CHAPTER 9
Theoretical Probability Models

Notes
Chapter 9 is a straightforward treatment of probability modeling using five standard distributions. The five distributions (binomial, Poisson, exponential, normal, beta) were chosen because they cover a variety of different probability-modeling situations. Tables of probabilities for four of the distributions (all except the exponential, probabilities for which are easily calculated) are provided in the appendices.

In addition to the five distributions treated in the main part of the chapter, the uniform distribution is developed in problems 9.27 - 9.29, the triangular distribution in 9.30, and the lognormal distribution in problem 9.36 and the Municipal Solid Waste case study. Depending on the nature of the course and the level of the students, instructors may wish to introduce other distributions.

One note of caution: Chapter 9 provides an introduction to probability distributions that are used in Chapter 10 (fitting model parameters and natural conjugate priors) as well as Chapter 11 (creating random variates in a simulation). In particular, if the course is intended to move on to Chapter 11, it is important to expose students to the uniform distribution, the triangular distribution, and some of the other distributions as well.

In addition to using the tables provided in the appendices, RISKview can be used for computing and seeing distributions. Step-by-step instructions for viewing theoretical distributions with RISKview are provided in the chapter. RISKview is a pop-up dialog box within @RISK that is activated using the Define Distribution button. The desired probabilities can be determined by sliding the delimiters (marked by inverted triangles) or by entering Left and Right X and P values in the statistics grid.

Topical cross-reference for problems

Bayes' theorem	9.17, 9.21
Beta distribution	9.4, 9.5, 9.22, 9.24
Binomial distribution	9.1, 9.5, 9.15 - 9.19, 9.29, 9.31, 9.34, 9.35, Overbooking
Central limit theorem	9.36, Municipal Solid Waste
Empirical rule	9.11
Exponential distribution	9.5, 9.8, 9.12, 9.14, 9.15, Earthquake Prediction
Linear transformations	9.14, 9.25, 9.26
Lognormal distribution	9.36, Municipal Solid Waste
Memoryless property	9.12, 9.13, 9.28
Normal distribution	9.2, 9.5 - 9.7, 9.11, 9.25, 9.26, 9.31, 9.36, Municipal Solid Waste
Pascal sampling	9.18
Poisson distribution	9.3, 9.5, 9.9, 9.10, 9.13, 9.15, 9.19 - 9.21, 9.32 - 9.35, Earthquake Prediction
PrecisionTree	9.35
Requisite models	9.23
RISKview	9.1-9.15, 9.18, 9.19, 9.22, 9.24-9.34
Sensitivity analysis	9.17, 9.24, Earthquake Prediction
Triangular distribution	9.30
Uniform distribution	9.27 - 9.29

Solutions

9.1. Find P(Gone 6 or more weekends out of 12)

$$= P_B(R \geq 6 \mid n = 12, p = 0.65)$$

$$= P_B(R' \leq 6 \mid n = 12, p = 0.35) = 0.915.$$

Being gone 6 or more weekends out of 12 is the same as staying home on 6 or fewer.

Using RISKview, start RISKview. Select a function as the distribution source, the binomial as the distribution type, 12 for the n parameter, and 0.65 for the p parameter. In the statistics grid, set the Left X value to 5.5, and the Right X value to 12. The Difference P value then shows the desired probability: 91.54%. This distribution is saved as a RISKview file titled "Problem 9.1.rvp".

9.2.

$$P(Loss) = P_N (X < 0 \mid \mu = 2000, \sigma = 1500) = P(Z < \frac{0 - 2000}{1500}) = P(Z < -1.33) = 0.0918.$$

P(Gain greater than 4000)

$$= P_N (X > 4000 \mid \mu = 2000, \sigma = 1500) = P(Z > \frac{4000 - 2000}{1500})$$

$$= P(Z > 1.33) = 1 - P(Z \leq 1.33) = 1 - 0.9082 = 0.0918.$$

Note that $P(Z \leq -1.33) = P(Z \geq 1.33)$ because of symmetry of the normal distribution.

Using RISKview, select a function as the distribution source, the normal as the distribution type, 2000 for the μ parameter, and 1500 for the σ parameter. To determine the probability of a loss, in the statistics grid, set the Left X value to 4000, and the Right X value to 0. The Difference P value then shows the desired probability: 9.12%. To determine the probability that the return will be greater than 4000, set the Left X value to 4000 and the Right X value to 8000. The Difference P value then shows the desired probability: 9.12%. This distribution is saved as a RISKview file titled "Problem 9.2.rvp".

9.3. P(No chocolate chips) = $P_P(X = 0 \mid m = 3.6) = 0.027$ from Appendix C.

P(Fewer than 5 chocolate chips) = $P_P(X < 5 \mid m = 3.6)$

$$= P_P(X \leq 4 \mid m = 3.6) = 0.706 \text{ from Appendix D.}$$

P(More than 10 chocolate chips) = $P_P(X > 10 \mid m = 3.6)$

$$= 1 - P_P(X \leq 10 \mid m = 3.6) = 1 - 0.999 = 0.001.$$

Using RISKview, select a function as the distribution source, the Poisson as the distribution type, and 3.6 for the λ parameter. To determine the probability of no chips, in the statistics grid, set the Left X value to -0.5, and the Right X value to 0.5. The Difference P value then shows the desired probability: 2.73%. To determine the probability that there are fewer than 5 chips, set the Left X value to -0.5 and the Right X value to 4.5. The Difference P value then shows the desired probability: 70.64%. To determine the probability that there are more than 10 chips, set the Left X value to 10.5 and the Right X value to 15. The Difference P value then shows the desired probability: 0.13%. This distribution is saved as a RISKview file titled "Problem 9.3.rvp".

9.4. P(Net Contribution > 4500) = P[Q (100,000)(0.40) - 8000 > 4500]

$$= P_\text{ß}(Q > \frac{4500 + 8000}{40,000} \mid r = 1, n = 4) = P_\text{ß}(Q > 0.313 \mid r = 1, n = 4)$$

$$= 1 - P_\text{ß}(Q \le 0.313 \mid r = 1, n = 4) = 1 - 0.67 = 0.33.$$

Using RISKview, select a function as the distribution source, the Beta as the distribution type, 1 for the α_1 parameter and 3 for the α_2 parameter. To determine the probability of the net contribution greater than $4500, in the statistics grid, set the Left X value to 0.313, and the Right X value to 1. The Difference P value then shows the desired probability: 32.42%. This distribution is saved as a RISKview file titled "Problem 9.4.rvp".

9.5. a. $P_\text{B}(R = 5 \mid n = 10, p = 0.22) = 0.037$ from Appendix A.
Using RISKview, select a function as the distribution source, the Binomial as the distribution type, 10 for the n parameter and 0.22 for the p parameter. In the statistics grid, set the Left X value to 4.5, and the Right X value to 5.5. The Difference P value then shows the desired probability: 3.75%.

$P_\text{B}(R \le 1 \mid n = 7, p = 0.04) = 0.971$ from Appendix B.
Change the n parameter to 7 and the p parameter to 0.04. In the statistics grid, set the Left X value to 1. The Left P value then shows the desired probability: 97.06%.

$P_\text{B}(7 < R \le 10 \mid n = 12, p = 0.50)$

$$= P_\text{B}(R \le 10 \mid n = 12, p = 0.50) - P_\text{B}(R \le 7 \mid n = 12, p = 0.50)$$

$$= 0.997 - 0.806 = 0.191 \text{ from Appendix B.}$$

Change the n parameter to 12 and the p parameter to 0.5. In the statistics grid, set the Left X value to 7, and the Right X value to 10.5. The Difference P value then shows the desired probability: 19.07%.

$$P_\text{B}(R = 4 \mid n = 6, p = 0.42) = \frac{6!}{4! \, 2!} (0.42)^4 (0.58)^2 = 0.157.$$

Change the n parameter to 6 and the p parameter to 0.42. In the statistics grid, set the Left X value to 3.5, and the Right X value to 4.5. The Difference P value then shows the desired probability: 15.7%.

$$P_\text{B}(R = 10 \mid n = 15, p = 0.63) = \frac{15!}{10! \, 5!} (0.63)^{10} (0.37)^5 = 0.205.$$

Change the n parameter to 15 and the p parameter to 0.63. In the statistics grid, set the Left X value to 9.5, and the Right X value to 10.5. The Difference P value then shows the desired probability: 20.51%.

$P_\text{B}(R \le 3 \mid n = 10, p = 0.35) = 0.514$ from Appendix B.
Change the n parameter to 10 and the p parameter to 0.35. In the statistics grid, set the Left X value to 3. The Left P value then shows the desired probability: 51.38%.

$P_\text{B}(R > 1 \mid n = 19, p = 0.06) = 1 - P_\text{B}(R \le 1 \mid n = 19, p = 0.06) = 1 - 0.683 = 0.317.$
Change the n parameter to 19 and the p parameter to 0.06. In the statistics grid, set the Left X value to 1, and the Right X value to 5. The Difference P value then shows the desired probability: 31.64%.

$P_\text{B}(R < 2 \mid n = 4, p = 0.67)$

$$= P_\text{B}(R = 0 \mid n = 4, p = 0.67) + P_\text{B}(R = 1 \mid n = 4, p = 0.67)$$

$$= \frac{4!}{0!\ 4!} (0.67)^0 (0.33)^4 + \frac{4!}{1!\ 3!} (0.67)^1 (0.33)^3$$

$= 0.108.$

Change the n parameter to 4 and the p parameter to 0.67. In the statistics grid, set the Left X value to 1.5. The Left P value then shows the desired probability: 10.82%. This distribution is saved as a RISKview file titled "Problem 9.5a.rvp".

b. $\text{Pp}(X = 3 \mid m = 2.0) = 0.180$ from Appendix C.

Using RISKview, select a function as the distribution source, the Poisson as the distribution type, and 2 for the λ parameter. In the statistics grid, set the Left X value to 3.5, and the Right X value to 2.5. The Difference P value then shows the desired probability: 18.04%.

$\text{Pp}(X > 4 \mid m = 5.2) = 1 - \text{Pp}(X \leq 4 \mid m = 5.2) = 1 - 0.406 = 0.594$ from Appendix D.

Change the λ parameter to 5.2. In the statistics grid, set the Left X value to 4, and the Right X value to 16. The Difference P value then shows the desired probability: 59.39%.

$\text{Pp}(X \leq 1 \mid m = 3.9) = 0.099$ from Appendix D.

Change the λ parameter to 3.9. In the statistics grid, set the Left X value to 1. The Left P value then shows the desired probability: 9.92%.

$$\text{Pp}(X = 4 \mid m = 1.75) = \frac{e^{-1.75} (1.75^4)}{4!} = 0.068.$$

Change the λ parameter to 1.75. In the statistics grid, set the Left X value to 3.5, and the Right X value to 4.5. The Difference P value then shows the desired probability: 6.79%.

$\text{Pp}(X \geq 17 \mid m = 15) = 1 - \text{Pp}(X \leq 16 \mid m = 15) = 1 - 0.664 = 0.336$ from Appendix D.

