ and c of the likelihood given by (10)

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \tag{32}$$

where **A_{11} is a 10×10 diagonal matrix** with diagonal elements

$$(4.56, 4.33, 2.89, 6, 1.5, 2.44, 6, 6, 6, 0.375) \tag{33}$$

and associated with **the contributing factors $X_1, \ldots, X_{10}$.** (The matrix A_{11} in (32) is a diagonal matrix since the paired comparison scenarios $\underline{X}^1$ and $\underline{X}^2$ **only**

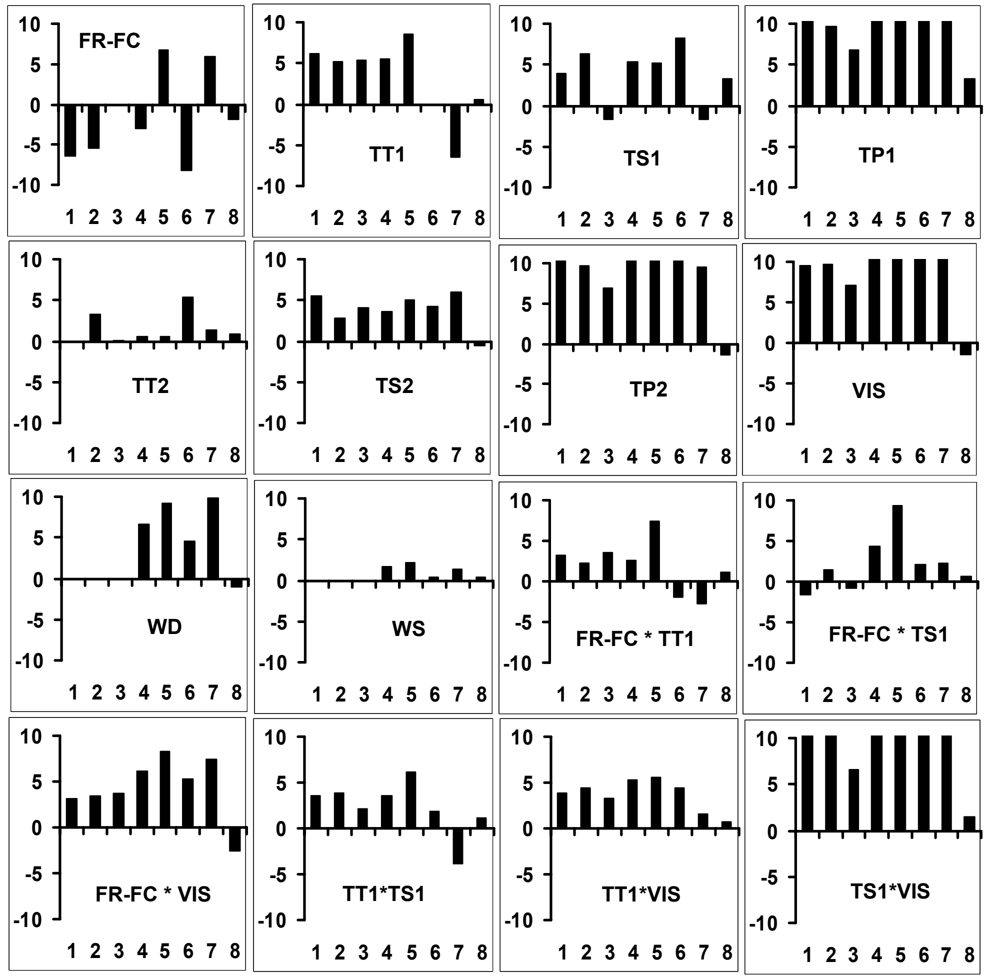
differed in one covariate (see ,e.g., the previous question). **The matrix $\boldsymbol{A}_{22}$** in (32) is a symmetric 6×6 matrix with elements

$$\begin{bmatrix} 3.45 & 0.33 & 0 & 1.44 & 0.76 & 0 \\ 0.33 & 3.45 & 0.44 & 0.33 & 0 & 1 \\ 0 & 0.44 & 4.11 & 0 & 1 & 2.39 \\ 1.44 & 0.33 & 0 & 1.89 & 0.36 & 0.08 \\ 0.76 & 0 & 1 & 0.36 & 3.02 & 2 \\ 0 & 1 & 2.39 & 0.08 & 2 & 6.67 \end{bmatrix} \tag{34}$$

and associated with the **interaction effects $\boldsymbol{X}_{11}, \ldots, \boldsymbol{X}_{16}$.** Finally, **the matrix $\boldsymbol{A}_{21} = \boldsymbol{A}_{12}^T$** is a sparse 10×6 matrix

$$\begin{bmatrix} 1 & 2.82 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2.26 & 0 & 2.12 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1.13 & 0 & 0 & 0 & 0 & 0 & 0 & 3.06 & 0 & 0 \\ 0 & 2.13 & 0.52 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.02 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 1.56 & 0 & 0 & 0 & 0 & 5.33 & 0 & 0 \end{bmatrix} \tag{35}$$

with **only positive elements** associated with the contributing factors X_1, X_2, X_3 and X_8 that are **included in the interaction effects** $X_{11}, \ldots, X_{16}$.



