Session 1: Exploratory Data Analysis, Probability Calculus, Random Variables

Lecture Notes by: J. René van Dorp

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Example 1:
The tragedy that befell the space shuttle *Challenger* and its astronauts in 1986 led to a number of studies to investigate the reasons for mission failure. Attention quickly focused on the behavior of the rocket engine's O-rings. Here is the data consisting of observations on $x = \text{O-rign temperature (°F)}$ for each test firing or actual launch of the shuttle rocket engine (Presidential Commission on the Space Shuttle Challenger Accident, Vol. 1, 1986: 129-131).

\[
\begin{array}{cccccccccccc}
84 & 49 & 61 & 40 & 83 & 67 & 45 & 66 & 70 & 69 & 80 & 58 \\
68 & 60 & 67 & 72 & 73 & 70 & 57 & 63 & 70 & 78 & 52 & 67 \\
53 & 67 & 75 & 61 & 70 & 81 & 76 & 79 & 75 & 76 & 58 & 31 \\
\end{array}
\]

Without any organization, it is difficult to get a sense of what a typical or representative temperature might be, whether values are highly concentrated about a typical value or quite spread out, whether there are any gaps in the data, what percentages of the values are in the 60's and so on.
Exploratory Data Analysis

A MINITAB stem-and-leaf display

- Gives a feel of the distribution shape without loss of data
- Reasonable breakpoints in units of tens, some modifications are possible
A MINITAB Histogram with 10 cells of equal width

- Data are put into cells and the frequency of each cell is displayed graphically as a rectangle about the midpoint of the cell.

- Cell definitions and their number are at the discretion of the modeler with the rule of thumb that number of cells $\approx \sqrt{\text{number of observations}}$
STATISTICAL REVIEW

Exploratory Data Analysis

- Width of cells need not be of the same size (but usually are, since it is the best procedure for distribution representation).

- Cell definitions may change histogram shape.

A MINITAB Histogram with 8 cells of equal width
With enough data, a histogram approximates distributional forms.

A MINITAB Histogram with Normal Distribution fit
Empirical Cumulative Distribution Function (CDF):

\[ F_n(x) = \Pr\{X \leq x\} = \begin{cases} 0 & \text{for } x < x^{(1)} \\ \frac{i}{n} & \text{for } x^{(i)} \leq x < x^{(i+1)} \end{cases} \]

where \( x^{(i)} \) = \( i \)-th smallest observation and \( n \) = sample size

Example 2:
Power companies need information about customer usage to obtain accurate forecast of demands. Investigators from Wisconsin Power and Light determined energy consumption (BTUs) during a particular period for a sample of 90 gas-heated homes. An adjusted consumption value was calculated as follows:

\[ \text{adjusted consumption} = \frac{\text{consumption}}{(\text{weather, in degree days})(\text{house area})} \]

This resulted in the following data, which are ordered from smallest to largest.
Exploratory Data Analysis

<table>
<thead>
<tr>
<th>Percent</th>
<th>2015</th>
<th>10 5</th>
<th>10 0</th>
</tr>
</thead>
</table>

Empirical CDF of Adjusted Power Consumption

![Empirical CDF of Adjusted Power Consumption](image)

A MINITAB Empirical CDF of Adjusted Power Consumption

<table>
<thead>
<tr>
<th>Adjusted Consumption</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>20</td>
</tr>
<tr>
<td>10</td>
<td>40</td>
</tr>
<tr>
<td>15</td>
<td>60</td>
</tr>
<tr>
<td>20</td>
<td>80</td>
</tr>
</tbody>
</table>

\[ \approx 12.62 \]
• **Comparative Box plots:**

  Compares typically the median values, first and third quartile and extreme values across different treatment groups.