Change the λ parameter to 15. In the statistics grid, set the Left X value to 16.5, and the Right X value to 40. The Difference P value then shows the desired probability: 33.59%.

$\text{Pp}(X < 7 \mid m = 10.0) = \text{Pp}(X \leq 6 \mid m = 10.0) = 0.130$ from Appendix D.

Change the λ parameter to 10. In the statistics grid, set the Left X value to 6.5. The Left P value then shows the desired probability: 13.01%.

$\text{Pp}(3 \leq X < 7 \mid m = 1.5) = \text{Pp}(X < 7 \mid m = 1.5) - \text{Pp}(X < 3 \mid m = 1.5)$

$= \text{Pp}(X \leq 6 \mid m = 1.5) - \text{Pp}(X \leq 2 \mid m = 1.5) = 0.999 - 0.809 = 0.190.$

Change the λ parameter to 1.5. In the statistics grid, set the Left X value to 6.5, and the Right X value to 2.5. The Difference P value then shows the desired probability: 19.03%.

$\text{Pp}(X \leq 2 \mid m = 3.56)$

$= \text{Pp}(X = 0 \mid m = 3.56) + \text{Pp}(X = 1 \mid m = 3.56) + \text{Pp}(X = 2 \mid m = 3.56)$

$$= \frac{e^{-3.56} (3.56^0)}{0!} + \frac{e^{-3.56} (3.56^1)}{1!} + \frac{e^{-3.56} (3.56^2)}{2!}$$

$= 0.310.$

Change the λ parameter to 3.56. In the statistics grid, set the Left X value to 2. The Left P value then shows the desired probability: 30.99%. This distribution is saved as a RISKview file titled "Problem 9.5b.rvp".

c. $P_E(T \leq 5 \mid m = 1) = 1 - e^{-5(1)} = 0.993$.

Using RISKview, select a function as the distribution source, the Exponential as the distribution type and 1 for the β parameter (note β is $1/m$). In the statistics grid, set the Left X value to 5. The Left P value then shows the desired probability: 99.33%.

$P_E(0.25 < T \leq 1 \mid m = 2) = e^{-0.25(2)} - e^{-1(2)} = 0.471$.
    Note that $P_E(T < 1 \mid m) = P_E(T \leq 1 \mid m)$.

Change the β parameter to 0.5. In the statistics grid, set the Left X value to 1, and the Right X value to 0.25. The Difference P value then shows the desired probability: 47.12%.

$P_E(T = 3.2 \mid m = 2) = 0$. For continuous random variables, the probability is zero that the random variable will take a specific value.

$P_E(T \geq 4 \mid m = 0.25) = P_E(T > 4 \mid m = 0.25) = e^{-4(0.25)} = e^{-1} = 0.368$.

Change the β parameter to 4. In the statistics grid, set the Left X value to 4, and the Right X value to 25. The Difference P value then shows the desired probability: 36.59%.

$P_E(T \geq 3.2 \mid m = 2) = e^{-3.2(2)} = 0.002$.

Change the β parameter to 0.5. In the statistics grid, set the Left X value to 3.2, and the Right X value to 5. The Difference P value then shows the desired probability: 0.16%. This distribution is saved as a RISKview file titled "Problem 9.5c.rvp".

$P_E(T \geq k \mid m = 1/k) = e^{-k(1/k)} = e^{-1} = 0.368$.

d. $P_N(Y \leq 12 \mid \mu = 10, \sigma = 4) = P(Z \leq \frac{12 - 10}{4})$

    $= P(Z \leq 0.50) = 0.6915$ from Appendix E.

Using RISKview, select a function as the distribution source, the Normal as the distribution type, 10 for the μ parameter, and 4 for the σ parameter. In the statistics grid, set the Left X value to 12. The Left P value then shows the desired probability: 69.15%.

$P_N(100 < Y \leq 124 \mid \mu = 133, \sigma = 15)$
    $= P_N(Y \leq 124 \mid \mu = 133, \sigma = 15) - P_N(Y \leq 100 \mid \mu = 133, \sigma = 15)$
    $= P(Z \leq \frac{124 - 133}{15}) - P(Z \leq \frac{100 - 133}{15})$
    $= P(Z \leq -0.60) - P(Z \leq -2.20) = 0.2743 - 0.0139 = 0.2604$ from Appendix E.

Change the μ parameter to 133 and the σ parameter to 15. In the statistics grid, set the Left X value to 124, and the Right X value to 100. The Difference P value then shows the desired probability: 26.03%.

$P_N(Y > 0 \mid \mu = -10, \sigma = 4) = P(Z \leq \frac{0 - (-10)}{4}) = P(Z > 2.50)$

    $= 1 - P(Z \leq 2.50) = 1 - 0.9938 = 0.0062$ from Appendix E.

Change the μ parameter to -10 and the σ parameter to 4. In the statistics grid, set the Left X value to 0, and the Right X value to 20. The Difference P value then shows the desired probability: 0.62%.

$P_N(Y = 12 \mid \mu = 10, \sigma = 4) = 0$.

$P_N(Y > 50 \mid \mu = 100, \sigma = 30) = P(Z \leq \frac{50 - 100}{30}) = 1 - P(Z < -1.67)$
    $= 1 - 0.0475 = 0.9525$ from Appendix E.

Change the µ parameter to 100 and the σ parameter to 30. In the statistics grid, set the Left X value to 50, and the Right X value to 250. The Difference P value then shows the desired probability: 95.22%.

$$P_N\ (Y > 20 \mid \mu = 15, \sigma = 4) = P(Z \le \frac{20 - 15}{4}\ ) = 1\text{-}\ P(Z \le 1.25) = 1 - 0.8944 = 0.1056.$$

Change the µ parameter to 15 and the σ parameter to 4. In the statistics grid, set the Left X value to 20, and the Right X value to 100. The Difference P value then shows the desired probability: 10.56%.

$$P_N\ (\text{-}44 \le Y \le 12 \mid \mu = 10, \sigma = 20)$$
$$= P_N\ (Y \le 12 \mid \mu = 10, \sigma = 20) - P_N\ (Y \le \text{-}44 \mid \mu = 10, \sigma = 20)$$
$$= P(Z \le \frac{12 - 10}{20}\ ) - P(Z \le \frac{\text{-}44 - 10}{20}\ ) = P(Z \le 0.10) - P(Z \le \text{-}2.70)$$
$$= 0.5398 - 0.0035 = 0.5363 \text{ from Appendix E.}$$

Change the µ parameter to 10 and the σ parameter to 20. In the statistics grid, set the Left X value to -44, and the Right X value to 12. The Difference P value then shows the desired probability: 53.64%.

$$P_N\ (Y < 12 \mid \mu = 10, \sigma = 4) = P_N\ (Y \le 12 \mid \mu = 10, \sigma = 4) = 0.6915 \text{ (calculated above in}$$
$$9.5.d).$$

Change the µ parameter to 10 and the σ parameter to 4. In the statistics grid, set the Left X value to 12. The Left P value then shows the desired probability: 69.15%. This distribution is saved as a RISKview file titled "Problem 9.5d.rvp".

e. $P_\beta(Q \le 0.9 \mid r = 9, n = 10) = 0.39$ from Appendix F.

Using RISKview, select a function as the distribution source, the Beta as the distribution type, 9 for the α1 parameter, and 1 for the α2 parameter. In the statistics grid, set the Left X value to 0.9. The Left P value then shows the desired probability: 38.74%.

$$P_\beta(Q \le 0.44 \mid r = 20, n = 30) = 0.01 \text{ from Appendix F.}$$

Change the α1 parameter to 20 and the α2 parameter to 10. In the statistics grid, set the Left X value to 0.44. The Left P value then shows the desired probability: 0.59%.

$$P_\beta(0.12 \le Q \le 0.25 \mid r = 4, n = 14)$$
$$= P_\beta(Q \le 0.25 \mid r = 4, n = 14) - P_\beta(Q \le 0.12 \mid r = 4, n = 14)$$
$$= 0.42 - 0.06 = 0.36 \text{ from Appendix F.}$$

Change the α1 parameter to 4 and the α2 parameter to 10. In the statistics grid, set the Left X value to 0.12, and the Right X value to 0.25. The Difference P value then shows the desired probability: 35.49%.

$$P_\beta(Q \ge 0.5 \mid r = 2, n = 5) = 1 - P_\beta(Q \le 0.5 \mid r = 2, n = 5) = 1 - 0.69 = 0.30.$$

Change the α1 parameter to 2 and the α2 parameter to 3. In the statistics grid, set the Left X value to 0.5, and the Right X value to 1.0. The Difference P value then shows the desired probability: 31.25%.

$$P_\beta(0.04 \le Q \le 0.38 \mid r = 8, n = 18)$$
$$= P_\beta(Q \le 0.38 \mid r = 8, n = 18) - P_\beta(Q \le 0.04 \mid r = 8, n = 18)$$
$$= 0.30 - 0 = 0.30.$$

Change the α1 parameter to 8 and the α2 parameter to 10. In the statistics grid, set the Left X value to 0.04, and the Right X value to 0.38. The Difference P value then shows the desired probability: 29.71%. This distribution is saved as a RISKview file titled "Problem 9.5e.rvp".

$$P_\beta(Q = 0.76 \mid r = 14, n = 18) = 0.$$

9.6. $P_N (Z \leq \underline{-1.645} \mid \mu = 0, \sigma = 1) = 0.05$

Using RISKview, select a function as the distribution source, the Normal as the distribution type, 0 for the μ parameter, and 1 for the σ parameter. In the statistics grid, set the Left P value (rather than the Left X) to 0.05. The Left X value then shows the desired z-value: -1.645.

$P_N (Z \leq \underline{0} \mid \mu = 0, \sigma = 1) = 0.50$

In the statistics grid, set the Left P value to 0.5. The Left X value then shows the desired z-value: 0.

$P_N (Z > \underline{0.675} \mid \mu = 0, \sigma = 1) = 0.25$, or $P_N (Z \leq 0.675 \mid \mu = 0, \sigma = 1) = 0.75$

In the statistics grid, set the Left P value to 0.75. The Left X value then shows the desired z-value: 0.675.

$P_N (Z > \underline{1.28} \mid \mu = 0, \sigma = 1) = 0.10$, or $P_N (Z \leq 1.28 \mid \mu = 0, \sigma = 1) = 0.90$

In the statistics grid, set the Left P value to 0.90. The Left X value then shows the desired z-value: 1.2815. This distribution is saved as a RISKview file titled "Problem 9.6.rvp".

9.7. To find μ, notice that μ will fall halfway between the first and third quartiles. Thus,

$$\mu = (125 + 275)/2 = 200.$$

To find σ, note that $x = 125$ must be the 0.25 fractile. Therefore,

$$P(Z \leq -0.675) = 0.25, \text{ or } P(Z \leq \frac{125 - 200}{\sigma}) = 0.25.$$

From this, $\dfrac{125 - 200}{\sigma} = -0.675$, and thus $\sigma = 111.11$.

