Solutions

5.1. Sensitivity analysis answers the question "What matters in this decision?" Or, "How do the results change if one or more inputs change?" To ask it still another way, "How much do the inputs have to change before the decision changes?"

We have framed the main issue in sensitivity analysis as "What matters" because of our focus on constructing a requisite model. Clearly, if a decision is insensitive to an input—the decision does not change as the input is varied over a reasonable range—then variation in that input does not matter. An adequate model will fix such an input at a "best guess" level and proceed.

5.2. This is a rather amorphous question. The decision apparently is, "In which house shall we live?" Important variables are the value of the current home, costs of moving, purchase price, financing arrangements (for the current house as well as for a new one), date of the move, transportation costs, and so on.

What role would sensitivity analysis play? The couple might ask whether the decision would change if they took steps to reduce driving time in the future (e.g., by deciding to have no more children). How does the attractiveness of the different alternatives vary as the family's size and the nature of future outings are varied? (For example, how much more skiing would the family have to do before living out of town is preferred?) Can the family put a price tag on moving into town; that is, is there an amount of money (price, monthly payment, etc.) such that if a house in town costs more than this amount, the family would prefer to stay in the country?

5.3. Another rather amorphous question. Students may raise such issues as:
- Is a retail business the right thing to pursue? (Is the right problem being addressed?)
- Does the father really want to be a retailer?
- Operating costs and revenues may vary considerably. These categories cover many possible inputs that might be subjected to sensitivity analysis.

To use sensitivity analysis in this problem, the first step would be to determine some kind of performance measure (NPV, cash flow, payback, profit). Then a tornado diagram could be constructed showing how the selected performance measure varies over the range of values for the inputs. The tornado diagram will suggest further modeling steps.

5.4. From the discussion in the text, the basic issue is whether some relevant uncertainty could be resolved during the life of the option. Some possibilities include:
- Obtaining a new job, promotion, or raise,
- Obtaining cash for a down payment,
- Learning about one's preferences. "Is this house right for me?"
- Are there hidden defects in the house that will require repairs?
- Are zoning decisions or other developments hanging in the balance?

If such an uncertainty exists, then the option may have value. If not, it may be a dominated alternative.

5.5. By answering the question, "What matters in this decision," sensitivity analysis helps identify elements of the decision situation that must be included in the model. If an input can vary widely without affecting the decision, then there is no need to model variation in that input. On the other hand, if the variation matters, then the input's uncertainty should be modeled carefully, as an uncertain variable or as a decision if it is under the decision maker's control. (See also the discussion for 5.1 above.)

5.6. Each line is a line of indifference where two of the alternatives have the same EMV. Imagine a point where two of the lines cross. At that point, all three of the alternatives must have the same EMV. Thus, the third line must also go through this point.

5.7.   Cost of protective action $= C$
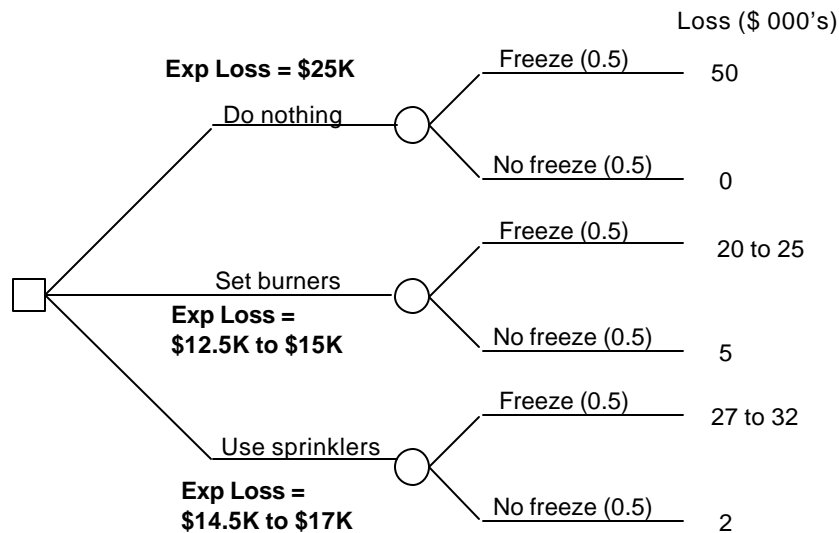Expected loss if no action taken $= pL$

Set $C = pL$, and solve for p: $p = \dfrac{C}{L}$.

Thus, if $p = \dfrac{C}{L}$, take protective action.

The only information needed is p and $\dfrac{C}{L}$. Note that the specific values of C and L are not required, only their relative values.

5.8. The best way to see if it is necessary to model the uncertainty about D is to revert to the regular cost-to-loss problem. If $pL < C < C + D$, then one would not take action, and if $pL > C + D$, then the optimal choice would be to take action. However, if $C < pL < C + D$ then the occurrence of damage D does matter. The choice between taking action and not is unclear, and one would want to include D in the decision tree.