Summary of Individual Expert Response for 8 WSF experts in terms of i-th element of the vector $\underline{b}$ (cf. (11) for each of the contributing factors $X_i, i = 1, \ldots, 10$ in Table 1 and interaction effects $X_i,\ i = 11, \ldots, 16$ in Table 2.

Table 4. Values for c (cf. (11)) for the 8 individual experts.

Expert Index	1	2	3	4	5	6	7	8
c	149.07	95.28	55.74	147.93	185.71	177.30	147.12	44.94

11.2. Posterior Analysis

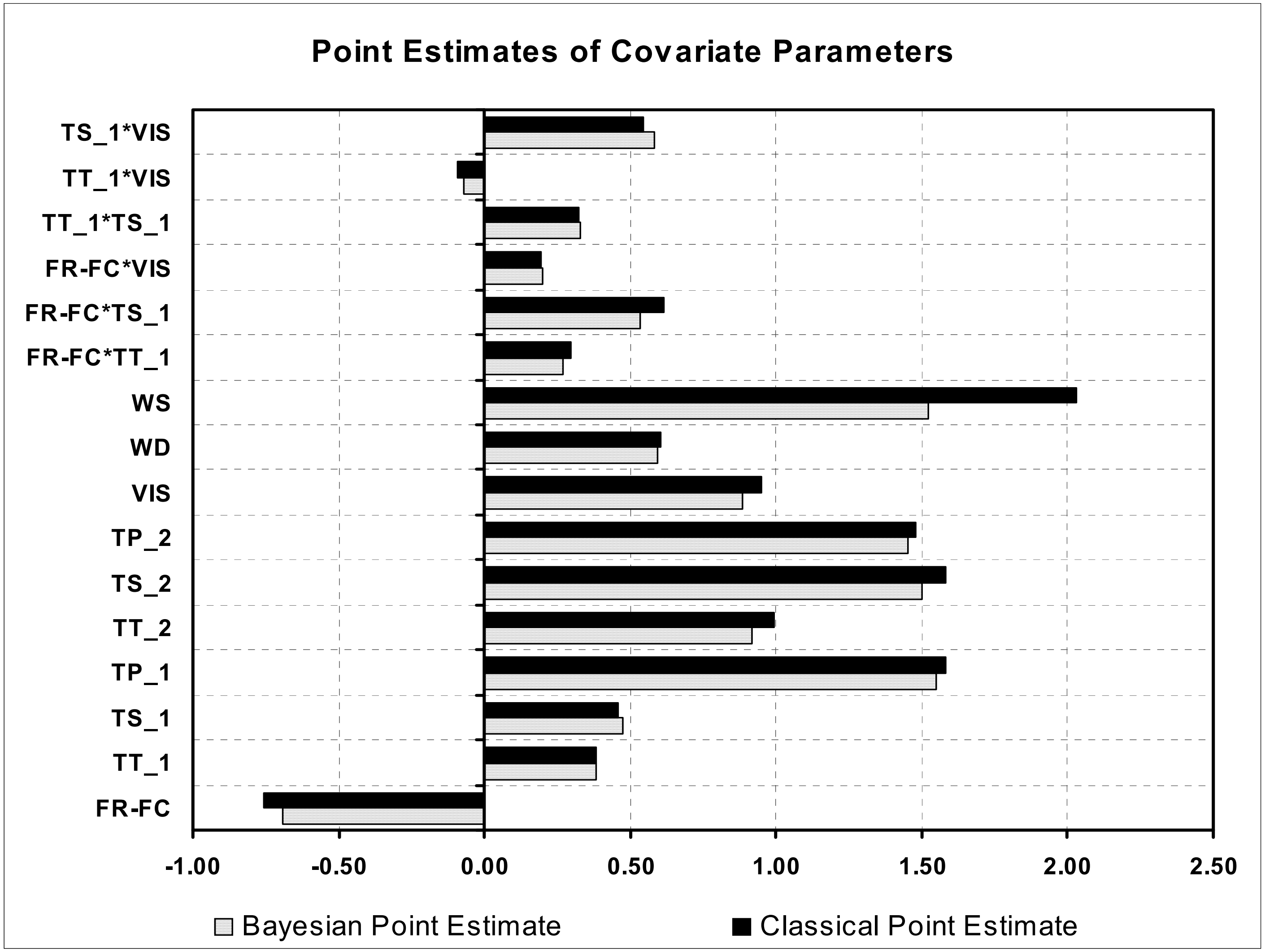
The resulting posterior parameters for the precision $r \sim Gamma(\frac{\alpha^u}{2}, \frac{\nu^u}{2})$ are

$$\alpha^u = 480.38,\ \nu^u = 530.95 \tag{36}$$

The posterior distribution of the parameter vector $\underline{\beta}$ is a multivariate t distribution with location vector $\underline{m}^u$ and precision matrix $\frac{\alpha^u}{\nu^u}\Delta^u$, where α^u, ν^u are given by (36),

$$\Delta^u = \Delta + 8A$$

and location vector $\underline{m}^u$ is depicted in the following figure.