9.8. If $P_E(T \geq 5 \mid m) = 0.24$, then $e^{-5m} = 0.24$. Take logs of both sides:

$$\ln[e^{-5}] = \ln(0.24)$$

$$-5m = -1.4271$$

$$m = 0.2854.$$

9.9. Looking up these probabilities in Appendix C gives $m = 5.4$.

9.10. If $P_P(X = 0 \mid m) = 0.175$, then

$$\frac{e^{-m} m^0}{0!} = 0.175$$

$$e^{-m} = 0.175 \text{ because } m^0 = 0! = 1.$$

$$m = -\ln(0.175) = 1.743.$$

9.11. $P_N (\mu - \sigma < Y \leq \mu + \sigma \mid \mu, \sigma) = P_N (Y \leq \mu + \sigma \mid \mu, \sigma) - P_N (Y \leq \mu - \sigma \mid \mu, \sigma)$

$$= P(Z \leq \frac{(\mu + \sigma) - \mu}{\sigma}) - P(Z \leq \frac{(\mu - \sigma) - \mu}{\sigma})$$

$$= P(Z \leq 1) - P(Z \leq -1) = 0.8413 - 0.1587 = 0.6826.$$

$$P_N (\mu - 2\sigma < Y \leq \mu + 2\sigma \mid \mu, \sigma) = P_N (Y \leq \mu + 2\sigma \mid \mu, \sigma) - P_N (Y \leq \mu - 2\sigma \mid \mu, \sigma)$$

$$= P(Z \leq \frac{(\mu + 2\sigma) - \mu}{\sigma}) - P(Z \leq \frac{(\mu - 2\sigma) - \mu}{\sigma})$$

$$= P(Z \leq 2) - P(Z \leq -2) = 0.9772 - 0.0228 = 0.9544.$$

Using RISKview, select a function as the distribution source, the Normal as the distribution type, 0 for the μ parameter, and 1 for the σ parameter. In the statistics grid, set the Left X value to -1, and the Right X value to 1. The Difference P value then shows the desired probability: 68.27%. Then set the Left X value to –2 and the Right X value to 2. The Difference P value then shows the desired probability: 95.45%. This distribution is saved as a RISKview file titled "Problem 9.11.rvp".

9.12. If the mean is 10 days, then $m = 1/10 = 0.1$.

a. $P_E(T \leq 1 \mid m = 0.1) = 1 - e^{-1(0.1)} = 0.0952.$
Using RISKview, select a function as the distribution source, the Exponential as the distribution type, and 10 for the λ parameter. In the statistics grid, set the Left X value to 1. The Left P value shows the desired probability: 9.52%.

b. $P_E(T \leq 6 \mid m = 0.1) = 1 - e^{-6(0.1)} = 0.4512.$
Change the Left X value to 6. The Left P value shows the desired probability: 45.12%.

c. $P_E(6 \leq T \leq 7 \mid m = 0.1) = e^{-6(0.1)} - e^{-7(0.1)} = 0.0522.$
Set the Left X value to 6 and the Right X value to 7. The Difference P value shows the desired probability: 5.22%.

d. $P(T \leq 7 \mid T \geq 6, m = 0.1)$

$$= \frac{P(T \leq 7 \text{ and } T \geq 6 \mid m = 0.1)}{P(T \geq 6 \mid m = 0.1)}$$

$$= \frac{P_E(6 \leq T \leq 7 \mid m = 0.1)}{P_E(T \geq 6 \mid m = 0.1)}$$

$$= \frac{0.0522}{1 - P_E(T \leq 6 \mid m = 0.1)}$$

$$= 0.0952.$$

This problem demonstrates the "memoryless" property of the exponential distribution; the probability of time T lasting another day is the same no matter how much time has already elapsed. See also problem 9.28. This distribution is saved as a RISKview file titled "Problem 9.12.rvp".

9.13. Because of independence among arrivals, the probability distribution for arrivals over the next 15 minutes is independent of how many arrived previously. Thus, for both questions,

$P_P(X = 1 \text{ in 15 minutes} \mid m = 6 \text{ per hour})$
   $= P_P(X = 1 \text{ in 15 minutes} \mid m = 1.5 \text{ per 15 minutes})$
   $= 0.335 \text{ from Appendix C.}$
Using RISKview, select a function as the distribution source, the Poisson as the distribution type, and 1.5 for the λ parameter. In the statistics grid, set the Left X value to 0.5 and the Right X value to 1.5. The

Difference P value then shows the desired probability: 33.47%. This distribution is saved as a RISKview file titled "Problem 9.13.rvp".

9.14. a. $E(T_A) = 5$ years and $E(T_B) = 10$ years, choose B.

b. $P(T_A \geq 5 \mid m = 0.2) = e^{-5(0.2)} = 0.368$

$P(T_B \geq 10 \mid m = 0.1) = e^{-10(0.1)} = 0.368$.

For exponential random variables, the probability is 0.368 that the random variable will exceed its expected value.

Using RISKview, select a function as the distribution source, the Exponential as the distribution type, and 5 for the λ parameter. In the statistics grid, set the Left X value to 5 and the Right X value to 50. The Difference P value then shows the desired probability: 36.78%. Then, change the λ parameter to 10, and the Left X value to 10 and the Right X value to 100. The Difference P value is the same: 36.78%. This distribution is saved as a RISKview file titled "Problem 9.14.rvp".

c.i. Average lifetime $= 0.5 (T_A) + 0.5 (T_B) = \overline{T}$.

$E(\overline{T}) = 0.5 (5) + 0.5 (10) = 7.5$.

$Var(\overline{T}) = 0.5^2 (25) + 0.5^2 (100) = 31.25$.

ii. Difference $\Delta T = T_B - T_A$

$E(\Delta T) = E(T_B) - E(T_A) = 10 - 5 = 5$

$Var(\Delta T) = 1^2 Var(T_B) + (-1)^2 Var(T_A) = 25 + 100 = 125$.

9.15. a. $P_E(T \geq 2$ hours $\mid m = 8.5$ cars per 10 hours$)$
       $= P_E(T \geq 2$ hours $\mid m = 0.85$ cars per hour$)$
       $= e^{-2(0.85)} = 0.183$.
Using RISKview, select a function as the distribution source, the Exponential as the distribution type, and 1.1765 for the β parameter. In the statistics grid, set the Left X value to 2 and the Right X value to 10. The Difference P value then shows the desired probability: 18.25%. This distribution is saved as a RISKview file titled "Problem 9.15a.rvp".

b. $P_P(X = 0$ in 2 hours $\mid m = 8.5$ cars per 10 hours$)$
       $= P_P(X = 0$ in 2 hours $\mid m = 1.70$ cars per 2 hours$)$
       $= 0.183$.
Using RISKview, select a function as the distribution source, the Poisson as the distribution type, and 1.70 for the λ parameter. In the statistics grid, set the Left X value to 0. The Left P value then shows the desired probability: 18.25%. This distribution is saved as a RISKview file titled "Problem 9.15b.rvp".

This probability must be the same as the answer to part a because the outcome is the same. The first sale happening after 2 hours pass $(T \geq 2)$ is the same as no sales $(X = 0)$ in the first two hours.

c. $E(Bonus) = \$20 \, P_P(X = 13 \mid m = 8.5) + \$30 \, P_P(X = 14 \mid m = 8.5)$

$$+ \$50 \ \text{P}_\text{P}(X = 15 \mid m = 8.5) + \$70 \ \text{P}_\text{P}(X \geq 16 \mid m = 8.5)$$

$$= \$20 \ (0.0395) + \$30 \ (0.024) + \$50 \ (0.0145) + \$70 \ (0.0138)$$
$$= \$3.20$$

Using RISKview, select a function as the distribution source, the Poisson as the distribution type, and 8.5 for the λ parameter. To find the probability that $X = 13$, set the Left X value to 12.5 and the Right X value to 13.5. The Difference P value then shows the desired probability: 3.95%. To find the probability that $X = 14$, set the Left X value to 13.5 and the Right X value to 14.5. The Difference P value then shows the desired probability: 2.4%. To find the probability that $X = 15$, set the Left X value to 14.5 and the Right X value to 15.5. The Difference P value then shows the desired probability: 1.36%. To find the probability that $X \geq 16$, set the Left X value to 15.5 and the Right X value to 25. The Difference P value then shows the desired probability: 1.38%. This distribution is saved as a RISKview file titled "Problem 9.15c.rvp".

d. P(Pay \$20 bonus) $= \text{P}_\text{P}(X = 13 \mid m = 8.5) = 0.0395$. But this is the probability for any given day. There are 8 such days, for each of which P(Pay \$20 bonus) $= 0.0395$. Now we have a binomial problem. Find the probability of 2 out of 8 "successes" when P(success) $= 0.0395$:

$$\text{P}_\text{B}(R = 2 \mid n = 8, p = 0.0395) = 0.033.$$

Using RISKview, select a function as the distribution source, the Binomial as the distribution type, 8 as the n parameter, and 0.395 as the p parameter. Set the Left X value to 2. The Left P value then shows the desired probability: 32.59%. This distribution is saved as a RISKview file titled "Problem 9.15d.rvp".

9.16. a. Assuming independence from one questionnaire item to the next, we can calculate the probability using the binomial distribution:

$$\text{P}_\text{B}(R \leq 2 \mid n = 10, p = 0.90)$$

$$= \text{P}_\text{B}(R' \geq 8 \mid n = 10, p = 0.10)$$

$$= 1 - \text{P}_\text{B}(R' \leq 7 \mid n = 10, p = 0.10)$$

$$= 1 - 0.945$$

$$= 0.055.$$

b. Why might the occurrences *not* be independent? Imagine the following: You have just learned that the first answers fell outside of the ranges that you specified. Would you want to modify the ranges of the remaining five? Most of us would. And that would mean that, given data regarding hits or misses on some, you might change the probability of a hit or miss on the others. Thus, the data — whether each item is a hit or a miss — may not be independent, but only because of uncertainty about the level of calibration of the probability assessor! For a more complete discussion, see Harrison, J.M. (1977), "Independence and Calibration in Decision Analysis," *Management Science*, 24, 320-328.

