Chapter 4
Making Choices
Texaco Versus Pennzoil

In early 1984, Pennzoil and Getty Oil agreed to the terms of a merger. But before any formal documents could be signed, Texaco offered Getty a substantially better price, and Gordon Getty, who controlled most of the Getty Stock, reneged on the Pennzoil deal and sold to Texaco. Naturally, Pennzoil felt as if it had been dealt with unfairly and immediately files a lawsuit against Texaco alleging that Texaco had interfered illegally in the Pennzoil-Getty negotiations. Pennzoil won the case: in late 1985, it was awarded $11.1 billion, the largest judgment ever in the United States. A Texas appeal court reduced the judgement to $2 billion, but interest and penalties drove the total back up to $10.3 billion. James Kinnear, Texaco’s Chief executive officer, had said that Texaco would file for bankruptcy if Pennzoil obtained court permission to secure the judgment by filing liens against Texaco’s assets.
Furthermore, Kinnear had promised to fight the case all the way to the **U.S. Supreme Court** if necessary, arguing in part that Pennzoil had not followed Security and Exchange Commission regulations in its negotiations with Getty. In April 1987, just before Pennzoil began to file liens, **Texaco offered to Penzoil $2 billion dollars** to settle the entire case. Hugh Liedtke, chairman of Pennzoil, indicated that his advisors were telling him that a settlement **between $3 billion and $5 billion would be fair.**

**What should Hugh Liedtke do?**

1. Accept $2 Billion
2. Refuse $2 Billion and counter offer $5 Billion
Texaco Versus Pennzoil – Decision Tree

Accept $2 Billion

Texaco Accepts $5 Billion

Final Court Decision

High

Medium

Low

Max Settlement Amount ($ Billion )

2

5

10.3

5

2

10.3

3

2

5

0

0

5

3

0

Accept $3 Billion

Counteroffer

$5 Billion

Texaco Refuses

Counteroffer

Texaco Accepts $5 Billion

Final Court Decision

High

Medium

Low

Accept $3 Billion

Counteroffer

$5 Billion

Texaco Refuses

Counteroffer

Accept $3 Billion

Counteroffer

$5 Billion

Texaco Refuses

Counteroffer

Refuse

Final Court Decision

High

Medium

Low

Refuse

Final Court Decision

High

Medium

Low

Accept $3 Billion

Counteroffer
Texaco Versus Pennzoil - Continued

- Given tough negotiation positions of the two executives, their could be **an even chance (50%)** that Texaco will refuse to negotiate further.

- Liedtke and advisor figure that it is **twice as likely** that Texaco would counter offer $3 billion than accepting the $5 billion. Hence, because there is a **50% of refusal**, there must be a **33% chance** of a Texaco counter offer and a **17% chance** of Texaco accepting $5 billion.

- What are the probabilities of the final court decision?
  - Liedtke **admitted** that Pennzoil could lose the case. Thus there is a significant possibility the outcome would be zero. It’s probability is assessed at **30%**.
  - Given the strength of the Pennzoil case it is also possible that the court will **upheld the judgment** as it stands. It’s probability is assessed at **20%**.
  - Finally, the possibility exists that the judgment could be **reduced somewhat to $5 billion**. Thus there must be a chance of **50%** of this happening.
Texaco Versus Pennzoil - Continued

• Given tough negotiation positions of the two executives, it could be **an even chance (50%)** that Texaco will refuse to negotiate further.

• Liedtke and advisor figures that it is **twice as likely** that Texaco would counter offer $3 billion than accepting the $5 billion. Hence, because there is a **50% of refusal**, there must be a **33% chance** of a Texaco counter offer and a **17% chance** of Texaco accepting $5 billion.

• What are the probabilities of the final court decision?
  - Liedtke **admitted** that Pennzoil could lose the case. Thus there is a significant possibility the outcome would be zero. It’s probability is assessed at **30%**.
  - Given the strength of the Pennzoil case it is also possible that the court will **upheld the judgment** as it stands. It’s probability is assessed at **20%**.
  - Finally, the possibility exists that the judgment could be **reduced somewhat to $5 billion**. Thus there must be a chance of **50%** of this happening.
Texaco Versus Pennzoil – Decision Tree

Accept $2 Billion

Texaco Accepts $5 Billion (0.17)

High (0.20) 10.3

Medium (0.50) 5

Low (0.30) 0

Final Court Decision

Counteroffer

Texaco Refuses (0.50)

Max Settlement Amount ($ Billion )

Accept $3 Billion

Refuse

Final Court Decision

High (0.20) 10.3

Medium (0.50) 5

Low (0.30) 0

Counteroffer

Texaco Counteroffers $3 Billion (0.33)

$5 Billion

Count offer
Decision Tree and Expected Monetary Value (EMV)