5.9.

Loss ($ 000's)

| | | Freeze (0.5) | 50 |
Exp Loss = $25K

Do nothing ◯

No freeze (0.5) | 0

Freeze (0.5) | 20 to 25

Set burners ◯

Exp Loss = $12.5K to $15K

No freeze (0.5) | 5

Freeze (0.5) | 27 to 32

Use sprinklers ◯

Exp Loss = $14.5K to $17K

No freeze (0.5) | 2

The expected loss from doing nothing is much greater than for either of the two measures, and so it is certainly appropriate to take some action. The expected loss for burners is almost entirely below that for sprinklers, the only overlap being between $14.5K and $15K. It would be reasonable to set the burners without pursuing the analysis further.

Another argument in favor of this is that most likely the same factors lead to more or less damage for both burners and sprinklers. With this reasoning, there would be a negligible chance that the burners would produce a high loss and the sprinklers a low loss.

A final note: Some students may solve this problem without calculating the expected loss, comparing the range of losses from burners or sprinklers if damage occurs with the $50K loss from doing nothing. However, if uncertainty about the weather is ignored altogether, the appropriate analysis has the loss ranging from $0 to $50K for no action, $5 to $25K for the burners, and $2 to $32K for the sprinklers. Because the three ranges overlap so much, no obvious choice can be made. It is, therefore, appropriate and necessary to include the probability of adverse weather and calculate the expected losses. This decision tree is modeled in the Excel file "Problem 5.9.xls" and the sensitivity analysis dialog box has the parameters saved.

5.10. This decision tree (shown in Figure 4.47 in the text) is modeled in the Excel file "Problem 5.10.xls". The model is a linked tree where the uncertainty node for the amount of fun is linked to cell F6 in the spreadsheet model ("Fun Level for Forest Job"), and the uncertainty node for the amount of work is linked to cell G7 in the spreadsheet model ("Salary Level for In-town Job"). The outcome nodes for the Forest Job are linked to cell F8 and the outcome nodes for the In-Town Job to cell G8. The user can then vary the weights to see that Sam will still prefer the forest job. The sensitivity analysis gives the following results:

Expected Overall Score

| k_S	Forest Job	In-Town Job
0.50	71.25	57.50
0.75	76.125	56.25

Thus, regardless of the precise value of k_S, the optimal choice is the forest job. In fact, a much stronger statement can be made; it turns out that for no value of k_S between zero and one is the in-town job preferred. Smaller values of k_S favor the in-town job, but even setting $k_S = 0$ leaves the expected overall scores equal to 60 and 61.5 for the in-town and forest jobs, respectively.

Another way to show the same result is to realize that the expected overall scores are linear in the weights and in the expected scores for the individual attributes. Because the forest job has higher expected scores on both attributes, there cannot exist a set of weights that makes the in-town job have the higher overall expected score.
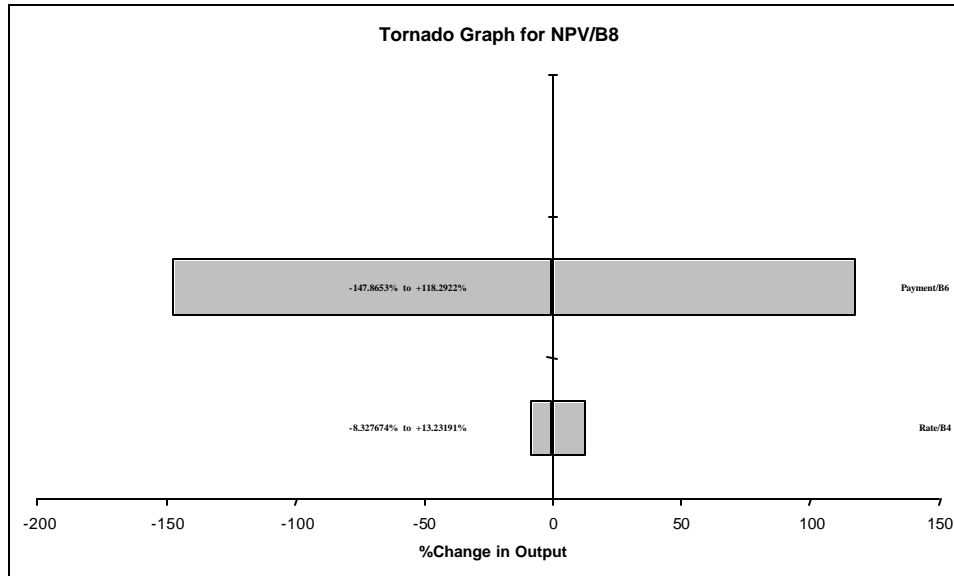
5.11. Using the base values of $5000 for the cash flows and 11% for the interest rate,

