QUESTION 9.28

$$T = Length\,of\,Strike\,in\,days$$

$$T \sim Uniform[0, 10.5]$$

$$f(t|a,b) = \frac{1}{(b-a)},\ a = 0,\ b = 10.5$$

a.

$$Pr(T \leq 1) = \int_0^1 f(u|a=0, b=10.5)du = \frac{1-a}{b-a} = \frac{1}{10.5} \approx 0.095$$

b.

$$Pr(T \leq 6) = \int_0^6 f(u|a = 0, b = 10.5)du = \frac{6 - a}{b - a} = \frac{6}{10.5}$$

c.

$$Pr(6 \leq T \leq 7) = \int_6^7 \frac{1}{b - a} du = \frac{7 - 6}{b - a} = \frac{1}{10.5}$$

d.

$$Pr(T \leq 7 | T > 6) = \frac{Pr(T \leq 7, T > 6)}{Pr(T > 6)} =$$

$$\frac{Pr(6 \leq T \leq 7)}{1 - Pr(T \leq 6)} = \frac{\frac{1}{10.5}}{1 - \frac{6}{10.5}} \approx 0.22$$

In Question 9.12 use:

$$T = Length\, of\, Strike\, in\, days$$

$$f(t|\lambda) = \lambda exp(-\lambda t), t \geq 0$$

$$E[T] = 10 = \frac{1}{\lambda} \Leftrightarrow \lambda = \frac{1}{10}$$

$$Pr(T \leq 1) = \int_0^1 \lambda exp(-\lambda u) du = 1 - exp(-0.1) \approx 0.095$$

$\boldsymbol{b.}$

$$Pr(T \leq 6) = \int_0^6 \lambda exp(-\lambda u)du = 1 - exp(-0.1 \times 6) \approx 0.45$$

$\boldsymbol{c.}$

$$Pr(6 \leq T \leq 7) = \int_6^7 \lambda exp(-\lambda u)du =$$

$$exp(-0.1 \times 7) - exp(-0.1 \times 6) \approx 0.05$$

$d.$

$$Pr(T \leq 7|T > 6) = \frac{Pr(T \leq 7, T > 6)}{Pr(T > 6)} =$$

$$\frac{Pr(6 \leq T \leq 7)}{1 - Pr(T \leq 6)} = \frac{0.05}{1 - 0.45} \approx 0.095$$

Solution Question 9.31

a. Define: L = Width of a Post Card

Then: $L \sim Normal(\mu, \sigma)$ where $\mu = 5.9 in$ and $\sigma = 0.0365\, in$

Pr(Card does not fit in Envelope)=

$$Pr(L > 5.975 | \mu = 5.9, \sigma = 0.0365) =$$

$$1 - Pr(L \leq 5.975 | \mu = 5.9, \sigma = 0.0365) =$$

$$1 - Pr(\tfrac{L-5.9}{0.0365} \leq \tfrac{5.975-5.9}{0.0365} | \mu = 5.9, \sigma = 0.0365) = 1 - Pr(Z \leq 2.05)$$

Where: $Z \sim Normal(0, 1)$

Utilizing the Table on Page 709:

$$Pr(L > 5.975 | \mu = 5.9, \sigma = 0.0365) = 1 - 0.9798 = 0.0202$$

b. Define: X = Number of Cards in a box of 20 that do not fit in envelope

Then: $X \sim Binom(20, p = 0.02)$

$$Pr(X \geq 2 | n = 20, p = 0.02) = 1 - Pr(X \leq 1 | n = 20, p = 0.02)$$
$$= 1 - 0.94 = 0.06$$ (Table Page 694)

Hence, there is 6% probability that 2 or more cards will not fit.