When objective is measured in **dollars**

First Suggestion:
Solve decision problem by choosing that alternative that maximizes the EMV

Expected value of discrete random variable $Y$:

$$E_Y[Y] = \sum_{i=1}^{n} y_i \cdot \Pr(Y = y_i) = \sum_{i=1}^{n} y_i \cdot p_i$$
A double-risk dilemma

<table>
<thead>
<tr>
<th>y</th>
<th>Pr(Y=y)</th>
<th>y*Pr(Y=y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$24.00</td>
<td>0.2</td>
<td>$4.80</td>
</tr>
<tr>
<td>-$1.00</td>
<td>0.8</td>
<td>-$0.80</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$4.00 = EMV</td>
</tr>
</tbody>
</table>

EMV = $4

Interpretation EMV: Playing the same lottery a lot of times will result over time in an average pay-off equal to the EMV

EMV = $4.5

Table:

<table>
<thead>
<tr>
<th>y</th>
<th>Pr(Y=y)</th>
<th>y*Pr(Y=y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10.00</td>
<td>0.45</td>
<td>$4.50</td>
</tr>
<tr>
<td>$0.00</td>
<td>0.55</td>
<td>$0.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$4.50 = EMV</td>
</tr>
</tbody>
</table>
Texaco Versus Pennzoil – Decision Tree

Solve tree using EMV by folding back the tree
Step 1: Calculate EMV of court decision uncertainty node

<table>
<thead>
<tr>
<th>y</th>
<th>Pr(Y=y)</th>
<th>y*Pr(Y=y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.300</td>
<td>0.2</td>
<td>$2.06</td>
</tr>
<tr>
<td>5.000</td>
<td>0.5</td>
<td>$2.50</td>
</tr>
<tr>
<td>0.000</td>
<td>0.3</td>
<td>$0.00</td>
</tr>
</tbody>
</table>

EMV = $4.56
Step 2: Evaluate decision regarding Texaco’s counter offer

Decision Tree and Expected Monetary Value (EMV)

EMV = 4.56

Refuse

High (0.20)
10.3

Medium (0.50)
5

Low (0.30)
0

Final Court Decision

Accept $3 Billion
3
Step 3: Calculate EMV Texaco’s reaction uncertainty node

<table>
<thead>
<tr>
<th>y</th>
<th>Pr(Y=y)</th>
<th>y*Pr(Y=y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.000</td>
<td>0.17</td>
<td>$0.85</td>
</tr>
<tr>
<td>4.560</td>
<td>0.5</td>
<td>$2.28</td>
</tr>
<tr>
<td>4.560</td>
<td>0.33</td>
<td>$1.50</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$4.63 = EMV</td>
</tr>
</tbody>
</table>

EMV = $4.63

Decision Tree and Expected Monetary Value (EMV)
Step 4: Evaluate the immediate decision

Optimal decision: Counteroffer $5 Billion

Optimal decision strategy: Counteroffer $5 Billion and if Texaco counteroffers $3 Billion, then refuse this counteroffer.
Folding back the Decision Tree from right to left using EMV

EMV = 4.63

Accept $2 Billion

EMV = 4.56

Texaco Accepts $5 Billion (0.17)

Max Result

EMV = 4.56

High (0.20)
10.3

Medium (0.50)
5

Low (0.30)
0

Final Court Decision

Texaco Refuses (0.50)

EMV = 4.56

Countertoffer

$5 Billion

Final Court Decision

Refuse

EMV = 4.56

High (0.20)
10.3

Medium (0.50)
5

Low (0.30)
0

Final Court Decision

Countertoffer

Texaco Counteroffers $3 Billion (0.33)

EMV = 4.56

Accept $3 Billion

EMV = 4.63
Definitions Decision Path and Strategy

**Definition decision path:**
A path starting at the left most node up to the values at the end of a branch by selecting one alternative from decision nodes or by following one outcome from uncertainty nodes. Represents a possible future scenario.

**Definition decision strategy:**
The collection of decision paths connected to one branch of the immediate decision by selecting one alternative from each decision node along these paths. Represents specifying at every decision in the decision problem what we would do, if we get to that decision (we may not get there due to outcome of previous uncertainty nodes).

**Optimal decision strategy:**
That decision strategy which results in the highest EMV if we maximize profit and the lowest EMV if we minimize cost.
Counting Strategies

How many decision strategies in Example 1?

Example 1

How many decision strategies in Example 2?

Example 2
Counting Strategies

How many decision strategies in Example 3?
Counting Strategies

How many decision strategies in Example 1?

Example 1

Strategy 1
Strategy 2
Strategy 3

Strategy 1
Strategy 2
Strategy 3

How many decision strategies in Example 2?