9.17. a. A hit is defined as market share $= p = 0.3$. A flop is market share $= p = 0.1$. Given $n = 20$ and $x = 4$, we can find, for example,

$$P(x = 4 \mid n = 20, p = 0.3) = \text{P}_\text{B}(R = 4 \mid n = 20, p = 0.3) = 0.130$$
and
$$P(x = 4 \mid n = 20, p = 0.1) = \text{P}_\text{B}(R = 4 \mid n = 20, p = 0.1) = 0.090.$$

Thus,

$$P(\text{Hit} \mid X = 4) = \frac{P(X = 4 \mid \text{Hit})\, P(\text{Hit})}{P(X = 4 \mid \text{Hit})\, P(\text{Hit}) + P(X = 4 \mid \text{Flop})\, P(\text{Flop})}$$

$$= \frac{0.13\,(0.2)}{0.13\,(0.2) + 0.09\,(0.8)} = 0.265.$$

b. For P(Hit) = 0.4,

$$P(\text{Hit} \mid X = 4) = \frac{0.13\,(0.4)}{0.13\,(0.4) + 0.09\,(0.6)} = 0.491.$$

c. For P(Hit) = 0.5,

$$P(\text{Hit} \mid X = 4) = \frac{0.13\,(0.5)}{0.13\,(0.5) + 0.09\,(0.5)} = 0.591.$$

d. For P(Hit) = 0.75,

$$P(\text{Hit} \mid X = 4) = \frac{0.13\,(0.75)}{0.13\,(0.75) + 0.09\,(0.25)} = 0.813.$$

e. For P(Hit) = 0.9,

$$P(\text{Hit} \mid X = 4) = \frac{0.13\,(0.9)}{0.13\,(0.9) + 0.09\,(0.9)} = 0.929.$$

f. For P(Hit) = 1.0,

$$P(\text{Hit} \mid X = 4) = \frac{0.13\,(1.0)}{0.13\,(1.0) + 0.09\,(0)} = 1.0.$$
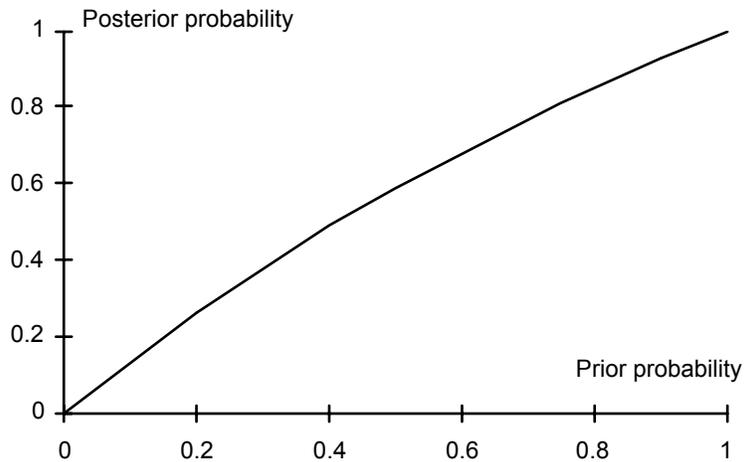
The very slight bow relative to the 45° line indicates that 4 favorable responses out of 20 people lead to a slight but undramatic revision of the prior probability.

9.18. a. $P_B(R > 15 \mid n = 20, p = 0.6) = P_B(R' = 15 - R < 5 \mid n = 20, p = 0.4)$
   $= P_B(R' \le 4 \mid n = 20, p = 0.4) = 0.051.$

To use RISKview, select a Binomial distribution with parameters n = 20 and p = 0.4. Set the Left X value to 15 and the Right X value to 20. The Difference P value shows the desired probability: 5.10%.

b. P(First 12 in favor) = $(0.6)^{12} = 0.0022$

P(Stop after 13th) = P(11 out of 12 in favor and 13th in favor)

$= P_B(R = 11 \mid n = 12, p = 0.6) \times 0.6$

$= P_B(R' = 12 - R = 1 \mid n = 12, p = 0.4) \times 0.6$

$= 0.017 (0.6) = 0.0102.$

c. P(Stop after 18th) = P(11 out of 17 in favor and 18th in favor)

$= P_B(R = 11 \mid n = 17, p = 0.6) \times 0.6$

$= P_B(R' = 17 - R = 1 \mid n = 17, p = 0.4) \times 0.6$

$= 0.002 (0.6) = 0.0012.$

9.19. a. $P_P(X > 2 \mid m = 1.1) = 1 - P_P(X \le 2 \mid m = 1.1) = 1.00 - 0.90 = 0.1.$

To use RISKview, select a Poisson distribution with the parameter $\lambda = 1.1$. Set the Left X value to 2.5 and the Right X value to 4.5. The Difference P value shows the desired probability: 9.41%.

b. Now we have to obtain 18 or more that conform out of 20, given that the probability of conformance equals 0.90 and the probability that a box is rectified is 0.1. This is a binomial model:

P(18 or more pass) = P(2 or fewer are rectified)

$= P_B(R \le 2 \mid n = 20, p = 0.1) = 0.677.$

To use RISKview, select a Binomial distribution with parameters n = 20 and p = 0.0941. Set the Left X value to 2. The Left P value shows the desired probability: 71.04%.

c. Let X be the random number of cases out of 20 that are rectified. Then X cases will have no nonconforming bottles. Also, (20 - X) may have up to 11 non-conforming bottles; we know that at least one bottle in each case is OK. Given X cases are rectified, the expected number of nonconforming bottles out of 20×11 bottles is

$0 (X) + 11 (0.1) (20 - X) = 1.1 (20 - X).$

The expected number of cases rectified are $np = 20(0.1) = 2$. Therefore E(X) = 2, and

E(# of nonconforming bottles) = 1.1 (20 - 2) = 1.1 (18) = 19.8.

9.20. The Poisson process is often used to model situations involving arrivals. The issue here is whether the rate (m) is constant over time. We might expect this rate to vary during a day — higher during lunch hour, for example. It might be possible to break the day into a few different periods, each with its own rate, and use a separate Poisson process for each period.

9.21. a.
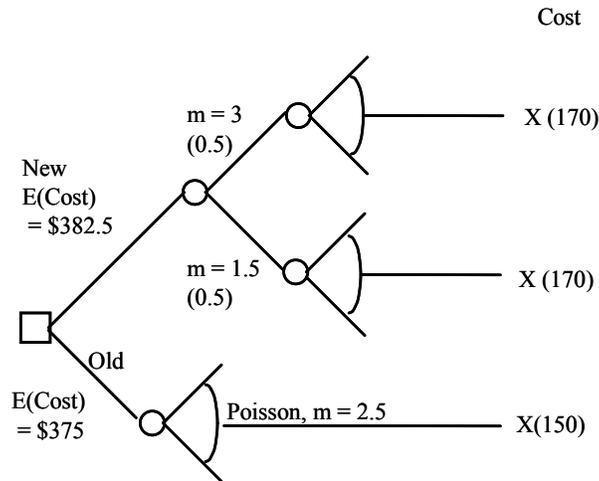
The older machines appear to have the edge. They have both lower expected cost and less uncertainty.

b. Find $P(m = 1.5$ per month $| X = 6$ in 3 months$) = P(m = 4.5$ per 3 months $| X = 6$ in 3 months$)$:

$P(m = 4.5$ per 3 months $| X = 6$ in 3 months$)$

$$= \frac{P(X = 6 \mid m = 4.5)\, P(m = 4.5 \text{ per 3 months})}{P(X = 6 \mid m = 4.5)\, P(m = 4.5\,) + P(X = 6 \mid m = 9)\, P(m = 9)}$$

$$= \frac{\text{Pp}(X = 6 \mid m = 4.5)\, 0.5}{\text{Pp}(X = 6 \mid m = 4.5)\, 0.5 + \text{Pp}(X = 6 \mid m = 9)\, 0.5}$$

$$= \frac{0.128\,(0.50)}{0.128\,(0.50) + 0.091\,(0.50)} = 0.5845.$$

c. Now the expected cost of the new machines would be

$E(\text{Cost}) = 0.5845\,(\$255) + 0.4155\,(\$510) = \$360.96.$

This is less than $375, so the new information suggests that the new machines would be the better choice.

d. The source of the information often provides us with some indication of credibility or biasedness. We might discount information from the distributor because of incentives to sell the product. A trade magazine report might be viewed as more credible or less biased. Does this mean Bayes' theorem is inappropriate? Not entirely, but very simple applications that do not account for bias or differential credibility may be.

    One straightforward way to incorporate the idea of biasedness or credibility would be to think in terms of "equivalent independent information." For example, a positive report from a source you think has a positive bias might be adjusted downward. If the distributor claimed "6 out of 10" preferred the product, say, this might be adjusted down to 5 of 10 or 4 of 10. In other cases, it might be judged that the information presented is to some extent redundant with information already obtained, in which case the reported sample size might be adjusted downward. Thus "6 out of 10" from a redundant sample might be judged to be equivalent to an independent test in which the results were "3 of 5." The "adjusted" or "equivalent independent" sample results could then be used in Bayes' theorem.

9.22. a. $f_\beta(q \mid r = 1, n = 3) = \dfrac{(3\text{-}1)!}{(1\text{-}1)!\,(3\text{-}1\text{-}1)!}\; q^{r\text{-}1}\,(1\text{-}q)^{n\text{-}r\text{-}1}$

$$= \dfrac{2}{1\,(1)}\; q^0\,(1 - q)^1$$

$$= 2\,(1\text{-}\,q).$$

Using RISKview, select a function as the distribution source, the Beta as the distribution type, 1 for the $\alpha1$ parameter, and 2 for the $\alpha2$ parameter.   This distribution is saved as the file "Problem 9.22.rvp".

b.
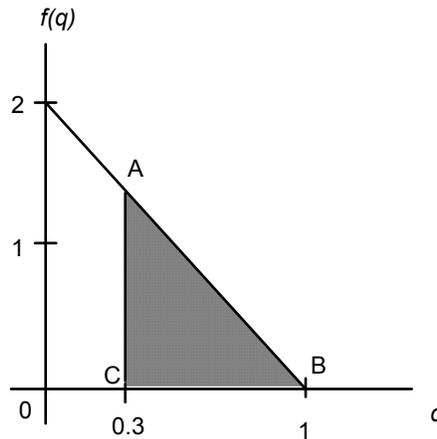
c. $P(Q > 0.30)$ is the area of the triangle ABC. The height AC is $f(0.3) = 2\,(1 - 0.3) = 1.4$. Therefore, the area of the triangle is $(0.7)\,(1.4)\,(0.5) = 0.490$. Because $P(q > 0.30) = 0.490$, it is also the case that $P(q \leq 0.30) = 0.51$, which means that the median is slightly less than 0.30.

    $P(Q > 0.50)$ is found the same way. The height of the triangle is $f(0.5) = 2\,(1 - 0.5) = 1$. Therefore $P(Q > 0.50) = 1\,(0.5)\,(0.5) = 0.25$. Thus, $P(Q < 0.50) = 1 - 0.25 = 0.75$. Thus, $q = 0.50$ is the third quartile for this beta distribution, just as it is for your subjective distribution.

To use RISKview, set the Left X value to 0.3. The Left P value show the desired probability: 51%.

d. $E(\text{profit}) = \$40{,}000\;[E(Q) - \$8500] = \$40{,}000\,(0.33) - \$8500 = \$4833$.

e. $P(\text{Loss}) = P(Q < 0.21) = 1 - P(Q \geq 0.21)$. Find $P(Q \geq 0.21)$ using the same method as in part c. The height of the triangle is $f(0.21) = 2\,(1 - 0.21) = 1.58$. So the area is $(1.58)\,(0.79)\,(0.5) = 0.624$. Thus, $P(\text{Loss}) = 1 - 0.624 = 0.376$. This is a pretty high probability of a loss!