**Example 3:**
Specimens of three different types of rope wire were selected, and the fatigue limit (MPa) was determined for each specimen, resulting in the accompanying data:

<table>
<thead>
<tr>
<th>Type 1:</th>
<th>350</th>
<th>350</th>
<th>350</th>
<th>358</th>
<th>370</th>
<th>370</th>
<th>370</th>
<th>371</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>371</td>
<td>372</td>
<td>372</td>
<td>384</td>
<td>391</td>
<td>391</td>
<td>392</td>
<td></td>
</tr>
<tr>
<td>Type 2:</td>
<td>350</td>
<td>354</td>
<td>359</td>
<td>363</td>
<td>365</td>
<td>368</td>
<td>369</td>
<td>371</td>
</tr>
<tr>
<td></td>
<td>373</td>
<td>374</td>
<td>376</td>
<td>380</td>
<td>383</td>
<td>388</td>
<td>392</td>
<td></td>
</tr>
<tr>
<td>Type 3:</td>
<td>350</td>
<td>361</td>
<td>362</td>
<td>364</td>
<td>364</td>
<td>365</td>
<td>366</td>
<td>371</td>
</tr>
<tr>
<td></td>
<td>377</td>
<td>377</td>
<td>377</td>
<td>379</td>
<td>380</td>
<td>380</td>
<td>392</td>
<td></td>
</tr>
</tbody>
</table>
A MINITAB comparative box plot with observations

Boxplot of Type 1:, Type 2:, Type 3:
• **Individual value plots:** A comparative plot of the individual observations.

A MINITAB individual value plot with 95% confidence interval for the mean
**Time Series Plot:** Sequential plot of data vs time or sample number. Helpful in visualizing variability, trends, cycles, or dependence

A MINITAB times series plot of 30-year Mortgage Interest Rates
STATISTICAL REVIEW

Probability Calculus

- For all events $A \subseteq \Omega$: $0 \leq Pr(A) \leq 1$

- Compliment Rule: $Pr(\overline{A}) = 1 - Pr(A)$

- Additive Law: $Pr(A \cup B) = Pr(A) + Pr(B) - Pr(A \cap B)$
• Inclusion-Exclusion Principle:

\[
Pr(\bigcup_{i=1}^{n} A_i) = Pr(A_1 \cup \cdots \cup A_n) = \sum_{i=1}^{n} Pr(A_i) - \sum_{i=1}^{n} \sum_{j>i} Pr(A_i \cap A_j) + \sum_{i=1}^{n} \sum_{j>i} \sum_{k>j} Pr(A_i \cap A_j \cap A_k) - \cdots + (-1)^{n-1} Pr(\bigcap_{i=1}^{n} A_i)
\]

Example n=3:

\[
Pr(A_1 \cup A_2 \cup A_3) = Pr(A_1) + Pr(A_2) + Pr(A_3) - Pr(A_1 \cap A_2) - Pr(A_1 \cap A_3) - Pr(A_2 \cap A_3) + Pr(A_1 \cap A_2 \cap A_3)
\]
• **Multiplicative Law:** \( Pr(A \cap B) = Pr(A|B)Pr(B) = Pr(B|A)Pr(A) \)

**General form of Multiplicative Law:**

\[
Pr(\bigcap_{i=1}^{n} A_i) = Pr(A_1 \cap \cdots \cap A_n) = \\
Pr(A_1) \times Pr(A_2|A_1) \times Pr(A_3|A_1 \cap A_2) \times \cdots \\
Pr(A_{n-1}|\bigcap_{i=1}^{n-2} A_i) \times Pr(A_n|\bigcap_{i=1}^{n-1} A_i)
\]

• **Conditional Probability:**

\[
Pr(A|B) = \frac{Pr(A \cap B)}{Pr(B)} = \frac{Pr(B|A)Pr(A)}{Pr(B)}
\]
New Total Event based on the condition that we know that the Dow Jones went up

\[ \Pr(Stock \uparrow | Dow \uparrow) = \frac{\Pr(Stock \uparrow \cap Dow \uparrow)}{\Pr(Dow \uparrow)} \]
Informally: Conditioning on an event coincides with reducing the total event to the conditioning event.