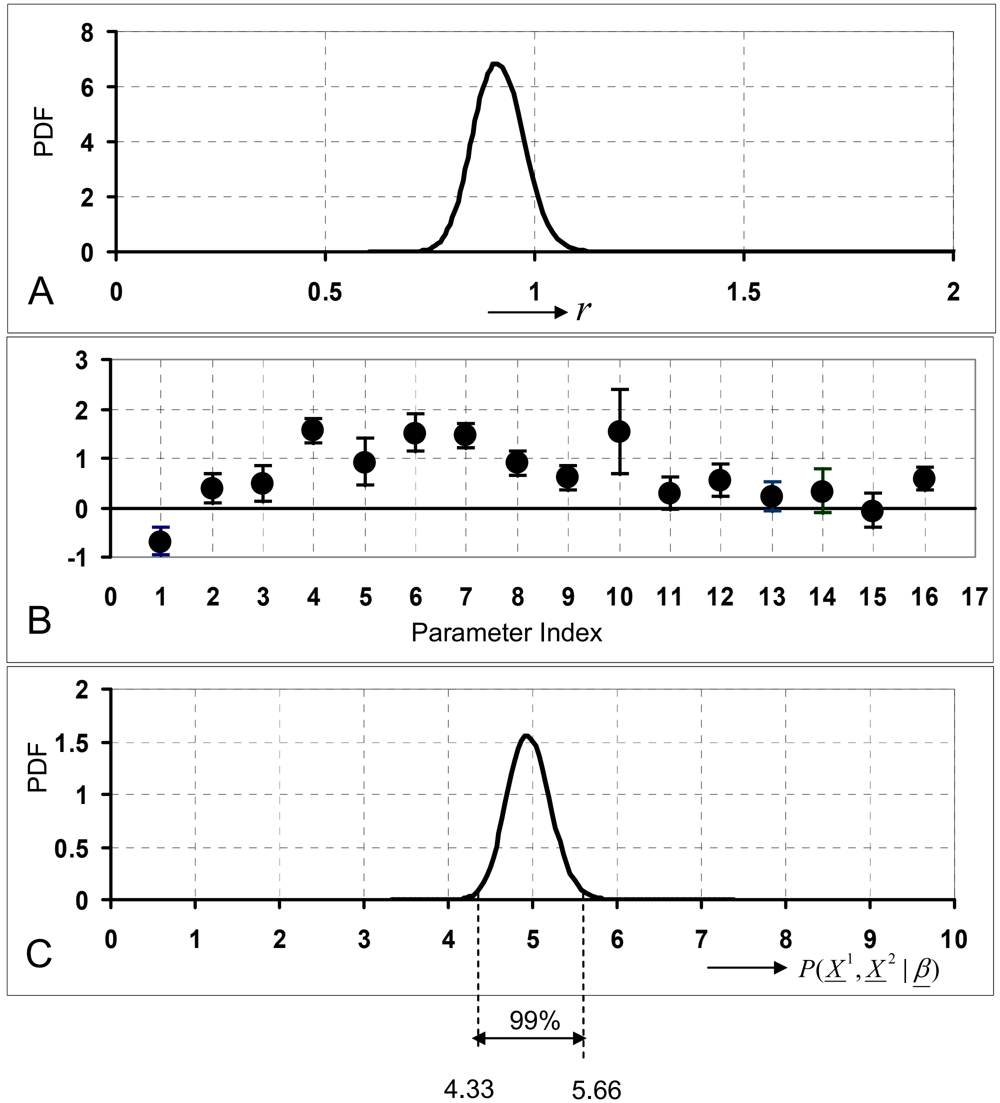


Comparison of Bayesian and Classical Point Estimates of the parameters β_i, $i = 1, \ldots, 16$.

- It can thus be concluded that **traffic proximity of the first and second interacting vessel** (X_4 and X_7, respectively), **traffic scenario of the second interacting vessel** X_7 and **wind speed** X_{10} are the largest contributing factors to accident risk. In addition, **the manner in which the first interacting vessel approaches the ferry route - ferry class combination** (X_{12}), i.e. crossing, passing or overtaking, and in what **visibility conditions** (X_{16}) are the largest interacting factors.

- **A remarkable agreement** should be noted between the **Bayesian** and **classical point estimates** provided in the figure above, except for a discrepancy associated with the contributing factor WS (Wind Speed).

