12.2.

\[
\begin{align*}
\text{EMV}(A) &= 7.00 \\
\text{EMV}(B) &= 6.00 \\
\text{EMV}(\text{Info}) &= 8.20 \\
\end{align*}
\]

\[\text{EVPI} = \text{EMV}(\text{Info}) - \text{EMV}(A) = 8.20 - 7.00 = 1.20.\]

This decision tree model is saved in the Excel file “Problem 12.2.xls”.

12.3. EVPI = 0 because one would still choose A regardless of A’s outcome. To state it slightly differently, no matter what you found out about A’s outcome, your decision would still be the same: Choose A. Because the information cannot change the decision, the expected value of the information equals zero.
12.4. a. The following decision trees are saved in the Excel file “Problem 12.4.xls”. Each part is shown in a separate worksheet.

Perfect Information about E
EMV(Info about E) = 6.24

EMV(A) = 3.00
EMV(B) = 3.20

EVP(I)(Information about E) = EMV(Info) - EMV(B) = 6.24 - 3.20 = 3.04
b.

Perfect Information about F

EMV(Info about F) = 4.4

EMV(A) = 3.0

EMV(B) = 3.2

B = 5
(0.7)

B = -1
(0.3)

EVPI(Information about F) = EMV(Info) - EMV(B) = 4.4 - 3.2 = 1.2
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EVPI(Information about both E and F) = EMV(Info) - EMV(B) = 6.42 - 3.2 = 3.22.

12.5. The basic influence diagram is:
For 12.5a, b, and c, add arrows appropriately representing the information obtained:

These models are saved in separate worksheets in the Excel file “Problem 12.5.xls”. To find the value of the information, you need to subtract the EV of the model without information from the EV of the model with information.

12.6. a. Of course, different people will have different feelings on this one. Personally, I would prefer that the doctor wait to inform me until after the other tests have been performed. (This may not be possible if the further tests require additional blood samples or other interventions; I would certainly know that something was going on.) Why wait? I would worry about the outcome of the other tests. I would just as soon not know that they were even being performed.

b. Suppose I know of no such defects. In this state of information, I can legitimately give my house a clean bill of health, given this state of knowledge. Now, suppose that I learn from the engineering report that the house has a defect and also that my buyer withdraws from the agreement to purchase. Now I would have to reveal the defect to any subsequent purchaser, and it would most likely result in a lower negotiated price. Under the circumstances, even though future buyers may also request an inspection that reveals the defect, I would rather not know the engineer’s report; this state of knowledge gives me a better chance at a better sales price.

c. The answer to this question really depends on your negotiation skill. If the seller knows that you have had the building appraised, he knows that you have a very clear bottom line, and he can be very tough in the negotiations to try to get you to make concessions until the price is right at the bottom line. If you have the appraisal in hand, you will have to do your best not to reveal anything about the appraised value through your behavior and sequence of counteroffers. Without the appraisal, you would not have such a clear view. (But then, you might end up purchasing the building for more than it is worth).
Perhaps the best situation is for your boss to have the appraisal but not to reveal it to you, the agent doing the negotiating. This way, you can negotiate as well as you can, and if the agreed-upon price is too high, the boss can disapprove.

12.7. This decision model is saved in the Excel file “Problem 12.7.xls”. The decision tree for the decision whether to drill or not is shown in the first worksheet. The decision tree for parts a and c:

\[
\begin{align*}
\text{EMV} &= \$10 \text{ K} \\
\text{EMV} &= \$19 \text{ K}
\end{align*}
\]

**a.** The expected value of drilling is $10 K, versus $0 for not drilling, so choose to drill.

**b.** The influence diagram representation is shown in the second worksheet. With the arc between the uncertainty node “Strike oil” and the decision node “Drill?” the influence diagram evaluates the expected value of the decision assuming perfect information. To see the expected value without information, delete the arc. The EVPI is the difference between these two EV's, or $19,000 - $10,000 = $9,000.

\[
\text{EVPI} = \text{EMV(Clairvoyant)} - \text{EMV(Drill)} = \$19\text{ K} - \$10\text{ K} = \$9\text{ K}.
\]

**c.** See the decision tree above or the decision tree model saved in the third worksheet.
d. We have:

\[
\begin{align*}
P(\text{“good”} \mid \text{oil}) &= 0.95 \quad P(\text{oil}) = 0.1 \\
P(\text{“poor”} \mid \text{dry}) &= 0.85 \quad P(\text{dry}) = 0.9
\end{align*}
\]