$$\text{NPV} = -14000 + \frac{5000}{1.11} + \frac{5000}{1.11^2} + \dots + \frac{5000}{1.11^6}$$
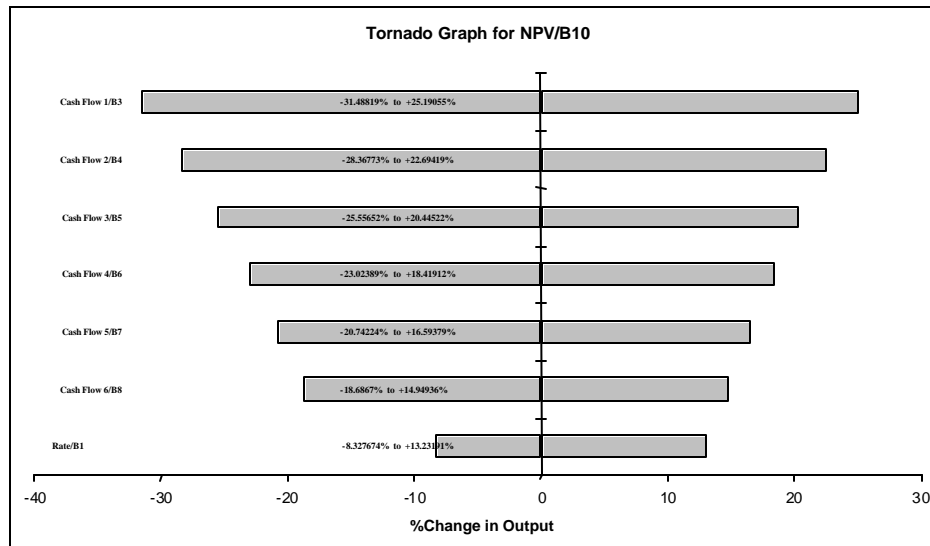
$$= \$7153.$$

When the cash flows are varied from $2500 to $7000, and the interest rate is varied from 9.5% to 12%, the following tornado diagram results:
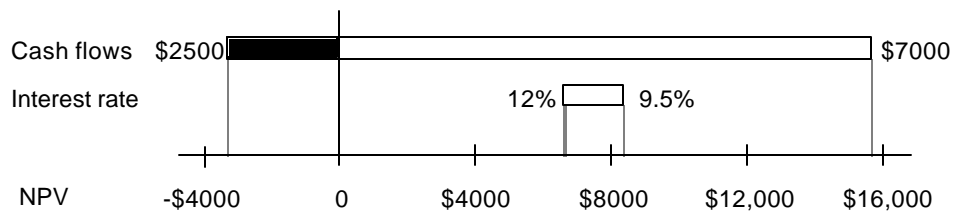
This graph assumes that while the cash flows may vary between $2500 and $7000, whatever the amount, it occurs each year. The range of possible interest rates appears not to pose a problem; NPV remains positive within a relatively narrow range. On the other hand, NPV is very sensitive to the range of cash flows. It would be appropriate to set the interest rate at 11% for the analysis but to model the uncertainty about the cash flows with some care. The tornado diagram generated by TopRank shows the difference as a percentage change of NPV, not by the absolute amount. You cannot get the absolute amounts from the graph, but if you click on the "Actuals" button in the results window, TopRank will provide these values. This model is shown in the Excel file "Problem 5.11.xls".

Tornado Graph for NPV/B8

-147.8653% to +118.2922%   Payment/B6

-8.327674% to +13.23191%   Rate/B4

%Change in Output

The problem can also be modeled where each year can vary independently between $2500 and $7000. This model is shown in the Excel file "Problem 5.11 alt.xls". In the student version, you cannot create both models in the same spreadsheet or have both files open at the same time. This will give you an error "Model Extends Beyond Allowed Region of Worksheet," and will crash the program and require restarting the computer.

Tornado Graph for NPV/B10

Cash Flow 1/B3   -31.48819% +25.19055%

Cash Flow 2/B4   -28.36773% to +22.69419%

Cash Flow 3/B5   -25.55652% to +20.44522%

Cash Flow 4/B6   -23.02389% to +18.41912%

Cash Flow 5/B7   -20.74224% to +16.59379%

Cash Flow 6/B8   -18.6867% to +14.94936%

Rate/B1   -8.327674% to +13.23191%

%Change in Output

Because of the discounting factor, the early years are more sensitive than the later years.

When using TopRank, the program automatically tries to create a name for each input and output cell by scanning the spreadsheet around the cell where the input/output is located and combining any row and column headings. These are often nonsensical, and can be changed by clicking in the name box in the inputs by outputs dialog box and entering the desired name. After the new names are entered, a what-if analysis must be run for the new names to take effect.

To use the graphs that are generated in reports, use the menu command "Graph in Excel." This command places the selected graph data points in Excel and thus a native format graph is created on the worksheet from the data points.