Or with RISKview, set Left X value to 0.21, then the Left P value is 37.59%. Set the Right X value to 1, then the Difference P value is 62.41%.

9.23. All we need to do is to create a model or representation of the uncertainty that adequately captures the essential features of that uncertainty. A requisite model of uncertainty will not be a perfect model, but the aspects that are important will be well modeled. For example, if a normal distribution is used to represent the weights of lab animals, the mean may be 10 grams with standard deviation 0.3 grams. In fact, all of the animals may weigh between 9 and 11 grams. According to the normal distribution, 99.92% of the animals should fall in this range.

9.24. a. The assessments give $P(Q < 0.08) = P(Q > 0.22) = 0.10$ and $P(Q < 0.14) = 0.50$. Therefore,

$P(0.08 < Q < 0.14) = P(Q < 0.14) - P(Q < 0.08) = 0.40.$

Likewise,

$P(0.14 < Q < 0.22) = P(Q < 0.22) - P(Q < 0.14) = 0.40.$

b. $n = 40$, $r = 6$.

$P_\beta(Q \leq 0.0829 \mid r = 6, n = 40) = 0.10$
$P_\beta(Q \leq 0.2249 \mid r = 6, n = 40) = 0.90$
$P_\beta(Q \leq 0.1441 \mid r = 6, n = 40) = 0.50$

To use RISKview, select a function as the distribution source, the Beta as the distribution type, 6 for the $\alpha 1$ parameter, and 34 for the $\alpha 2$ parameter. Set the Left X value to 0.0829, then the Left P value is 10.02%. Set the Left X value to 0.2249, then the Left P value is 90%. Set the Left X value to 0.1441, then the Left P value is 49.97%.

c. $E(Q) = \dfrac{6}{40} = 0.15$. However, there is more than a 50% chance that market share will be less than 0.15.

9.25. a. No, because the second investment is substantially riskier as indicated by the higher standard deviation.

b. It makes some sense. We need a single peak (mode) and a reasonably symmetric distribution for the normal distribution to provide a good fit. Returns can be positive or negative, and we might expect deviations around a central most-likely value to be evenly balanced between positive and negative deviations.

c. $P(R_1 < 0\%) = P_N (R_1 \leq 0 \mid \mu = 10, \sigma = 3)$

$= P(Z \leq \dfrac{0 - 10}{3}) = P(Z < -3.33) = 0.0004.$

To use RISKview, select a function as the distribution source, the Normal as the distribution type, 0.1 for the μ parameter, and 0.3 for the σ parameter. Set the Left X value to 0, then the Left P value is 0.04%.

$P(R_2 < 0\%) = P_N (R_2 \leq 0 \mid \mu = 20, \sigma = 12)$

$= P(Z \leq \dfrac{0 - 20}{12}) = P(Z < -1.67) = 0.0475.$

To use RISKview, select a function as the distribution source, the Normal as the distribution type, 0.2 for the μ parameter, and 0.12 for the σ parameter. Set the Left X value to 0, then the Left P value is 4.78%.

$P(R_1 > 20\%) = P_N (R_1 > 20 \mid \mu = 10, \sigma = 3)$

$= P(Z > \dfrac{20 - 10}{3}) = P(Z > 3.33) = 0.0004.$

To use RISKview, select a function as the distribution source, the Normal as the distribution type, 0.1 for the μ parameter, and 0.3 for the σ parameter. Set the Left X value to 0.2 and the Right X value to 0.5, then the Difference P value is 0.04%.

$P(R_2 < 10\%) = P_N (R_2 < 10 \mid \mu = 20, \sigma = 12)$

$= P(Z \leq \dfrac{10 - 20}{12}) = P(Z < -0.83) = 0.2033.$

To use RISKview, select a function as the distribution source, the Normal as the distribution type, 0.2 for the μ parameter, and 0.12 for the σ parameter. Set the Left X value to 0.1, then the Left P value is 20.23%.

d. Find $P(R_1 > R_2) = P(R_1 - R_2 > 0) = P(\Delta R > 0)$

$$= P_N (\Delta R > 0 \mid \mu = 10 - 20, \sigma = \sqrt{3^2 + 12^2 - 0.5\,(3)\,(12)}\ )$$

$$= P_N (\Delta R > 0 \mid \mu = -10, \sigma = 11.62)$$

$$= P(Z \le \frac{0 - (-10)}{11.62}\ ) = P(Z > 0.86) = 1 - P(Z < 0.86) = 0.1949.$$

e. The probability distributions could be used as the basis for a model about the uncertainty regarding their returns. This uncertainty can be included in a decision tree with appropriate portfolio alternatives.

9.26. Let X denote return (in per cent), M = McDonalds, and S = US Steel. We have prior probability $P(M)$ = 0.80.

a. $P(6 < X < 18 \mid M) = P_N (6 < X < 18 \mid \mu = 14, \sigma = 4)$

$$= P(\frac{6 - 14}{4} < Z < \frac{18 - 14}{4}\ ) = P(-2 < Z < 1) = 0.8185.$$

To use RISKview, select a function as the distribution source, the Normal as the distribution type, 0.14 for the μ parameter, and 0.04 for the σ parameter. Set the Left X value to 0.06 and the Right X value to 0.18. The Difference P value is the desired probability: 81.86%.

$P(6 < X < 18 \mid S) = P_N (6 < X < 18 \mid \mu = 12, \sigma = 3)$

$$= P(\frac{6 - 12}{3} < Z < \frac{18 - 12}{3}\ ) = P(-2 < Z < 2) = 0.9544.$$

To use RISKview, change the μ parameter to 0.12 and the σ parameter to 0.03. Set the Left X value to 0.06 and the Right X value to 0.18. The Difference P value is the desired probability: 95.45%.

b. $P(6 < X < 18) = P(6 < X < 18 \mid M)\,P(M) + P(6 < X < 18 \mid S)\,P(S)$
$$= 0.8185\,(0.8) + 0.9544\,(0.2) = 0.84568.$$

c. $P(X > 12 \mid M) = P_N (X > 12 \mid \mu = 14, \sigma = 4) = P(Z > \frac{12 - 14}{4}\ ) = P(Z > -0.5) = 0.6915.$

To use RISKview, change the μ parameter to 0.14 and the σ parameter to 0.04. Set the Left X value to 0.12 and the Right X value to 0.5. The Difference P value is the desired probability: 69.15%.

$P(X > 12 \mid S) = P_N (X > 12 \mid \mu = 12, \sigma = 3) = P(Z > \frac{12 - 12}{3}\ ) = P(Z > 0) = 0.5.$

To use RISKview, change the μ parameter to 0.12 and the σ parameter to 0.03. Set the Left X value to 0.12 and the Right X value to 0.5. The Difference P value is the desired probability: 50%.

$$P(M \mid X > 12) = \frac{P(X > 12 \mid M)\,P(M)}{P(X > 12 \mid M)\,P(M) + P(X > 12 \mid S)\,P(S)}$$

$$= \frac{0.6915\,(0.8)}{0.6915\,(0.8) + 0.5\,(0.2)} = 0.847.$$

d. $E(\text{Return}) = 0.5\,E(X \mid M) + 0.5\,E(X \mid S) = 0.5\,(14) + 0.5\,(12) = 13\%.$

$\text{Var}(\text{Return}) = 0.5^2\,\text{Var}(X \mid M) + 0.5^2\,\text{Var}(X \mid S) = 0.25\,(4^2) + 0.25\,(3^2) = 6.25.$

9.27. a. This problem introduces the uniform probability distribution. The density function is

$$f_U(x \mid b, a) = \frac{1}{b-a} \text{ when } a \leq x \leq b, \text{ and } 0 \text{ otherwise:}$$

f(x)
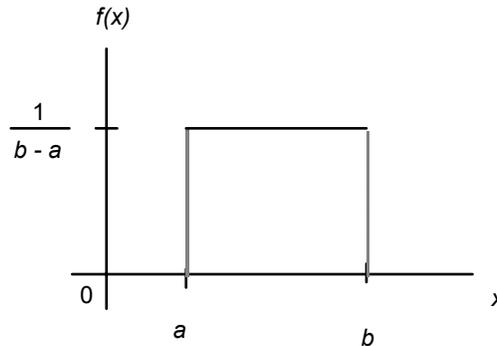
The area under the density function is just the area of the rectangle: $\frac{1}{b-a}(b-a) = 1.$

b. $P_U(X < c \mid a, b) = \frac{c-a}{b-a}$. When $a = 3$ and $b = 5$, then

$$P_U(X < 4.5 \mid a = 3, b = 5) = \frac{4.5 - 3}{5 - 3} = \frac{1.5}{2.0} = 0.75.$$

To use RISKview, select a function as the distribution source, the Uniform as the distribution type, 3 for the min parameter, and 5 for the max parameter. Set the Left X value to 0 and the Right X value to 4.5. The Difference P value is the desired probability: 75%.

c. $P(X < 4.3 \mid a = 3, b = 5) = \frac{4.3 - 3}{5 - 3} = \frac{1.3}{2.0} = 0.65.$

Set the Left X value to 0 and the Right X value to 4.3. The Difference P value is the desired probability: 65%.

$$P(0.25 < X < 0.75 \mid a = 0, b = 1) = \frac{0.75 - 0.25}{1 - 0} = 0.5.$$

Change the min parameter to 0 and the max parameter to 1. Set the Left X value to 0.25 and the Right X value to 0.75. The Difference P value is the desired probability: 50%.

$$P(X > 3.4 \mid a = 0, b = 10) = \frac{10 - 3.4}{10 - 0} = 0.66.$$

Set the min parameter to 0 and the max parameter to 10. Set the Left X value to 3.4 and the Right X value to 10. The Difference P value is the desired probability: 66%.

$$P(X < 0 \mid a = -1, b = 4) = \frac{0 - (-1)}{4 - (-1)} = \frac{1}{5} = 0.2.$$

Set the min parameter to –1 and the max parameter to 4. Set the Left X value to 0. The Left P value is the desired probability: 20%.

d.
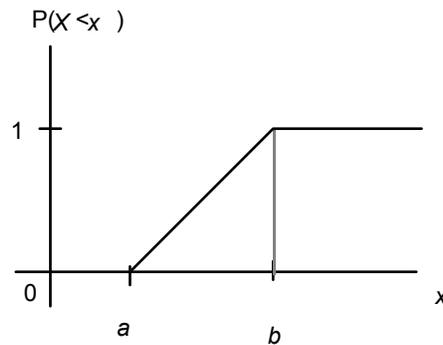
e. $E(X) = \dfrac{a+b}{2} = \dfrac{3+5}{2} = 4.$ $Var(X) = \dfrac{(b-a)^2}{12} = \dfrac{(5-3)^2}{12} = \dfrac{4}{12} = 0.33.$

9.28. a.
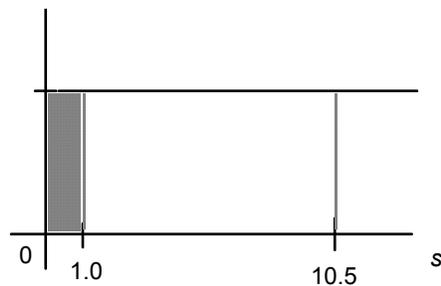
$P(S < 1) = (\text{height} \times \text{width}) = 1 \infty \dfrac{1}{10.5} = 0.0952.$

To use RISKview, select a function as the distribution source, the Uniform as the distribution type, 0 for the min parameter, and 10.5 for the max parameter. Set the Left X value to 1. The Left P value is the desired probability: 9.52%.

b. $P(S < 6) = 6 \times \dfrac{1}{10.5} = 0.5714.$

Set the Left X value to 6. The Left P value is the desired probability: 57.14%.

c. $P(6 < S < 7) = 1 \times \dfrac{1}{10.5} = 0.0952.$

Set the Left X value to 6 and the Right X value to 7. The Difference P value is the desired probability: 9.52%.

d. $P(S \le 7 \mid S > 6) = \dfrac{P(6 < S < 7)}{P(S > 6)} = \dfrac{0.0952}{1 - 0.5714} = 0.2222.$

Note that this result is different from the probability in part a. The uniform distribution does not have a "memoryless" property like the exponential does (problem 9.12).

