

SOLUTION QUESTION 7.20:

A company owns two different computers, which are in separate buildings and operated entirely separately. Based on past history, Computer 1 is **expected to breakdown 5.0 times a year**, with **a variance of 6**, and **costing \$200 per breakdown**. Computer 2 is **expected to breakdown 3.6 times per year**, with **a variance of 7**, and **costing \$165 per breakdown**. What is the company's expected cost for computer breakdowns and the variance of the breakdown cost? What assumption must you make to find the variance? Is this a reasonable assumption?

SOLUTION: Definitions:

X_1 = Annual Number of Breakdowns of Computer 1

c_1 = Cost per Breakdown of Computer 1 (\$200)

X_2 = Annual Number of Breakdowns of Computer 2

c_2 = Cost per Breakdown of Computer 2 (\$165)

TC = Annual Total Cost of Computer Breakdowns

Note that, X_1 and X_2 are random variables (i.e. their values are uncertain) and c_1 and c_2 are constant values.

From the text, we conclude:

$$TC = c_1X_1 + c_2X_2 \text{ (A LINEAR RELATIONSHIP!)}$$

Conclusion: TC is a random variable (its value is uncertain).

A. Because of the Linear Relationship we have:

$$E[TC] = E[c_1X_1 + c_2X_2] = c_1E[X_1] + c_2E[X_2]$$

From the text we have: $c_1 = \$200$, $c_2 = 165$, $E[X_1] = 5$ and $E[X_2] = 3.6$. Hence:

$$E[TC] = \$200 \times 5 + \$165 \times 3.6 = \$ 1594$$

B. With an assumption of independence between X_1 and X_2 we have:

$$\text{Var}[TC] = \text{Var}[c_1X_1 + c_2X_2] = (c_1)^2\text{Var}[X_1] + (c_2)^2\text{Var}[X_2]$$

From the text we have: $c_1 = \$200$, $c_2 = 165$, $\text{Var}[X_1] = 6$ and $\text{Var}[X_2] = 7$. Hence:

$$\text{Var}[TC] = (\$200)^2 \times 6 + (165)^2 \times 7 = (\$)^2 430575$$

$$\text{Standard Deviation TC: } \sqrt{430575} = \$656.18$$

IS THIS ASSUMPTION OF INDEPENDENCE REASONABLE?