- The next figure displays the posterior distribution of the relative probability $P(\underline{X}^1, \underline{X}^2 \,|\, \underline{\beta})$ associated with our previous pair wise comparison question.



Posterior on $(\underline{\beta}\,, r)$ and $P(\underline{X}^1, \underline{X}^2 \,|\, \underline{\beta}\,)$ of Previous Question.

- **The median point estimate of $P(\underline{X}^1, \underline{X}^2 \mid \underline{\beta})$** equals **4.94.** Hence, Situation 2 in Figure 3 is approximately 5 times more likely to result in a collision than Situation 1 given that a navigation equipment failure occurred on the ferry.

- Compare the **50% posterior credibility interval of $P(\underline{X}^1, \underline{X}^2 \mid \underline{\beta})$ of $[4.78, 5.13]$** to the **50% prior one of $[0.18, 5.52]$.** In addition, the **99% posterior credibility interval of $[4.33, 5.66]$** is indicated in the figure above which is remarkably narrow compared to the prior **75% credibility interval of $[2.012 \cdot 10^{-5},\ 4.971 \cdot 10^{4}]$**

- Utilizing **posterior distributional results for the parameter vector $\underline{\beta}$** credibility statements can be made for any arbitrary paired comparison. For example, setting **Situation 1** in (2) to the **best possible scenario ($\underline{X}^1 = \underline{0}$)** and **Situation 2** to the **worst possible scenario ($\underline{X}^2 = \underline{1}$)** a **99% credibility interval of $P(\underline{X}^1, \underline{X}^2 \mid \underline{\beta})$** equals **$[31142, 36749]$.** Therefore, collision risk in the worst possible scenario differs at least by **4 orders of magnitude** to that of the best possible scenario **while taking uncertainty** of the expert judgments **into account.**

12. COMMENTS ON EXPERT JUDGMENT METHOD

- **Bayesian aggregation method** has been developed using responses from multiple experts to a **paired comparison questionnaire** to assess the distribution of **relative accident probabilities. The classical analysis** conducted during the WSF risk assessment **only resulted in point estimates** of relative accident probabilities.

For more detailed information see:

P. Szwed, J. R. van Dorp, J. R. W. Merrick, T. A. Mazzuchi and A. Singh (2004). A Bayesian Paired Comparison Approach for Relative Accident Probability Assessment with Covariate Information. *European Journal of Operational Research*, Vol. 161 (1), pp. 240-255.

PREVIOUS EXPERT AGGREGATION METHOD UTILIZED **SEQUENTIAL UPDATING** WHICH ESSENTIALLY MEANS THAT GIVEN THE PARAMETER VECTOR $\underline{\beta}$ AND THE PRECISION r THE EXPERTS ARE **STATISTICALLY INDEPENDENT.**