Example: The probability of drawing an ace of spades in a deck of 52 cards equals $1/52$. However, if I tell you that I have an ace in my hands, the probability of it being the ace of spades equals $1/4$.

$$Pr(Spades|Ace) = \frac{Pr(Spades \cap Ace)}{Pr(Ace)}$$

$$= \frac{1/52}{4/52} = 1/4$$

Note also that:

$$Pr(B|A) = \frac{Pr(A \cap B)}{Pr(A)} = \frac{Pr(A|B)Pr(B)}{Pr(A)}$$
**Law of Total Probability**: Let $A_1, \ldots, A_n$ be collectively exhaustive and mutually exclusive events, i.e.;

$$A_i \cap A_j = \emptyset \text{ for all possible combinations } i, j$$

$$Pr\left( \bigcup_{i=1}^{n} A_i \right) = Pr(A_1 \cup \cdots \cup A_n) = \Omega$$

$$Pr(B) = \sum_{i=1}^{n} Pr(B \cap A_i) = \sum_{i=1}^{n} Pr(B | A_i) Pr(A_i)$$
Bayes Law: Let $A_1, \ldots, A_n$ be collectively exhaustive and mutually exclusive events, i.e.;

$$A_i \cap A_j = \emptyset \text{ for all possible combinations } i, j$$

$$Pr\left( \bigcup_{i=1}^{n} A_i \right) = Pr(A_1 \cup \cdots \cup A_n) = \Omega$$

$$Pr(A_j|B) = \frac{Pr(B|A_j)Pr(A_j)}{\sum_{i=1}^{n} Pr(B|A_i)Pr(A_i)}$$

Proof: From the rule for conditional probabilities we have

$$Pr(A_j|B) = \frac{Pr(B|A_j)Pr(A_j)}{Pr(B)}$$

and with the LOTP it follows here that: $Pr(B) = \sum_{i=1}^{n} Pr(B|A_i)Pr(A_i)$. □
Example 4: Given a batch of 100 items, 10 being defectives

- What is the probability of selecting a non defective item?
  
  \[ D \equiv \text{A randomly selected item is defective, } Pr(D) = \frac{10}{100} = 0.1 \]

  Apply the complement rule:  
  \[ Pr(\overline{D}) = 1 - Pr(D) = 1 - 0.1 = 0.9 \]

- What is the probability of selecting a defective on the first three draws?
  
  \[ D_i \equiv \text{The } i\text{-th item selected is defective} \]

  Apply the multiplicative law:
  \[
  Pr(D_1 \cap D_2 \cap D_3) = Pr(D_1)Pr(D_2|D_1)Pr(D_3|D_1 \cap D_2) = \\
  = \left( \frac{10}{100} \right) \left( \frac{9}{99} \right) \left( \frac{8}{98} \right) \approx .0007
  \]

- What is the probability of selecting a defective on the first or second draws?

  Apply the additive law and the law of total probability:
\[ Pr(D_1 \cup D_2) = Pr(D_1) + Pr(D_2) - Pr(D_1 \cap D_2) = \\
Pr(D_1) + Pr(D_2|D_1)Pr(D_1) + \\
Pr(D_2|\overline{D_1})Pr(\overline{D_1}) - Pr(D_2|D_1)Pr(D_1) = \\
Pr(D_1) + Pr(D_2|\overline{D_1})Pr(\overline{D_1}) = \\
\frac{10}{100} + \left(\frac{10}{99}\right)\left(\frac{90}{100}\right) = .1091 \]

- What is the probability that a defect was drawn on the first draw given two defects were drawn by the third draw?

\[ ND_{i,j} = \text{Total of } i \text{ defects drawn in } j \text{ draws} \]

