Solutions

7.1. We often have to make decisions in the face of uncertainty. Probability is a formal way to cope with and model that uncertainty.

7.2. An uncertain quantity or random variable is an event that is uncertain and has a quantitative outcome (time, age, $, temperature, weight, . . .). Often a non-quantitative event can be the basis for defining an uncertain quantity; specific non-quantitative outcomes (colors, names, categories) correspond to quantitative outcomes of the uncertain quantity (light wavelength, number of letters, classification number). Uncertain quantities are important in decision analysis because they permit us to build models that may be subjected to quantitative analysis.

7.3.

$P(A \text{ and } B) = 0.12$ $\qquad$ $P(\overline{B}) = 0.35$

$P(A \text{ and } \overline{B}) = 0.29$ $\qquad$ $P(B \mid A) = \frac{0.12}{0.41} = 0.293$

$P(A) = 0.41$ $\qquad$ $P(A \mid B) = \frac{0.12}{0.65} = 0.185$

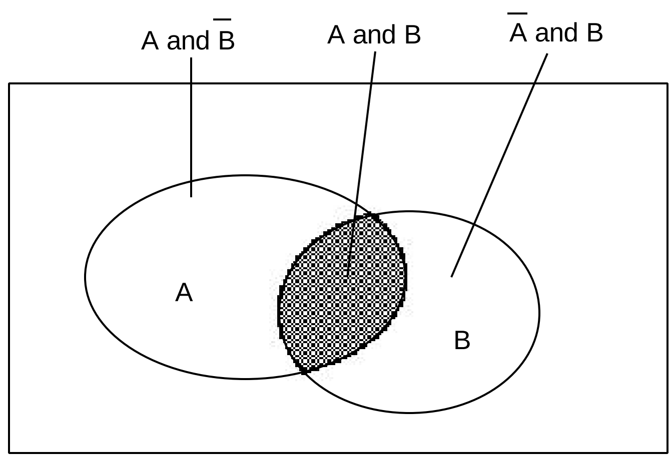
$P(B) = 0.65$ $\qquad$ $P(\overline{A} \mid \overline{B}) = \frac{0.06}{0.35} = 0.171$

7.4. $P(A \text{ or } B) = P(A \text{ and } B) + P(A \text{ and } \overline{B}) + P(\overline{A} \text{ and } B)$
$= 0.12 + 0.53 + 0.29 = 0.94$

or $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$
$= 0.41 + 0.65 - 0.12 = 0.94$

or $P(A \text{ or } B) = 1 - P(\overline{A} \text{ and } \overline{B}) = 1 - 0.06 = 0.94$

7.5.



From the diagram, it is clear that

$$P(A) = P(A \text{ and } B) + P(A \text{ and } \overline{B})$$

and

$$P(B) = P(A \text{ and } B) + P(\overline{A} \text{ and } B).$$

But $P(A \text{ or } B)$ clearly equals $P(A \text{ and } B) + P(A \text{ and } \overline{B}) + P(\overline{A} \text{ and } B)$ because of property 2. Thus,

P(A or B) = P(A and B) + P(A and $\bar{B}$) + P($\bar{A}$ and B)
= P(A) + P(B) - P(A and B).

7.6.a. Joint. P(left-handed and red-haired) = 0.08
b. Conditional P(red-haired | left-handed) = 0.20
c. Conditional P(Cubs win | Orioles lose) = 0.90
d. Conditional P(Disease | positive) = 0.59
e. Joint P(success and no cancer) = 0.78
f. Conditional P(cancer | success)
g. Conditional P(food prices up | drought)
h. Conditional P(bankrupt | lose crop) = 0.50
i. Conditional, but with a joint condition: P(lose crop | temperature high and no rain)
j. Conditional P(arrest | trading on insider information)
k. Joint P(trade on insider information and get caught)

7.7. For Product B, EMV = \$8M(0.38) + \$4M(0.12) + 0(0.50) = \$3.52M

Var(B) = $0.38(8 - 3.52)^2 + 0.12(4 - 3.52)^2 + 0.50(0 - 3.52)^2$
= 13.8496 "Millions-of-dollars squared"

Standard Deviation for B = $\sigma_B = \sqrt{13.8496}$ = \$3.72M.

For Product C, there is no variation. Thus, Var(C) = 0 and $\sigma_C = 0$.

7.8.