9.29. a.
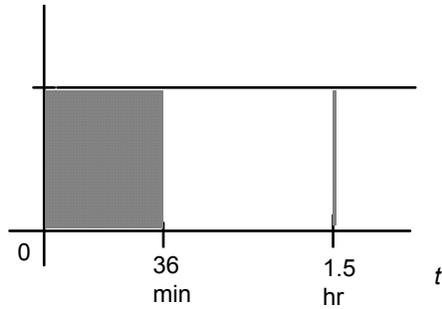
$$P(T \le 36) = \frac{36}{90} = 0.40.$$

To use RISKview, select a function as the distribution source, the Uniform as the distribution type, 0 for the min parameter, and 1.5 for the max parameter. Set the Left X value to 0.6. The Left P value is the desired probability: 40%.

b. Use a binomial model with $p = 0.4$. Each of 18 customers has a 0.4 chance of being a "success" (shopping for 36 minutes or less). We require the probability that 10 or more are "successes" out of 18:

$$P_B(R \ge 10 \mid n = 18, p = 0.4) = 1 - P_B(R \le 9 \mid n = 18, p = 0.4) = 1 - 0.865 = 0.135.$$

To use RISKview, select a function as the distribution source, the Binomial as the distribution type, 18 for the n parameter, and 0.4 for the p parameter. Set the Left X value to 9 and the Right X value to 18. The Difference P value is the desired probability: 13.47%.

9.30. a.
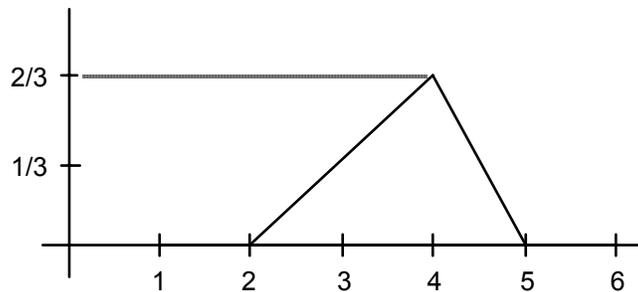
b. The probability we want is the shaded area in the graph below:
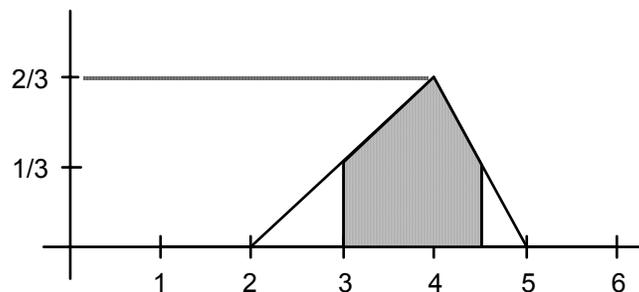
The easiest way to calculate this area is to calculate the areas of the two triangles on the ends and subtract these areas from 1.00. Recalling that the area of a triangle is one-half times its height times its base,

$$P_T(x \mid 3 \le x \le 4.5) = 1 - P_T(x \mid x \le 3) - P_T(x \mid 4.5 \le x)$$

$$= 1 - [1 \ (1/3) \ (1/2)] - [\ 0.5 \ (1/3) \ (1/2)]$$

$$= 1 - 1/6 - 1/12$$

$$= 9/12 = 3/4$$

To use RISKview, select a function as the distribution source, the Triangular as the distribution type, 2 for the min parameter, 4 for the most likely value, and 5 for the max parameter. Set the Left X value to 3 and the Right X value to 4.5. The Difference P value is the desired probability: 75%.

c. This problem requires us to find a point on the x axis such that the area under the triangle on the left is equal to 0.50 (and hence the area to the right also is 0.50):
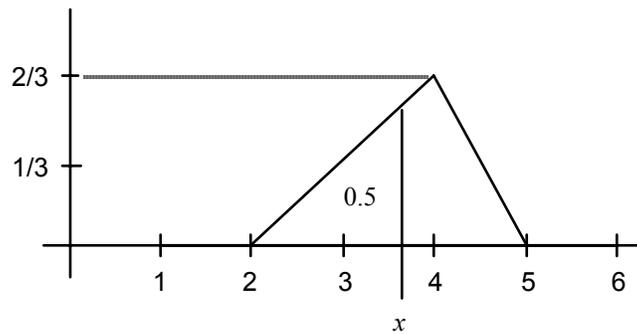
Finding this area requires us to find an $x_{0.5}$ that satisfies the equation defining the area of the triangle. The base is $x_{0.5}$ - 2, and the height is the height of the density curve at $x_{0.5}$, or $x_{0.5}/3$ - 2/3), so the equation is:

$$0.50 = \frac{(x_{0.5} - 2)(x_{0.5}/3 - 2/3)}{2}$$

$$= \frac{(x_{0.5} - 2)(x - 2)}{6}$$

$$3 = (x_{0.5} - 2)^2$$

$$x_{0.5} = 2 + \sqrt{3} = 3.73.$$

Using RISKview, set the Left P value to 50%, then the Left X value shows the desired x: 3.7321.

d. This problem requires the same techniques as in part c. The lower quartile is defined by the value of $x_{0.25}$ such that

$$0.25 = \frac{(x_{0.25} - 2)(x_{0.25}/3 - 2/3)}{2}$$

Solving by exactly the same steps as above gives $x_{0.25} = 2 + \sqrt{1.5} = 3.22$.

The equation for the upper quartile would use the line for the upper end of the density function, $-2x/3 + 10/3$. Thus, the equation would be

$$0.25 = \frac{(5 - x_{0.75})(-2x_{0.75}/3 + 10/3)}{2}$$

Solving this equation for $x_{0.75}$ gives

$$x_{0.75} = 5 - \sqrt{0.75} = 4.13.$$

Using RISKview, set the Left P value to 25%, then the Left X value shows the desired x: 3.2247. Set the Right P value to 75%, then the Right X value shows the desired x: 4.134.

9.31. Let L denote the uncertain length of an envelope.

a. $P_N (L > 5.975 \mid \mu = 5.9, \sigma = 0.0365) = P(Z > \dfrac{5.975 - 5.9}{0.0365}) = P(Z > 2.055) = 0.02.$

To use RISKview, select a function as the distribution source, the Normal as the distribution type, 5.9 for the μ parameter, and 0.0365 for the σ parameter. Set the Left X value to 5.975 and the Right X value to 8. The Difference P value is the desired probability: 1.99%.

b. We will use a binomial model, in which $p = P$(Envelope fits) = 0.98. We need

$P_B(R \leq 18 \mid n = 20, p = 0.98) = P_B(R \geq 2 \mid n = 20, p = 0.02)$

$= 1 - P_B(R \leq 1 \mid n = 20, p = 0.02) = 1 - 0.94 = 0.06.$

That is, about 6% of the boxes will contain 2 or more cards that do not fit in the envelopes.

To use RISKview, select a function as the distribution source, the Binomial as the distribution type, 20 for the n parameter, and 0.98 for the p parameter. Set the Left X value to 18. The Left P value is the desired probability: 5.99%.

9.32. a. Reasons for using the Poisson distribution include:
- Machines often break down independently of one another.
- The probability of a breakdown is small.
- The rate of breakdowns is roughly constant over time.
- Machines can break down at any point in time.

b. There are many ways to solve this problem. The simplest is to look in the tables for a Poisson distribution that gives approximately the same probabilities. Using $m = 0.70$ provides a close fit:

$P(X = 0) = 0.50$	$P_P(X = 0 \mid m = 0.70) = 0.497$
$P(X = 1) = 0.35$	$P_P(X = 1 \mid m = 0.70) = 0.348$
$P(X = 2$ or $3) = 0.15$	$P_P(X = 2 \, 04 \, 3 \mid m = 0.70) = 0.150$
$P(X \geq 4) = 0$	$P_P(X \geq 4 \mid m = 0.70) = 0.006$

Another approach is to use the Poisson formula on one of the probabilities. The probability that $X = 0$ is the easy one:

$$P(X = 0) = 0.5 = \frac{e^{-m} m^0}{0!} = e^{-m}.$$

Solve for m to get $m = 0.693$.

Using RISKview, select Poisson as the distribution type and use the spin buttons to alter the λ parameter until the distribution is similar to the desired probabilities ($\lambda = 0.7$).

c. The expected number of breakdowns is m, whatever value was chosen.

9.33.
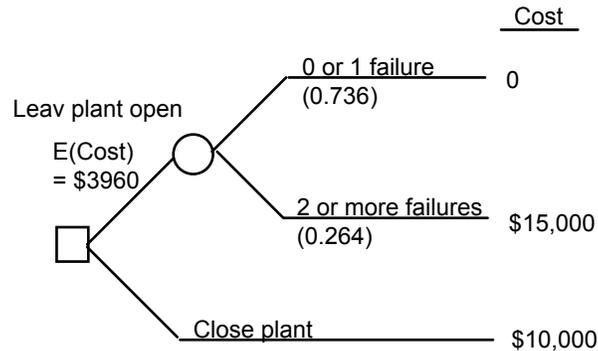
$$P(0 \text{ or } 1 \text{ failure}) = P_P(X = 0 \mid m = 1) + P_P(X = 1 \mid m = 1) = 0.368 + 0.368 = 0.736.$$
$$P(2 \text{ or more failures}) = 1 - P(0 \text{ or } 1 \text{ failure}) = 1 - 0.736 = 0.264.$$

Because E(Cost) for leaving the plant open is $3960 < $10,000, the choice would be to leave the plant open.