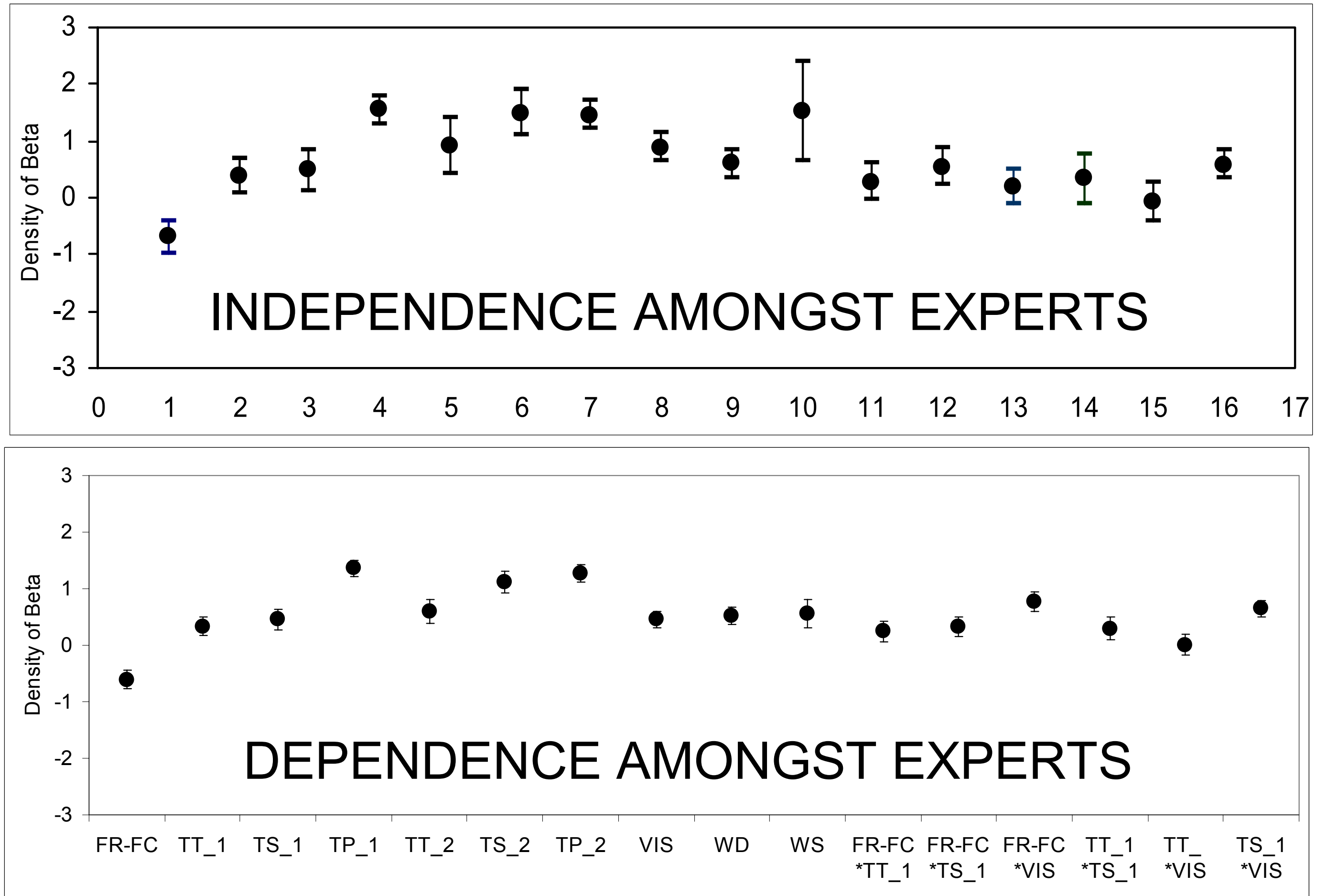
THE LATTER ASSUMPTION CAN BE RELAXED! ESSENTIALLY BY UTILIZING A MULTIVARIATE BAYESIAN REGRESSION APPROACH TO THE PAIRED COMPARISON QUESTIONS INVOLVING A **PRIOR MULTIVARIATE NORMAL** ON $(\underline{\beta} \mid \Sigma)$ WHERE Σ IS THE INVERSE VARIANCE COVARIANCE MATRIX AND **A WISHART PRIOR** OR Σ.

(see, e.g., Press, S. J. 1982. *Applied Multivariate Analysis Using Bayesian and Frequentist Methods and Inference*. 2nd Edition. Robert E. Krieger Publishing Company, Malabar, Florida).

For more detailed information on our specific application see:

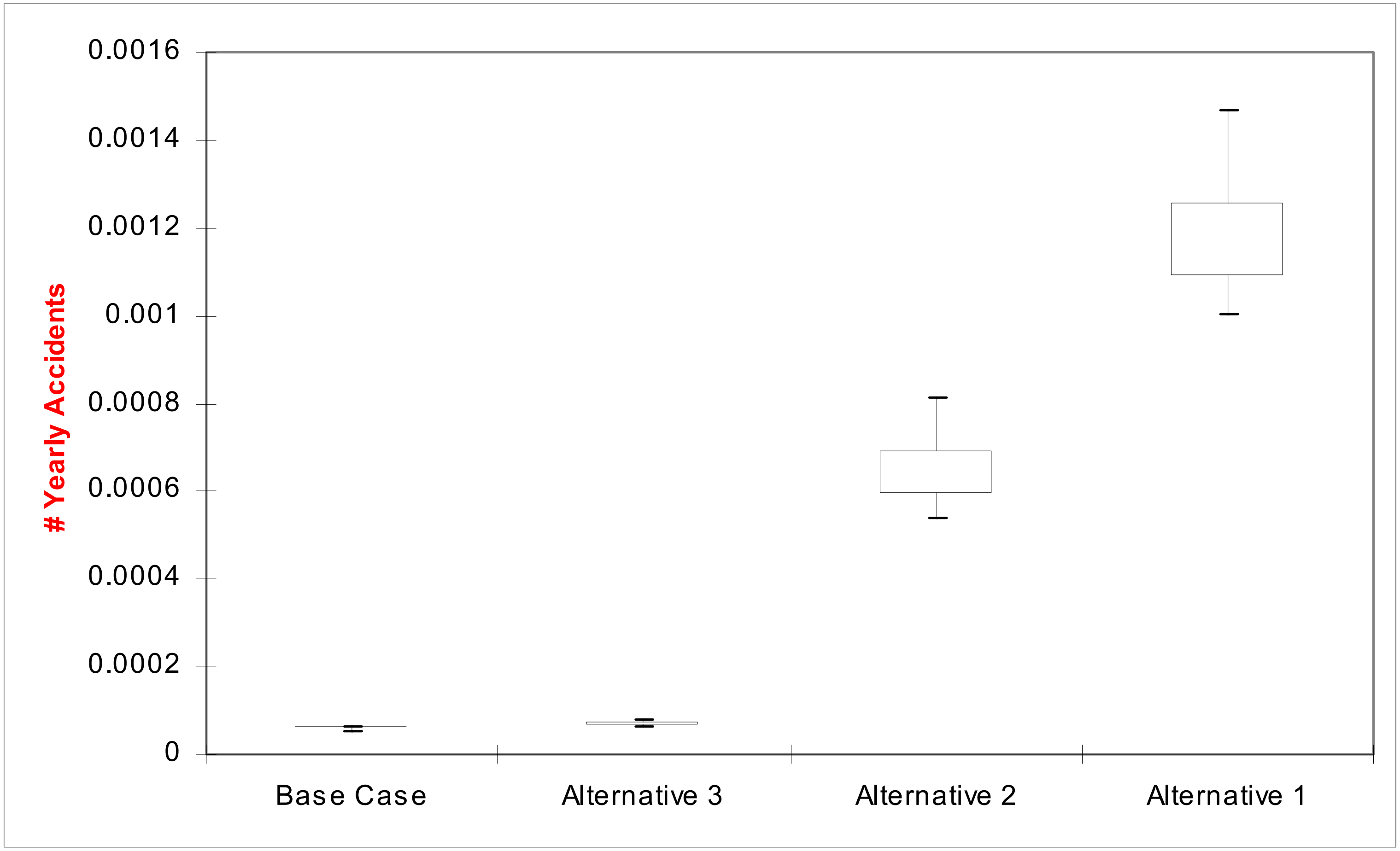
Merrick, J. R. W., J. R. van Dorp and A. Singh (2003). Analysis of Correlated Expert Judgments from Pairwise Comparisons. Re-submitted to *Decision Analysis*, September 2004, First Revision.