	A	$\bar{A}$	
B	0.2772	0.1450	0.4222
$\bar{B}$	0.1428	0.4350	0.5778
	0.42	0.58	1

P(A) = 0.42 is given, so P($\bar{A}$) = 1 - P(A) = 1 - 0.42 = 0.58

P($\bar{B}$ | A) = 1- P(B | A) = 1 - 0.66 = 0.34

P($\bar{B}$ | $\bar{A}$) = 1- P(B | $\bar{A}$) = 1 - 0.25 = 0.75

P(B) = P(B | A) P(A) + P(B | $\bar{A}$) P($\bar{A}$) = 0.66(0.42) + 0.25(0.58) = 0.4222

P($\bar{B}$) = 1 - P(B) = 1- 0.4222 = 0.5778

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)} = \frac{P(B|A)P(A)}{P(B)} = \frac{0.66(0.42)}{0.4222} = 0.6566$$

P($\bar{A}$ | B) = 1 - P(A | B) = 1 - 0.6566 = 0.3434

$$P(A \mid \bar{B}) = \frac{P(A \text{ and } \bar{B})}{P(\bar{B})} = \frac{P(\bar{B}|A)P(A)}{P(\bar{B})} = \frac{0.34(0.42)}{0.5778} = 0.2471$$

$P(\bar{A} \mid \bar{B}) = 1 - P(A \mid \bar{B}) = 1 - 0.2471 = 0.7529$

7.9. $P(\bar{A}) = 1 - P(A) = 1 - 0.10) = 0.90$

$P(\bar{B} \mid A) = 1 - P(B \mid A) = 1 - 0.39 = 0.61$

$P(\bar{B} \mid \bar{A}) = 1 - P(B \mid \bar{A}) = 1 - 0.39 = 0.61$

$P(B) = P(B \mid A)\, P(A) + P(B \mid \bar{A})\, P(\bar{A}) = 0.39(0.10) + 0.39(0.90) = 0.39$

$P(\bar{B}) = 1- P(B) = 1 - 0.39 = 0.61$

At this point, it should be clear that A and B are independent because $P(B) = P(B \mid A) = P(B \mid \bar{A}) = 0.39$.

Thus, $P(A) = P(A \mid B) = P(A \mid \bar{B}) = 0.10$, and $P(\bar{A}) = P(\bar{A} \mid B) = P(\bar{A} \mid \bar{B}) = 0.90$. (Actually, the fact that A and B are independent can be seen in the statement of the problem.)

7.10.

$P(X=2, Y=10) = P(Y=10 \mid X=2)\, P(X=2) = 0.9\ (0.3) = 0.27$

Likewise,

$P(X=2, Y=20) = P(Y=20 \mid X=2)\, P(X=2) = 0.1\ (0.3) = 0.03$
$P(X=4, Y=10) = P(Y=10 \mid X=4)\, P(X=4) = 0.25\ (0.7) = 0.175$
$P(X=4, Y=20) = P(Y=20 \mid X=4)\, P(X=4) = 0.75\ (0.7) = 0.525$

$E(X) = 0.3(2) + 0.7(4) = 3.4$

$P(Y = 10) = P(Y=10 \mid X=2)\, P(X=2) + P(Y=10 \mid X=4)\, P(X=4) = 0.27 + 0.175 = 0.445$
$P(Y = 20) = 1-0.445 = 0.555$

$E(Y) = 0.445\ (10) + 0.555\ (20) = 15.55$

Now calculate:

X	*Y*	*X* - E(*X*)	*Y* - E(*Y*)	(*X*-E(*X*))(*Y*-E(*Y*))	P(*X*, *Y*)
2	10	-1.4	-5.55	7.77	0.27
2	20	-1.4	4.45	-6.23	0.03
4	10	0.6	-5.55	-3.33	0.175
4	20	0.6	4.45	2.67	0.525

The covariance is the expected value of the cross products in the next-to-last column. To calculate it, use the joint probabilities in the last column:
$\mathrm{Cov}(X, Y) = 0.27\ (7.77) + 0.03\ (-6.23) + 0.175\ (-3.33) + 0.525\ (2.67) = 2.73$

Calculate the standard deviations by squaring the deviations in the third and fourth columns (for *X* and *Y*, respectively), finding the expected value of the squared deviations, and finding the square root:

$$s_X = \sqrt{0.3\ (-1.4^2) + 0.7\ (0.6^2)} = 0.917$$