Solution Question 9.15

a. Define: $T \equiv$ Time between two car sales

Let: m = Average number of Car Sales per hour

Assumptions meet the assumptions of the Poisson Process
(Also the subscripts E and P indicate exponential distribution and Poisson Distribution in this Question)

$m =$ 8.5 cars per 10 hours - 0.85 cars per hour

$$Pr(T \leq t) = 1 - e^{-m \cdot t}$$

Hence, $Pr(T > 2\, hours) = e^{-0.85 \cdot 2} = e^{-1.7} = 0.183$

b. Define: $X \equiv$ Number of Car Sales in t hours

Then: $$Pr(X = k) = \frac{(m \cdot t)^k}{k!} e^{-m \cdot t}$$

Let: $t = 2\, hours$ and: $k = 0$

Remember: $m =$ 8.5 cars per 10 hours - 0.85 cars per hour

It thus follows that: $$Pr(X = 0) = \frac{(0.85 \cdot 2)^0}{0!} e^{-0.85 \cdot 2} = e^{-1.7} = 0.183$$

Note that the results in question a and b should be the same since if the time until the first customer arrives is more than two hours it immediately follows that no customers have arrived in the first two hours and vice versa.

c. Here we let t be 10 hours (That is a full day)

$$Pr(Bonus = \$20) = Pr(X = 13) = \frac{(0.85 \cdot 10)^{13}}{13!} e^{-0.85 \cdot 10} = 0.04$$

(Table Page. 700)

$$Pr(Bonus = \$30) = Pr(X = 14) = \frac{(0.85 \cdot 10)^{14}}{14!} e^{-0.85 \cdot 10} = 0.024$$

(Table Page. 700)

$$Pr(Bonus = \$50) = Pr(X = 15) = \frac{(0.85 \cdot 10)^{15}}{15!} e^{-0.85 \cdot 10} = 0.014$$

(Table Page. 700)

$$Pr(Bonus = \$70) = Pr(X \geq 16) = 1 - Pr(X \leq 15)$$

$$= 1 - \sum_{k=0}^{15} \frac{(0.85 \cdot 10)^{k}}{k!} e^{-0.85 \cdot 10} = 1 - 0.986 = 0.014$$

(Table Page. 705)

$$E[Bonus] = 0.04 \times \$20 + 0.024 \times \$30 + 0.014 \times \$50 + 0.014 \times \$70$$
$$= \$3.20$$

Solution Question 9.21

Define: X = Number of breakdowns of old machines in $[0, t]$

where: t has the dimension "months"

Define: Y = Number of breakdowns of new machines in $[0, t]$

where: t has the dimension "months"

Then: $X \sim Poisson(m \cdot t)$ $Y \sim Poisson(n \cdot t)$

Where: $m = 2.5\, per\, months$ and $\begin{cases} Pr(n = 1.5\, per\, month) = 50\% \\ Pr(n = 3\, per\, month) = 50\% \end{cases}$

Repair of a new machine costs \$170 and of an old machine \$150

a. Let t be 1 month (hence we are comparing machine on a month to month bases). We have for the expected repair cost the following

$$E[X|m = 2.5, t = 1] = m \times t = 2.5 \; machines$$
$$\Rightarrow E[Cost] = 2.5 \times \$150 = \$375 \text{ per month}$$

$$E[Y|n = 1.5, t = 1] = n \times t = 1.5 \; machines$$
$$E[Y|n = 3, t = 1] = n \times t = 3 \; machines$$
$$\text{Hence, } E[Y|t = 1] = 50\% \times 1.5 + 50\% \times 3 = 2.25$$

$$\Rightarrow E[Cost] = 2.25 \times \$170 = \$382.50 \text{ per month}$$

Hence, the old machines cost less on a month to month basis

b. Data = $(6,[0,3))$ (That is 6 breakdowns in 3 months)

$$Pr(n=1.5|Data) = \frac{Pr(Data|n=1.5)Pr(n=1.5)}{Pr(Data|n=1.5)Pr(n=1.5)+Pr(Data|n=3)Pr(n=3)}$$

$$Pr(Data|n=1.5) = Pr(Y=6|n=1.5,\ t=3) = \frac{(1.5\times 3)^6}{6!}e^{-1.5\times 3} = 0.128$$

(Table Page 698)

$$Pr(Data|n=3.0) = Pr(Y=6|n=3,\ t=3) = \frac{(3\times 3)^6}{6!}e^{-3\times 3} = 0.091$$

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$$Pr(n=1.5|Data) = \frac{0.128\times 0.5}{0.128\times 0.5+0.091\times 10.5} = 0.5845$$

c. Let t be 1 month (hence we are comparing machine on a month to month bases). We have for the expected repair cost the following

$$E[X|m = 2.5, t = 1] = m \times t = 2.5 \; machines$$
$$\Rightarrow E[Cost] = 2.5 \times \$150 = \$375 \text{ per month}$$

$$\begin{cases} Pr(n = 1.5 \, per \, month|Data) = 58.45\% \\ Pr(n = 3 \, per \, month|Data) = 41.55\% \end{cases}$$

$$E[Y|n = 1.5, t = 1] = n \times t = 1.5 \; machines$$
$$E[Y|n = 3, t = 1] = n \times t = 3 \; machines$$

Hence, $E[Y|t = 1, \; Data] = 58.45\% \times 1.5 + \; 41.55\% \times 3 = 2.12$

$$\Rightarrow E[Cost] = 2.12 \times \$170 = \$360.96 \text{ per month}$$

Hence, based on the data you would now prefer the new machines