We can find \(P(\text{“good”})\) and \(P(\text{“poor”})\) with the law of total probability:

\[
P(\text{“good”}) = P(\text{“good”} \mid \text{oil}) P(\text{oil}) + P(\text{“good”} \mid \text{dry}) P(\text{dry})
\]

\[
= 0.95 (0.1) + 0.15 (0.9)
\]

\[
= 0.23
\]

\[
P(\text{“poor”}) = 1 - P(\text{“good”})
\]

\[
= 1 - 0.23
\]

\[
= 0.77.
\]

Now we can find

\[
P(\text{oil} \mid \text{“good”}) = \frac{P(\text{“good”} \mid \text{oil}) P(\text{oil})}{P(\text{“good”} \mid \text{oil}) P(\text{oil}) + P(\text{“good”} \mid \text{dry}) P(\text{dry})}
\]

\[
= \frac{0.95 (0.1)}{0.95 (0.1) + 0.15 (0.9)}
\]

\[
= 0.41
\]

\[
P(\text{dry} \mid \text{“good”}) = 1 - P(\text{oil} \mid \text{“good”})
\]

\[
= 0.59
\]

\[
P(\text{oil} \mid \text{“poor”}) = \frac{P(\text{“poor”} \mid \text{oil}) P(\text{oil})}{P(\text{“poor”} \mid \text{oil}) P(\text{oil}) + P(\text{“poor”} \mid \text{dry}) P(\text{dry})}
\]

\[
= \frac{0.05 (0.1)}{0.05 (0.1) + 0.85 (0.9)}
\]

\[
= 0.0065
\]

\[
P(\text{dry} \mid \text{“poor”}) = 0.9935.
\]

Now the decision tree is:
The influence diagram solution:

**Without Geologist:**

**With Geologist:**

EVII = EMV(Consult Geologist) - EMV(Drill) = $16.56 K - $10 K = $6.56 K.

Because EVII is less than $7000, which the geologist would charge, this is a case where the expected value of the geologist’s information is less than what it would cost. Don’t consult her.

The corresponding influence diagram is shown in the fourth worksheet and the decision tree is in the fifth worksheet. Note, the values in the spreadsheet have slightly different values due to round-off error. Bayesian probability calculations are very sensitive to the significant digits carried through the calculations.
Influence diagrams:

Note that because we are minimizing cost in this problem, we need to find the expected cost savings due to the information. For that reason, $\text{EVPI} = \text{EC(Make processor)} - \text{EC(Information)} = $42.395 - $41.725 = $0.670 \text{ per unit.}$
This decision tree is shown in the first worksheet of the Excel file “Problem 12.8.xls”. Reference nodes are used to simplify the representation of the tree by referring to the cost uncertainty node associated with the “Buy” decision.

h.
EVPI = EC(Make processor) - EC(Information) = $42.395 - $41.8555 = $0.5355 per unit.

This decision tree is shown in the second worksheet of the Excel file “Problem 12.8.xls”. Reference nodes are used to simplify the representation by referring to the Cost uncertainty associated with the “Make” decision.

c.
Influence diagram:

```
   Information about cost to buy processor:
   Make or Buy
   Cost to make
   Cost to buy
   Cost

   Make or Buy
   Cost

   Information about costs to buy and to make processor
   Make or Buy
   Cost to make
   Cost to buy
   Cost
```

EVPI = EC(Make processor) - EC(Information) = $42.395 - $41.0805 = $1.3145 per unit

This decision tree is shown in the third worksheet of the Excel file “Problem 12.8.xls”. The decision tree, however, in its complete form has too many nodes for the student version (limit 50). The tree was manually trimmed near the bottom to satisfy this constraint.
Perfect information about making and buying processor
EC = $41.0805

Buy

EC = $44.70

Make

EC = $42.395

Make

Make = $35.00
(0.25)

Buy

Make = $35.00
(0.25)

Buy

Make = $35.00
(0.25)

Buy

Make = $35.00
(0.25)

Buy

Make = $35.00
(0.25)

Buy

Make = $42.50
(0.25)

Make

Buy

Make = $42.50
(0.25)

Make

Buy

Make = $42.50
(0.25)

Make

Buy

Make = $45.00
(0.37)

Make

Buy

Make = $45.00
(0.37)

Make

Buy

Make = $49.00
(0.13)

Make

Buy

Make = $49.00
(0.13)

Make

Buy

Make = $35.00
(0.25)

Buy

Make = $37.00
(0.10)

Buy

Make = $43.00
(0.40)