Apply Bayes law:

\[ Pr(D_1|ND_{2,3}) = \frac{Pr(ND_{2,3}|D_1)Pr(D_1)}{Pr(ND_{2,3}|D_1)Pr(D_1) + Pr(ND_{2,3}|\overline{D_1})Pr(\overline{D_1})} = \\
\frac{Pr(ND_{2,3} \cap D_1)}{Pr(ND_{2,3} \cap D_1) + Pr(ND_{2,3} \cap \overline{D_1})} \]
\[
\frac{Pr(D_1 \cap \overline{D}_2 \cap D_3) + Pr(D_1 \cap D_2 \cap \overline{D}_3)}{Pr(D_1 \cap \overline{D}_2 \cap D_3) + Pr(D_1 \cap D_2 \cap \overline{D}_3) + Pr(\overline{D}_1 \cap D_2 \cap D_3)} = \frac{(\frac{10}{100})(\frac{90}{99})(\frac{9}{98}) + (\frac{10}{100})(\frac{9}{99})(\frac{90}{98})}{(\frac{10}{100})(\frac{90}{99})(\frac{9}{98}) + (\frac{10}{100})(\frac{9}{99})(\frac{90}{98}) + (\frac{90}{100})(\frac{10}{99})(\frac{9}{98})} = \frac{2}{3}
\]

- **Probability Calculation Table for Bayes calculations:**

  Given \( A_1, \ldots, A_n \) are ME and CE and \( B_1, \ldots, B_m \) are ME

<table>
<thead>
<tr>
<th></th>
<th>TABLE 1</th>
<th></th>
<th>TABLE 2</th>
<th></th>
<th>TABLE 3</th>
<th></th>
<th>TABLE 4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( Pr(A_1) )</td>
<td>( \ldots )</td>
<td>( Pr(A_n) )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( A_1 )</td>
<td>( \ldots )</td>
<td>( A_n )</td>
<td>( A_1 )</td>
<td>( \ldots )</td>
<td>( A_n )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( B_1 )</td>
<td>( Pr(B_1</td>
<td>A_1) )</td>
<td>( \ldots )</td>
<td>( Pr(B_1</td>
<td>A_n) )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( B_2 )</td>
<td>( Pr(B_2</td>
<td>A_1) )</td>
<td>( \ldots )</td>
<td>( Pr(B_2</td>
<td>A_n) )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
</tr>
<tr>
<td>( B_m )</td>
<td>( Pr(B_m</td>
<td>A_1) )</td>
<td>( \ldots )</td>
<td>( Pr(B_m</td>
<td>A_n) )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
• Column 1 of Table 2 (the shaded portion of Table 2) is given by:

\[(\text{Column 1 of Table 1}) \times \text{(the probability at the top of Column 1 of Table 1)}\]

\[
\begin{bmatrix}
Pr(B_1|A_1) \\
Pr(B_2|A_1) \\
\vdots \\
Pr(B_m|A_1)
\end{bmatrix} \times Pr(A_1)
\]

• In general, Column \(i\) of Table 2 is given by:

\[(\text{Column } i \text{ of Table 1}) \times \text{(the probability at the top of column } i \text{ of Table 1)}\]

<table>
<thead>
<tr>
<th>TABLE 1</th>
<th>TABLE 2</th>
<th>TABLE 3</th>
<th>TABLE 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Pr(A_1))</td>
<td>(Pr(A_n))</td>
<td>(Pr(B_1</td>
<td>A_1))</td>
</tr>
<tr>
<td>(A_1)</td>
<td>(A_n)</td>
<td>(A_1)</td>
<td>(A_n)</td>
</tr>
<tr>
<td>(B_1)</td>
<td>(Pr(B_1</td>
<td>A_1))</td>
<td>(Pr(B_1</td>
</tr>
<tr>
<td>(B_2)</td>
<td>(Pr(B_2</td>
<td>A_1))</td>
<td>(Pr(B_2</td>
</tr>
<tr>
<td>\vdots</td>
<td>\vdots</td>
<td>\vdots</td>
<td>\vdots</td>
</tr>
<tr>
<td>(B_m)</td>
<td>(Pr(B_m</td>
<td>A_1))</td>
<td>(Pr(B_m</td>
</tr>
</tbody>
</table>

| \(Pr(B_m|A_1)\) | \(Pr(B_m|A_n)\) | \(Pr(B_m \cap A_1)\) | \(Pr(B_m \cap A_n)\) | \(A_1\) | \(A_n\) | \(A_1\) | \(A_n\) |
• Row 1 of Table 3 (the shaded portion of Table 3) is given by the sum of elements of Row 1 of Table 2

\[ Pr(B_1 \cap A_1) + Pr(B_1 \cap A_2) + \cdots + Pr(B_1 \cap A_n) \]