- A **surprising** result is that the **incorporation of dependence** here results in a reduction of **the predictive variance** of the elements of the parameters vector $\underline{\beta}$

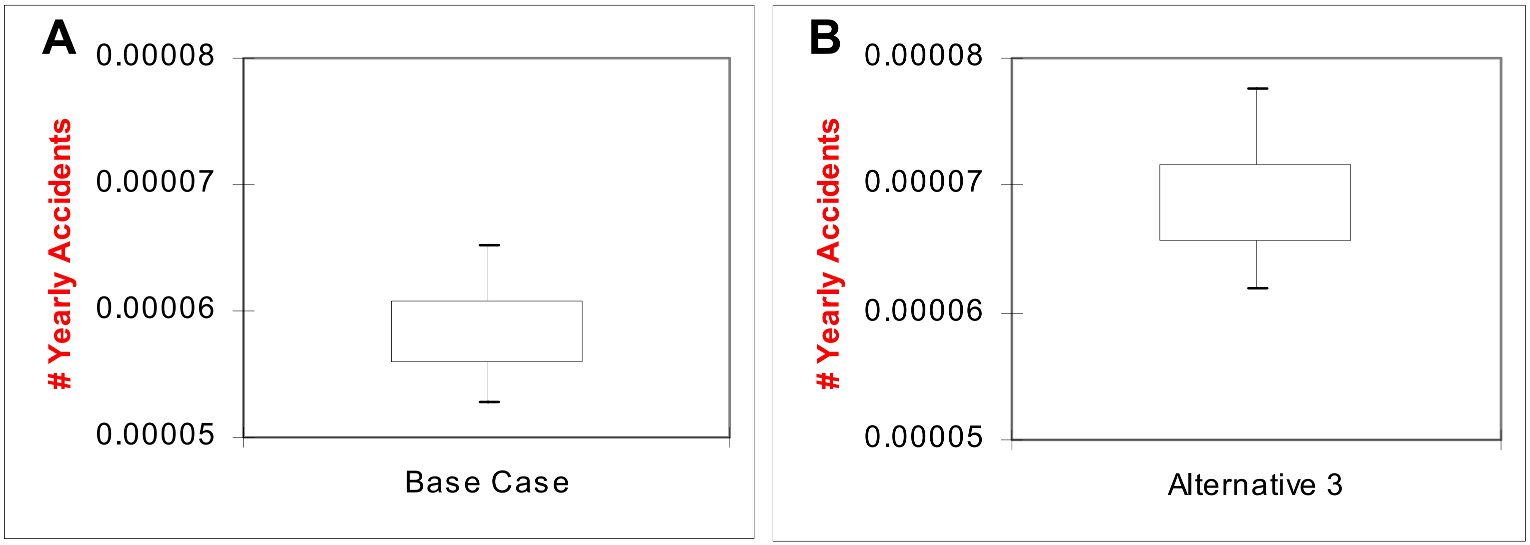


90% Creditibility Intervals of the elements of the parameter vector $\underline{\beta}$ of $Pr(Accident|Incident, \underline{X}) = P_0 Exp(\underline{\beta}^T \underline{X})$