Buy

Make = $46.00
(0.30)

Buy

Make = $50.00
(0.20)
12.9. The answer of course depends on the specific GPA and the distribution for the course score. Let’s assume GPA = 3.0, and the student has the following 3-point discrete approximation for the course score:

<table>
<thead>
<tr>
<th>Score</th>
<th>Probability</th>
<th>Salary</th>
</tr>
</thead>
<tbody>
<tr>
<td>67</td>
<td>0.185</td>
<td>$27,756</td>
</tr>
<tr>
<td>82</td>
<td>0.630</td>
<td>$28,776</td>
</tr>
<tr>
<td>90</td>
<td>0.185</td>
<td>$29,320</td>
</tr>
</tbody>
</table>

The decision tree, including obtaining information would be:

\[
\text{EVPI} = \text{EMV(Information)} - \text{EMV(Take Course)} = \$28,773 - \$28,688 = \$85.
\]

This decision tree is shown in the Excel file “Problem 12.9.xls”.

Note that in many cases (for example the illustrative solution to problem 8.11 in this manual), EVPI = 0 because taking the course deterministically dominates dropping the course.
12.10. a.

Note that because we are minimizing cost in this problem, we need to find the expected cost savings due to the information. For that reason, the formula for EVPI appears reversed:

\[
EVPI = E(\text{Cost for leaving plant open}) - E(\text{Cost for information}) = $3960 - $2640 = $1320.
\]

This decision tree is shown in the first worksheet in the Excel file “Problem 12.10.xls”.

b.

\[
EVPI = E(\text{Cost for leaving plant open}) - E(\text{Cost for information}) = $7920 - $2640 = $5280.
\]

This decision tree is shown in the second worksheet in the Excel file “Problem 12.10.xls”.

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Now EVPI = 0 because leaving the plant open deterministically dominates closing the plant. The manager would leave the plant open regardless of the number of broken machines. This decision tree is shown in the third worksheet of the Excel file “Problem 12.10.xls”.

12.11. a. Liedtke’s EVPI regarding Texaco’s response would be zero because Liedtke would still counteroffer $5 billion regardless of Texaco’s response.

Influence diagram:
EMV = EMV(Information) - EMV(Counteroffer $5 billion) = 4.63 - 4.63 = 0. This decision tree is shown in the first worksheet in the Excel file “Problem 12.11.xls”.

b.i.
Influence diagram:
Accept 2 billion

Counteroffer 5 billion

Payoff
($billion)

Accept 2

Counteroffer 5

Award = 10.3
(0.2)

Accept 2

Counteroffer 5

Award = 5
(0.5)

Accept 2

Counteroffer 5

Award = 0
(0.3)

Accept 2

Counteroffer 5

Payoff

Information about court award

EMV = 4.98

Accept 2 billion

Counteroffer 5 billion

EMV = 4.63

Accept 2

Counteroffer 5

Award = 10.3
(0.2)

Accept 2

Counteroffer 5

Award = 5
(0.5)

Accept 2

Counteroffer 5

Award = 0
(0.3)

Accept 2

Counteroffer 5

EMV = 9.4

Counter 3

Refuse (0.33)

Accept

EMV = 5

Counter 3

Refuse (0.33)

Accept

EMV = 1.84

Counter 3

Refuse (0.33)

Accept

EVPI = EMV(Information) - EMV(Counteroffer 5)
= $4.98 billion - $4.63 billion
= $0.35 billion.

This decision tree is shown in the second worksheet of the Excel file “Problem 12.11.xls”. Note, this assumes that Liedke is the only one who receives the information regarding the court award.

b.ii.
Influence diagram:
Evacancy 2 billion

Counteroffer 5 billion

EMV = 4.63

Counteroffer 5, get information later.
EMV = 4.93

Texaco accepts 5 billion

(0.17)

(0.2)

Texaco refuses

(0.50)

Final court decision

(0.3)

Award = 5

(0.5)

Accept 3

Refuse

(0.2)

Award = 10.3

(0.17)

Accept 3

Refuse

(0.5)

Award = 5

(0.5)

Accept 3

Refuse

(0.3)

Award = 0

(0.3)

Accept 3

Refuse

Payoff ($billion)

2

EVPI = EMV(Information) - EMV(Counteroffer $5 billion) = 4.93 - 4.63 = $0.30 billion. This decision tree is shown in the third worksheet of the Excel file “Problem 12.12.xls.”

c. Information later is less useful. In fact, we saw in part b(i) that, if the award turned out to be zero, Liedtke would accept the $2 billion that is on the table. Obtaining the information later does not allow him to use the information in this way.