• In general, Row \(i\) of Table 3 is given by the sum of elements of Row \(i\) of Table 2

<table>
<thead>
<tr>
<th>TABLE 1</th>
<th>TABLE 2</th>
<th>TABLE 3</th>
<th>TABLE 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Pr(A_1) )</td>
<td>( \cdots )</td>
<td>( A_1 )</td>
<td>( \cdots )</td>
</tr>
<tr>
<td>( A_1 )</td>
<td>( \cdots )</td>
<td>( A_1 )</td>
<td>( \cdots )</td>
</tr>
<tr>
<td>( B_1 )</td>
<td>( Pr(B_1</td>
<td>A_1) )</td>
<td>( Pr(B_1</td>
</tr>
<tr>
<td>( B_2 )</td>
<td>( Pr(B_2</td>
<td>A_1) )</td>
<td>( Pr(B_2</td>
</tr>
<tr>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
</tr>
<tr>
<td>( B_m )</td>
<td>( Pr(B_m</td>
<td>A_1) )</td>
<td>( Pr(B_m</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TABLE 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Pr(B_1) )</td>
</tr>
</tbody>
</table>

• Row 1 of Table 4 (the shaded portion of Table 4) is given by:

\[
\frac{\text{(Row 1 of Table 2)}}{\text{(Row 1 of Table 3)}} \times \frac{1}{Pr(B_1)}
\]

\[
= [ Pr(B_1 \cap A_1) \cdots Pr(B_1 \cap A_n) ] \times \frac{1}{Pr(B_1)}
\]
In general, Row $j$ of Table 4 is given by

$$(\text{Row } j \text{ of Table 2})/(\text{Row } j \text{ of Table 3})$$

- **Final Table**

| TABLE 1 | | TABLE 2 | | TABLE 3 | | TABLE 4 |
|---|---|---|---|---|---|
| $Pr(A_1)$ | $\ldots$ | $Pr(A_n)$ | | | |
| $A_1$ | $\ldots$ | $A_n$ | $A_1$ | $\ldots$ | $A_n$ |
| $B_1$ | $Pr(B_1|A_1)$ | $\ldots$ | $Pr(B_1|A_n)$ | $Pr(B_1 \cap A_1)$ | $\ldots$ | $Pr(B_1 \cap A_n)$ | $Pr(B_1)$ | $Pr(A_1|B_1)$ | $\ldots$ | $Pr(A_n|B_1)$ |
| $B_2$ | $Pr(B_2|A_1)$ | $\ldots$ | $Pr(B_2|A_n)$ | $Pr(B_2 \cap A_1)$ | $\ldots$ | $Pr(B_2 \cap A_n)$ | $Pr(B_2)$ | $\ldots$ |
| $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ |
| $B_m$ | $Pr(B_m|A_1)$ | $\ldots$ | $Pr(B_m|A_n)$ | $Pr(B_m \cap A_1)$ | $\ldots$ | $Pr(B_m \cap A_n)$ | $Pr(B_m)$ | $\ldots$ |
| *** | *** | $Pr(A_1)$ | $Pr(A_1)$ | *** | ??? | ??? |

*** This value will be 1 if $B_1, \ldots, B_m$ are also collectively exhaustive and $< 1$ otherwise.

?? Can be any value $> 0$
• The column and row sums can be used to check your results.

• In calculating Table 2 the multiplicative law of probability is applied.

• In calculating Table 3 the law of total probability is applied.

• In calculating Table 4 Bayes law is applied.