13. PUTTING IT ALL TOGETHER IN A PROOF OF CONCEPT CASE STUDY



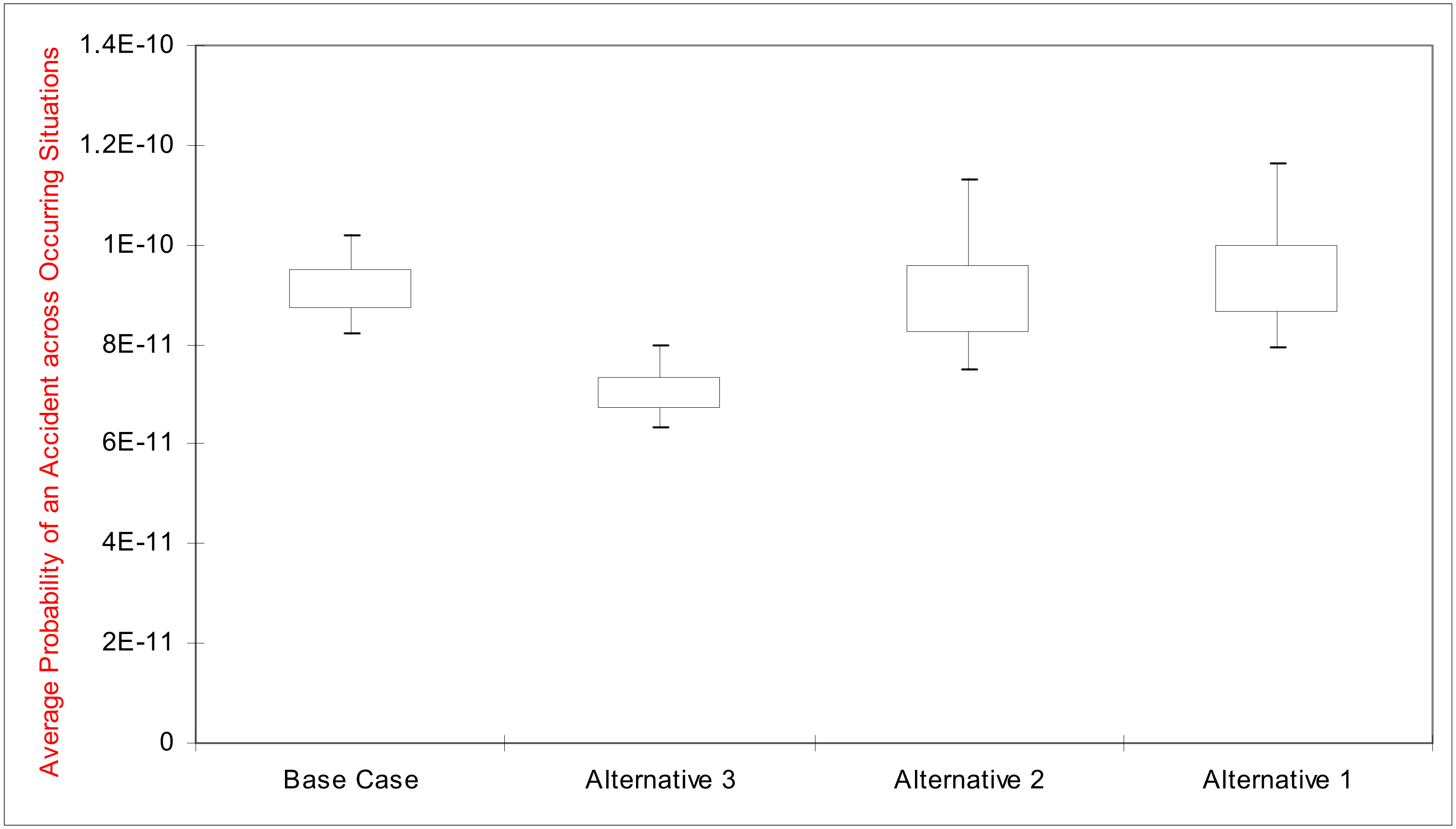
Whereas an **almost certain** ranking in terms of **the expected yearly number of situations**, this is not true for t**he expected yearly number of accidents**.