NOTE: The solution to part b(i) above is based on the assumption that only Liedtke obtains information about the court award, and not Kinnear. If Kinnear also finds out what the court award is to be, then Texaco’s reaction would change. The decision tree is shown below:
In this case, EVPI is actually negative (4.10 - 4.63 = -$0.53 billion). The fact that information has negative value here appears to be at odds with the text, which indicates that information can never have negative value. In fact, that statement implicitly assumes that the decision maker is not playing a strategic game against some other decision maker who could also take advantage of the information. If such is the case, then, as indicated by the solution to this problem, it is indeed possible for information to have negative value. In this case, it is conceivable that Liedtke might be willing to take some (costly) action to prevent the information about the court award from being revealed!
EVPI = EMV(Information) - EMV(Counteroffer $5 billion) = 5.23 - 4.63 = $0.60 billion.

Note that 0.60 > 0 + 0.35, or the sum of the two EVPIs for the individual bits of information in 12.10a and 12.10b(i). The information about Texaco’s reaction alone was not enough to be worthwhile. However, this information along with the court information helps Liedtke refine his strategy. Now he knows exactly what to do in each case, especially when the court award = 0.

This decision tree is saved in the Excel file “Problem 12.12.xls”.
12.13.a.

Influence diagram:

Basic Diagram:

Information about weather:

\[ \text{EVPI} = E(\text{Loss} | \text{Burner}) - E(\text{Loss} | \text{Information}) = 13.75 - 11.25 = $2.5 \text{ K} = $2500. \]

This decision tree is shown in the first worksheet of the Excel file “Problem 12.13.xls”.
b. The uniform distributions in this part of the problem are approximated as discrete distributions using the extended Pearson-Tukey method. Influence diagram:

\[
\begin{align*}
\text{EVPI} &= \text{E}(\text{Loss} | \text{Burners}) - \text{E}(\text{Loss} | \text{Information}) = 13.75K - 13.741K = $9
\end{align*}
\]

This decision tree to find the value of information is very large and exceeds the student version limit of 50 nodes. To represent it, we have split the decision tree into 2 separate trees: without information (in the second worksheet) and with information (in the third worksheet). Also, the tree with information still had too many nodes, so the uncertainty regarding the freeze is included in a formula in the consequence cell of each branch.

c. Better weather forecasts appear to be more important than knowing more about the specific losses from burners and sprinklers. In fact, the only reason that EVPI in 12.12b is positive is that the farmer would set sprinklers out instead of burners if he learned that the sprinkler loss is low and the burner loss is high. (However, this may actually be an unlikely scenario, given that the levels of losses in the two cases may be caused by the same factors. See the discussion for problem 5.9.)

12.14. The basic decision tree is shown in the first worksheet of the Excel file “Problem 12.14.xls”. The expected value of the decision with no information is $9,600. Influence diagrams:

Basic diagram:

Operating Cost

Information:
The decision tree for information on the Operating Cost is shown in the second worksheet of the Excel file “Problem 12.14.xls”. The decision tree for information on Capacity is in the third worksheet. The fourth worksheet contains the Bayes calculation to determine Capacity conditional on Hours Flown. This is necessary because the probabilities \( P(\text{Capacity}) \) and \( P(\text{Hours Flown} \mid \text{Capacity}) \) must be flipped using Bayes’ theorem in order to find EVPI for Hours Flown. This is because Hours Flown must appear before Capacity in the decision tree. The corresponding change in the influence diagram is the reversal of the arc between Hours Flown and Capacity prior to solving the diagram. Then the decision tree for information on Hours Flown is in the fifth worksheet.

\[
\text{EVPI(Operating Cost)} = 10,050 - 9,600 = 450. \\
\text{EVPI(Capacity)} = 12,088 - 9,600 = 2,488.  \\
\text{EVPI(Hours Flown)} = 10,276 - 9,600 = 676.
\]

Capacity is by far the most valuable variable. Additional effort and resources should be spent on determining what capacity will actually be.

12.15. Of course, the only available alternative is that the events are dependent. In this case, depending on which chance node is placed before the decision node in the tree, the branches for the subsequent one must display the appropriate conditional probabilities. Bayes’ theorem may be used to calculate the required conditional probabilities.

Having dependent events is an interesting possibility, because knowing the outcome of one event perfectly will reveal some (imperfect) information about the other event. That information is manifested as the conditional probabilities in the decision tree.