To use RISKview, select a function as the distribution source, the Poisson as the distribution type, and 1 for the λ parameter. To find the probability of no failures, set the Left X value to 0.5, then the Left P value is the desired probability: 36.79%. To find the probability of one failure, set the Left X value to 0.5 and the Right X value to 1.5, then the Difference P value is the desired probability: 36.79%.

9.34. The expected number of people to arrive in 30 minutes is 7.5. Thus, a Poisson Process with $m = 7.5$ for is a reasonable model for the arrival process. Because only 40% of the arrivals prefer the new recipe, and their preferences are judged to be independent of the arrivals, a Poisson process with $m = 7.5 \times 0.4 = 3.0$ is a reasonable model for the arrival of customers who prefer the new recipe. Thus, $P(X \geq 4 \mid m = 3.0) = 0.353$.

Using RISKview, select a function as the distribution source, the Poisson as the distribution type, and 3 for the λ parameter. To find the probability of more than 4 people, set the Left X value to 3.5 and the Right X value to 12. The Difference P value is the desired probability: 35.28%.

An alternative approach (and a more rigorous one) would use the binomial model explicitly:

P(4 or more people who prefer the new recipe arrive in 30 minutes)
$$= P_P(X = 4 \mid m = 7.5) \, P_B(R = 4 \mid n = 4, p = 0.4)$$
$$+ P_P(X = 5 \mid m = 7.5) \, P_B(R \geq 4 \mid n = 5, p = 0.4)$$
$$+ P_P(X = 6 \mid m = 7.5) \, P_B(R \geq 4 \mid n = 6, p = 0.4)$$
$$+ P_P(X = 7 \mid m = 7.5) \, P_B(R \geq 4 \mid n = 7, p = 0.4)$$
$$+ \ldots$$

These probabilities can be calculated quite easily using the Poisson and binomial formulas. The series must be calculated up to about $X = 16$ before the individual terms become insignificant.

9.35. a, b. Reasons for using the binomial distribution are the following: The number of machines to break down can be between 0 and 50. The probability of any machine breaking down is 0.004. The machines appear to break down independently of each other. Thus, a binomial distribution with $p = 0.004$ and $n = 50$ would be appropriate.

Reasons for using the Poisson distribution: The probability of an individual machine breaking down is small and can occur any time during the day. Breakdowns seem to occur independently of one another. The expected number of breakdowns in one day is $50 \times 0.004 = 0.2$, so a Poisson distribution with $m = 0.2$ would be appropriate.

c.

	# parts required (breakdowns)	Cost ($)

Stock 0 parts — E(Cost) = $13.00

0 (0.819)	0
1 (0.164)	65
2 (0.016)	130
3 (0.001)	195

Stock 1 part — E(Cost) = $11.24

0 (0.819)	10
1 (0.164)	10
2 (0.016)	75
3 (0.001)	140

Stock 2 parts — E(Cost) = $20.13

0 (0.819)	20
1 (0.164)	20
2 (0.016)	20
3 (0.001)	85
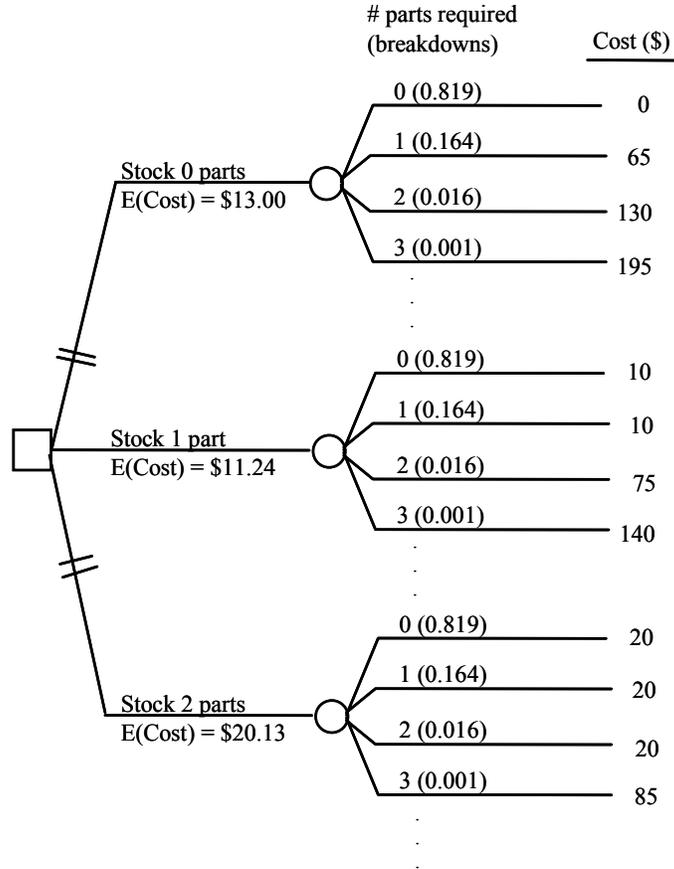
Calculations for E(Cost):

E(Cost | Stock 0) = E(Breakdowns) $\times$ $65.00 = 0.2 \times$ $65.00 = $13.00

E(Cost | Stock 1) = $10 + $65 $\times \sum_{x=1}^{50} [x - 1]P(X = x)$

$\quad$ = $10 + $65 $\times [\sum_{x=1}^{50} x\, P(X = x) - \sum_{x=1}^{50} P(X = x)]$

$\quad$ = $10 + $65 $\times$ [E(# breakdowns) - {1 - P(X = 0)}]

$\quad$ = $10 + $65 $\times$ [0.2 - (1 - 0.819)]

$\quad$ = $11.235.

E(Cost | Stock 2) = $20 + $65 $\times \sum_{x=2}^{50} [x - 2]P(X = x)$

$$= \$20 + \$65 \times [\sum_{x=2}^{50} x\, P(X = x) - 2 \sum_{x=2}^{50} P(X = x)]$$

$$= \$20 + \$65 \times [\sum_{x=1}^{50} x\, P(X = x) - P(X = 1) - 2\, \{1 - P(X = 0) - P(X = 1)\}]$$

$$= \$20 + \$65 \times [E(\# \text{ breakdowns}) - P(X = 1) - 2\, \{1 - P(X = 0) - P(X = 1)\}]$$

$$= \$20 + \$65 \times [0.2 - 0.164 - 2\, \{1 - 0.819 - 0.164\}]$$

$$= \$20.13.$$

This decision tree is modeled in the Excel file "Problem 9.35.xls". The Binomial distribution is included in the spreadsheet model using the Excel function BINOMDIST. For example, to find the probability of no breakdowns use the formula "=BINOMDIST(0,50,0.004,FALSE)".

More possible outcomes are included in the spreadsheet model (up to 7 parts required due to breakdowns) therefore there is some round-off difference between the example above and the answers in the spreadsheet. The preferred alternative is to stock 1 part with an expected cost of $12.20.

9.36. a. $E(X) = e^{[10 + 0.5\, (0.09)]} = \$23{,}040.$

$$\text{Var}(X) = e^{2(10)}\, (e^{0.09} - 1)\, (e^{0.09}) = 49{,}992{,}915.95$$

$$\sigma_X = \$7{,}071.$$

b. $P_L\, (X > 50{,}000 \mid \mu = 10, \sigma = 0.3) = P_N\, (Y = \ln(X) > \ln(50{,}000) \mid \mu = 10, \sigma = 0.3)$

$$= P_N\, (Y > 10.8198 \mid \mu = 10, \sigma = 0.3)$$

$$= P(Z > \frac{10.8198 - 10}{0.3}) = P(Z > 2.73) = 0.0032.$$

c. First, find the probability distribution for $Q = 200X$. According to the hint, Q will be approximately normal with mean $\mu = 200\, (23,0404) = \$4.608$ million, variance $\sigma^2 = 200\, (49{,}992{,}915.95) = 9{,}998{,}583{,}190$, and standard deviation $\sigma = \$99{,}992.91$, or about $0.1 million.
    Now find the 0.95 fractile of this normal distribution. That is, find the value q such that $P_N\, (Q \leq q \mid = 4.608, \sigma = 0.1) = 0.95$. We know that $P(Z \leq 1.645) = 0.95$. Therefore,

$$P(Z \leq \frac{q - 4.608}{0.1}) = 0.95$$

only if
$$\frac{q - 4.608}{0.1} = 1.645$$
or
$$q = 4.608 + 1.645\, (0.1) = \$4.772 \text{ million.}$$

Thus, if the company has on hand $4.772 million, they should be able to satisfy all claims with 95% probability.

Case Study: Overbooking

1. Use a binomial distribution, and let $Res = n = $ number of reservations sold.

$$P_B(R > 16 \mid Res = 17, p = 0.96) = P_B(R = 0 \mid n = 17, p = 0.04) = 0.4996.$$

$$P_B(R > 16 \mid Res = 18, p = 0.96) = P_B(R \leq 1 \mid n = 18, p = 0.04) = 0.8393.$$

$$P_B(R > 16 \mid Res = 19, p = 0.96) = P_B(R \leq 2 \mid n = 19, p = 0.04) = 0.9616.$$

This model is shown in the Excel file "Overbooking I.xls". The probabilities for possible arrivals given a certain number of reservations are shown in a table in the second worksheet in the file. For example, the probability that 17 people show-up given 17 reservations are taken (i.e., 0 no-shows) can be found using the Excel formula " =BINOMDIST(0,17,0.04,TRUE)". RISKview can be used to determine this value, but you would then need to manually transfer the values from RISKview into the spreadsheet model, while Excel provides useful formulas for many of the distributions. This case and the following cases use the built-in Excel functions rather than finding the value in RISKview and typing it into the spreadsheet.

2. $E(R \mid Res = 16) = \$225\,(16) = \$3600$

$$E(C_1 \mid Res = 16) = \$900 + \$100 \sum_{x=0}^{16} x\, P_B(X = x \mid n = 16, p = 0.96)$$
$$= \$900 + \$100\, E(X)$$
$$= \$900 + \$100\,(15.36)$$
$$= \$2436.$$

$E(C_2 \mid Res = 16) = 0$ (No extra passengers can arrive.)

$E(\text{Profit} \mid Res = 16) = \$3600 - \$2436 - \$0 = \$1164.$

These calculations are shown in the first worksheet in the Excel file.