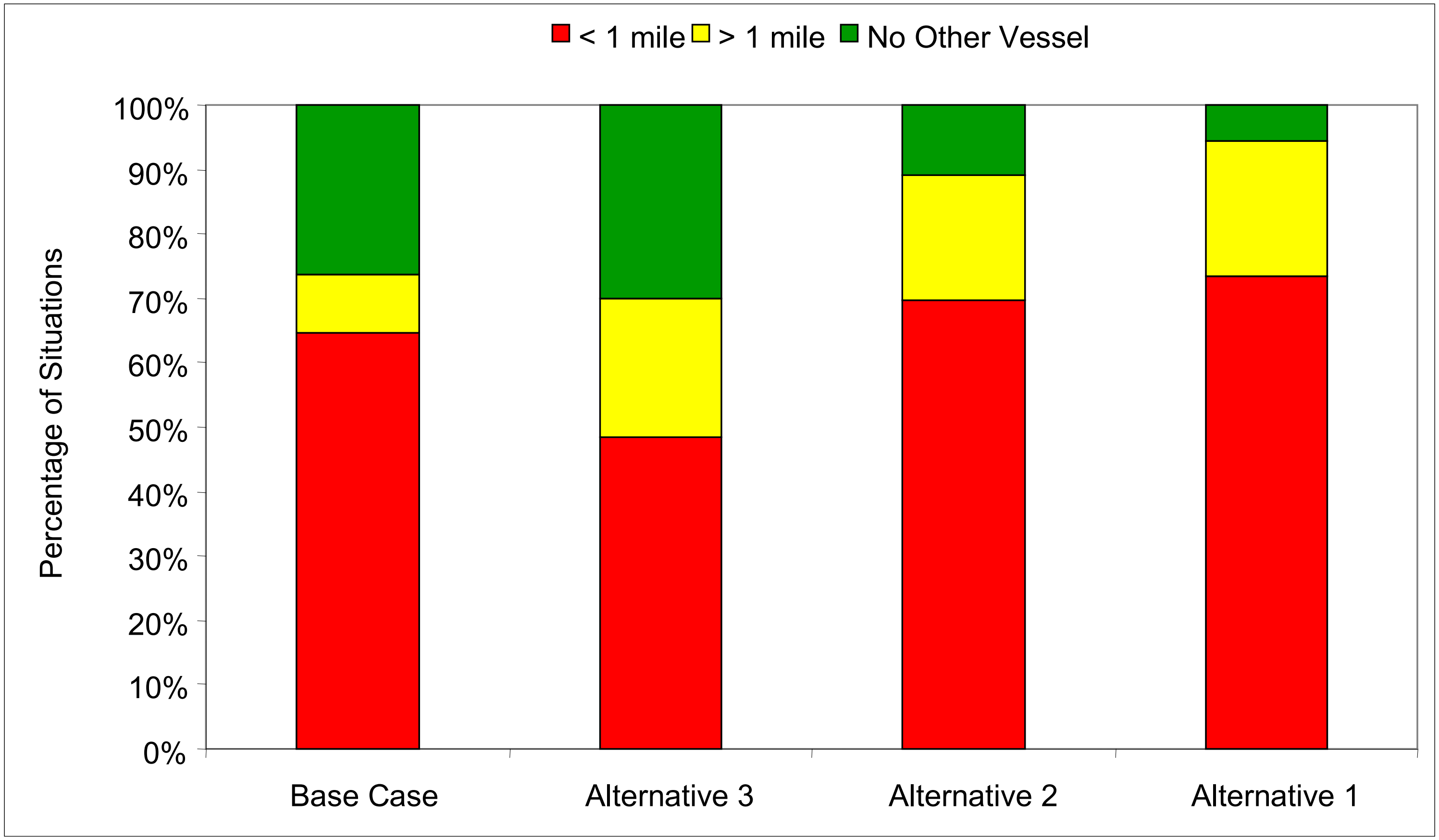
SEPARATE BOX PLOTS A-B OF PREVIOUS SLIDE

The box plots for the Base Case and Alternative 3 show that the range of their distributions do indeed overlap and the best we can say is that Alternative 3 **stochastically dominates** the Base Case in the sense that **their cumulative distribution functions do not cross.**

Reason for **Stochastic Dominance** and **not Deterministic Dominance** of the Alternative 3 and the Base Case is **a reduction in the average accident probability per occurring situation.**



Reason for **a reduction in the average accident probability per occurring situation** is a reduction in Alternative 3 of the number of second interacting vessels within a 1 mile distance. (Recall that Traffic Proximity of second interacting vessel was a dominant factor in the accident probability in a given interaction.)



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TENTATIVE OBSERVATION:

The reduction in accident probability per interaction could be an indication of what might be achieved when looking at the **SF Bay Ferry operation** as a **System of interconnected Ferry Routes** and by **designing a Comprehensive Ferry Schedule** that aims to **reduce the number of interactions and the number of vessels that are interacting in given situation**.

For more detailed information on this proof of concept case study see:

Merrick, J. R. W. and J. R. van Dorp (2004). Speaking the Truth in Maritime Risk Assessment. Submitted to *Management Science*, August 2004.

QUESTIONS?

14. MARITIME RISK ASSESSMENT LINKS

Faculty Home Page of J. Rene van Dorp:
http://www.seas.gwu.edu/~dorpjr

and

Faculty Home Page of Jason R.W. Merrick:
http://www.people.vcu.edu/~jrmerric

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