Example:

\[ Pr(A_1|B_1) = \frac{Pr(B_1 \cap A_1)}{Pr(B_1)} \]

\[ = \frac{Pr(B_1 \cap A_1)}{Pr(B_1 \cap A_1) + \ldots + Pr(B_1 \cap A_n)} \]

\[ = \frac{Pr(B_1|A_1)Pr(A_1)}{Pr(B_1|A_1)Pr(A_1) + \ldots + Pr(B_1|A_n)Pr(A_n)} \]
Example 5 - Quality Control Problem:
Let the probability of an item being defective, $p$, be 0.01, 0.05 or 0.10 with probability 0.6, 0.3, 0.1 respectively. If two samples are selected and tested what is the probability that $p$ is 0.01, 0.05, and 0.10 given 0, 1, or 2 defects are found.

Random Variable Definition: $X$ is the number of defects in a sample of 2

$$Pr(X = x | P) = \begin{cases} 
(1 - P)^2 & x = 0 \\
2P(1 - P) & x = 1 \\
P^2 & x = 2 
\end{cases}$$

$$Pr(P = p) = \begin{cases} 
0.6 & p = 0.01 \\
0.3 & p = 0.05 \\
0.1 & p = 0.10 
\end{cases}$$

<table>
<thead>
<tr>
<th>$p$</th>
<th>$X=0$</th>
<th>$X=1$</th>
<th>$X=2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>p=0.01</td>
<td>0.98</td>
<td>0.02</td>
<td>0.00</td>
</tr>
<tr>
<td>p=0.05</td>
<td>0.90</td>
<td>0.09</td>
<td>0.00</td>
</tr>
<tr>
<td>p=0.10</td>
<td>0.81</td>
<td>0.18</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Thus, for example: $Pr(P = 0.05 | X = 1) = 0.488$
Example 6: When a student attempts to log on to a computer time-sharing system, either all ports are busy ($F$), in which case the student will fail to obtain access, or else there is at least one port free ($S$), in which case the student will be successful in accessing the system.

Total Event: $\Omega = \{S, F\}$

Definition: For a given total event $\Omega$, a random variable (rv) is any rule that associates a number with each outcome in $\Omega$. In mathematical language, a random variable is a function whose domain is the sample space and whose range is the real numbers.

$$X(S) = 1 \quad X(F) = X(S) = 0$$
Example 7: Consider the experiment in which batteries are examined until a good
\(S\) is obtained.

Total Event: \(\Omega = \{S, FS, FFS, FFFS, \ldots \}\)

Define a rv \(X\) as follows:

\[ X = \text{the number of batteries examined before the experiment terminates.} \]

Then:

\[ X(S) = 1, \quad X(FS) = 2, \quad X(FFS) = 3, \text{ etc.} \]

The argument of the random variable function is typically omitted. Hence, one writes

\[ Pr(X = 2) = Pr(\text{The second battery works}) \]

Note that the above statement only has meaning with the above definition of the
random variable. It is good practice to always include the definition of a random
variable in words.
The nature of random variables can be **discrete** and **continuous**.

**Definition:** A **discrete** random variable is an rv whose possible values either constitute a finite set or else can be listed in an infinite sequence in which there is a first element, a second element, and so on. A random variable is **continuous** if its set of possible values consists of an entire interval on the number line.

**Example 7:** Suppose we select married couples at random and do a blood test on each person until we find an husband and wife who both have the same Rh factor.

\[ X = \text{the number of blood tests to be performed} \]

Then:

\[ X \in \{2, 4, 6, 8, \ldots \} \]

Since the possible values can be listed in a sequence, \( X \) is a discrete rv.
**Definition:** A random variable is said to be **continuous** if its set of possible values is an entire interval of numbers — that is, for some $A < B$, any number $X$ between $A$ and $B$ is possible.

**Example 8:** If in the study of ecology of a lake, we make depth measurements at randomly chosen locations, then

$$X = \text{the depth at a randomly chosen location},$$

is a **continuous** rv. Here $A$ is the minimum depth and $B$ is the maximum depth.

**Example 9:** If a chemical compound is randomly selected and its $pH$ $X$ is determined, then $X$ is a continuous rv because and $pH$ between 0 and 14 is possible. If more is known about the compound selected for analysis, then the set of possible values might be a subinterval of $[0, 14]$, such as $5.5 \leq x \leq 6.5$, but $X$ would still be **continuous**.