3. $E(R \mid Res = 17) = \$225\,(17) = \$3825$

$$E(C_1 \mid Res = 17) = \$900 + \$100 \sum_{x=0}^{16} x\, P_B(X = x \mid n = 17, p = 0.96)$$

$$+ \$1600\, P_B(X = 17 \mid n = 17, p = 0.96)$$

$$= \$900 + \$782.70 + \$799.34 = \$2482.04$$

$$E(C_2 \mid Res = 17) = \$325\, P_B(X = 17 \mid n = 17, p = 0.96)$$

$$= \$325\,(0.4996) = \$162.37$$

$E(\text{Profit} \mid Res = 17) = \$3825.00 - \$2482.04 - \$162.37 = \$1180.59.$

$$E(R \mid Res = 18) = \$225\,(18) = \$4050$$

$$E(C_1 \mid Res = 18) = \$900 + \$100 \sum_{x=0}^{16} x\, P_B(X = x \mid n = 18, p = 0.96)$$

$$+ \$1600\, P_B(X \geq 17 \mid n = 18, p = 0.96)$$

$$= \$900 + \$253.22 + \$1342.89 = \$2496.11$$

$E(C_2 \mid Res = 18) = \$325 \, P_B(X = 17 \mid n = 18, p = 0.96)$

$+ \$650 \, P_B(X = 18 \mid n = 18, p = 0.96)$

$= \$325 \, (0.3597) + \$650 \, (0.4796) = \$428.65$

$E(\text{Profit} \mid Res = 18) = \$4050.00 - \$2496.11 - \$428.65 = \$1125.24.$

$E(R \mid Res = 19) = \$225 \, (19) = \$4275$

$E(C_1 \mid Res = 19) = \$900 + \$100 \sum_{x=0}^{16} x \, P_B(X = x \mid n = 19, p = 0.96)$

$+ \$1600 \, P_B(X \geq 17 \mid n = 19, p = 0.96)$

$= \$900 + \$60.74 + \$1538.56 = \$2499.30$

$E(C_2 \mid Res = 19) = \$325 \, P_B(X = 17 \mid n = 19, p = 0.96)$

$+ \$650 \, P_B(X = 18 \mid n = 19, p = 0.96)$

$+ \$975 \, P_B(X = 19 \mid n = 19, p = 0.96)$

$= \$325 \, (0.1367) + \$650 \, (0.3645) + \$975 \, (0.4604) = \$730.26$

$E(\text{Profit} \mid Res = 19) = \$4275.00 - \$2499.30 - \$730.26 = \$1045.44.$

The optimum amount to overbook is by one seat. By selling 17 reservations, Mockingbird obtains the highest expected profit.

Case Study: Earthquake Prediction
The only reasonable way to perform the calculations necessary in this case is to use a computer. A recursive formula for calculating Poisson probabilities is helpful:

$$P_P(X = k + 1 \mid m) = \frac{e^{-m} m^{k+1}}{(k+1)!} = \frac{e^{-m} m^k}{k!} \left(\frac{m}{k+1} \right) = P_P(X = k \mid m) \left(\frac{m}{k+1} \right).$$

A recursive formula like this can be entered and calculated easily in a spreadsheet. It saves laborious calculations of many factorial terms. Also, Excel provides many useful functions to solve for the probabilities. This model is shown in the Excel file "Earthquake Prediction.xls".

1. $P_P(X \leq 10 \text{ in next year} \mid m = 24.93/\text{year}) = P_P(X \leq 10 \mid m = 24.93) = 0.000613$
Or this value can be found using the Excel formula "=POISSON(10,24.93,TRUE)"

$P_P(X \leq 7 \text{ in next 6 months} \mid m = 24.93/\text{year}) = P_P(X \leq 7 \mid m = 12.465) = 0.071069$
This value can be found using the Excel formula "=POISSON(7,12.465,TRUE)"

$P_P(X > 3 \text{ in next month} \mid m = 24.93/\text{year}) = 1 - P_P(X \leq 3 \mid m = 2.0775) = 0.157125$
This value can be found using the Excel formula "=1-POISSON(3,2.0775,TRUE)"

These formulas are shown in the first worksheet of the Excel file.

2.

f(x)

Density function for earthquake magnitude
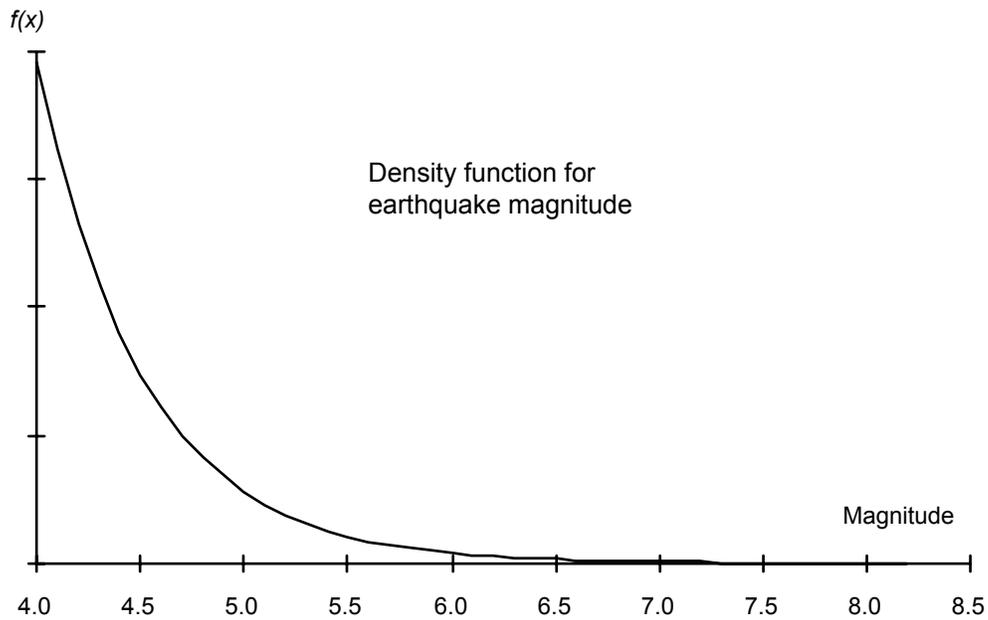
$P_E(M \leq 6.0 \mid m = 1.96) = 1 - e^{-1.96(6.0\text{-}4.0)} = 0.980$

$P_E(5.0 \leq M \leq 7.5 \mid m = 1.96) = e^{-1.96(5.0\text{-}4.0)} - e^{-1.96(7.5\text{-}4.0)} = 0.140$

$P_E(M \geq 6.4 \mid m = 1.96) = e^{-1.96(6.4\text{-}4.0)} = 0.009$

Range	Probability
4.0 - 5.0	0.859
5.0 - 6.0	0.121
6.0 - 7.0	0.017
7.0 - 8.0	0.002
8.0 +	0.001

Sensitivity analysis: Find *m* that corresponds to the data in Table 9.1:

Magnitude (x)	Empirical $P(M \geq x)$	Exponential rate m
8.0+	$\frac{1}{2493} = 0.0004$	1.96
7.0	$\frac{13}{2493} = 0.0052$	1.75
6.0	$\frac{93}{2493} = 0.0373$	1.64
5.0	$\frac{493}{2493} = 0.1978$	1.62

Thus, *m* could be less than 1.96. What value should be chosen for the model? A value of 1.96 makes sense if the concern is primarily with large earthquakes. Furthermore, this choice will result in larger probabilities of large earthquakes, a kind of "worst case" scenario.

These calculations are shown in the second worksheet of the Excel file "Earthquake Prediction.xls". Also, the probability as a function of m is implemented such that m can be varied to see the impact on the probability.

3. Performing the calculations as described in the case gives

P(At least 1 quake with $M > 8$ in next year) ≈ 0.01

P(At least 1 quake with $M > 8$ in next 5 years) ≈ 0.124.

These calculations are shown in the third worksheet of the Excel file "Earthquake Prediction.xls".

4. This question asks students to think about preparation for serious earthquakes. Clearly, the potential exists for substantial damage. Other than the issues raised in the case, one would want to look at training for residents, protection for utilities, emergency measures for providing power and water to hospitals, and distributing water, food, shelter, and other necessities in the event of a severe quake. This question may stimulate an interesting discussion, especially if some of the students have experienced a large quake.

Case Study: Municipal Solid Waste
1. The pollutant levels follow lognormal distributions, and so the log-pollutant levels follow normal distributions with corresponding parameters. Thus, we need to find the probability that the log-pollutant level exceeds the logarithm of the corresponding established level from Table 9.3. This can be done using the Normal Distribution Probability Tables or with Excel functions. This case is modeled in the spreadsheet "Municipal Solid Waste.xls".

For a small plant:
 • Dioxin/Furan (DF) permit level = 500 ng/Nm^3

P(DF > 500 ng/Nm^3) = P$_N$ (ln(DF) > ln(500) | $\mu = 3.13$, $\sigma = 1.2$)

$$= P(Z \le \frac{\ln(500) - 3.13}{1.2}) = P(Z > 2.57) = 0.0051.$$

Or with the Excel function:

=1-NORMDIST(LN(500),3.13, 1.2, TRUE)
= 0.0051

 • Particulate Matter (PM) permit level = 69 mg/dscm

P(PM > 69 mg/dscm) = P$_N$ (ln(PM) > ln(69) | $\mu = 3.43$, $\sigma = 0.44$)

$$= P(Z \le \frac{\ln(69) - 3.43}{0.44}) = P(Z > 1.83) = 0.0338.$$

Or with the Excel function:

=1-NORMDIST(LN(69),3.43, 0.44, TRUE)
= 0.0338

For a medium plant:

- DF permit level = 125 ng/Nm^3

$P(DF > 125 \text{ ng/Nm}^3) = P_N (\ln(DF) > \ln(125) \mid \mu = 3.13, \sigma = 1.2)$

$= P(Z \leq \dfrac{\ln(125) - 3.13}{1.2}) = P(Z > 1.42) = 0.0785.$

Or with the Excel function:

=1-NORMDIST(LN(125),3.13, 1.2, TRUE)
= 0.0785

- Particulate Matter: Analysis is the same as for the small plant.

2. SO$_2$ permit level = 30 ppmdv

For a single observation, the analysis follows question 1:

$P(SO_2 > 30 \text{ ppmdv}) = P_N (\ln(SO_2) > \ln(30) \mid \mu = 3.2, \sigma = 0.39)$

$= P(Z \leq \dfrac{\ln(30) - 3.2}{0.39}) = P(Z > 0.52) = 0.3030.$

Or with the Excel function:

=1-NORMDIST(LN(30),3.2, 0.39, TRUE)
= 0.3030

For a geometric average of 24 independent observations of SO$_2$, denoted by $\overline{S}$,

$P(\overline{S} > 30 \text{ ppmdv}) = P_N (\ln(\overline{S}) > \ln(30) \mid \mu = 3.2, \sigma = \dfrac{0.39}{\sqrt{24}})$

$= P(Z \leq \dfrac{\ln(30) - 3.2}{0.39 / \sqrt{24}}) = P(Z > 2.53) = 0.0057.$

Or with the Excel function:

=1-NORMDIST(LN(30),3.2, 0.0796, TRUE)
= 0.0057

Naturally, this is a much smaller probability than for the single-observation case. It is due to the effect of the Central Limit Theorem on the distribution of averages.

3. They appear to be able to satisfy the less strict requirements for a small plant more easily. However, for a medium plant they also appear to be in good shape; the highest probability of noncompliance is for Dioxins and Furans (0.0785). A larger plant might prove to be better in the long run because of extra capacity and the stricter incineration standards. In addition, the officials should consider the effects of recycling program growth, measures that would require recyclable packaging, and other potential impacts on waste disposal needs.