Example 2

Strategy 1
Strategy 2 (11)
Strategy 3 (00)
Strategy 4 (10)
Strategy 5 (01)

Strategy 1
Strategy 2
Strategy 3
Strategy 4
Strategy 5

Strategy 1
Strategy 2
Strategy 3
Strategy 4
Strategy 5
How many decision strategies in Example 3?

Strategy 1
Strategy 2 (111)
Strategy 3 (001)
Strategy 4 (101)
Strategy 5 (011)
Strategy 6 (110)
Strategy 7 (000)
Strategy 8 (100)
Strategy 9 (010)
Decision Strategies Texaco-Pennzoil Case

How many decision strategies do we have in the Texaco – Penzoil decision tree?

First strategy: “Accept $2 billion”
Second strategy: “Counter $5 billion and if Texaco counter offers $3 billion refuse this counteroffer of $3 Billion”
Third strategy: “Counter $5 billion and if Texaco counter offers $3 billion accept this counteroffer of $3 Billion”
Risk Profiles and Cumulative Risk Profiles

**RISK PROFILES** = Graph that shows probabilities for each of the possible outcomes *given a particular decision strategy*.

**Note:** Risk Profile is a probability mass function for the discrete random variable $Y$ representing the outcomes for the given decision strategy.

**CUMMULATIVE RISK PROFILES** = Graphs that shows cumulative probabilities associated with a risk profile.

**Note:** Cumulative risk profile is a cumulative distribution function for the discrete random variable $Y$ representing the outcomes for *the given decision strategy*. 
## Risk Profiles

### First strategy: “Accept $2 billion”

| Outcome x ($Billion) | Pr(Outcome|D) |
|----------------------|------------|
| 2                    | 1          |

Accept $2 Billion
Risk Profiles

Second strategy: “Counter $5 billion and if Texaco counter offers $3 billion refuse this counteroffer of $3 Billion”

![Risk Profile Diagram]

<table>
<thead>
<tr>
<th>Calculation</th>
<th>Prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.17</td>
<td>0.170</td>
</tr>
<tr>
<td>0.50*0.20</td>
<td>0.100</td>
</tr>
<tr>
<td>0.50*0.50</td>
<td>0.250</td>
</tr>
<tr>
<td>0.50*0.30</td>
<td>0.150</td>
</tr>
<tr>
<td>0.33*0.20</td>
<td>0.066</td>
</tr>
<tr>
<td>0.33*0.50</td>
<td>0.165</td>
</tr>
<tr>
<td>0.33*0.30</td>
<td>0.099</td>
</tr>
<tr>
<td>Total</td>
<td>1.000</td>
</tr>
</tbody>
</table>
Second strategy: “Counter $5 billion and if Texaco counter offers $3 billion refuse this counteroffer of $3 Billion”

| Outcome x ($Billion) | Calculation         | Pr(Outcome| D) |
|----------------------|---------------------|--------|
| 0                    | 0.150+0.099         | 0.249  |
| 5                    | 0.170+0.250+0.165   | 0.585  |
| 10.3                 | 0.100+0.066         | 0.166  |

Risk Profile D="Counter $5 Billion, refuse counteroffer of $3 Billion if given"
Third strategy: “Counter $5 billion and if Texaco counter offers $3 billion accept this counteroffer of $3 Billion”

![Risk Profiles Diagram](image-url)
### Risk Profiles

**Third strategy:** “Counter $5$ billion and if Texaco counter offers $3$ billion accept this counteroffer of $3$ Billion”

| Outcome (Billion) | Calculation     | Pr(Outcome| D) |
|-------------------|-----------------|--------|
| 0                 | 0.15            | 0.15   |
| 3                 | 0.33            | 0.33   |
| 5                 | $0.170+0.250$   | 0.42   |
| 10.3              | 0.1             | 0.1    |

Risk Profile D="Counter $5$ Billion, Accept Counter Offer of $3$ Billion if given"

![Risk Profile Graph](image-url)
Cumulative Risk Profiles

First strategy: “Accept $2 billion”

| Outcome x ($Billion) | Pr(Outcome|D) |
|----------------------|-------------|
| 2                    | 1           |

| Outcome x ($Billion) | Pr(Outcome ≤ x|D) |
|----------------------|------------------|
| 2                    | 1                |
Second strategy: “Counter $5$ billion and if Texaco counter offers $3$ billion refuse this counteroffer of $3$ Billion”

| Outcome x ($\text{Billion}$) | $\Pr(\text{Outcome}|D)$ |
|-------------------------------|--------------------------|
| 0                             | 0.249                    |
| 5                             | 0.585                    |
| 10.3                          | 0.166                    |

Cumulative Risk Profile D="Counter $5$ Billion, refuse counter offer of $3$ Billion if given"

| Outcome x ($\text{Billion}$) | $\Pr(\text{Outcome} \leq x|D)$ |
|-------------------------------|---------------------------------|
| 0                             | 0.249                           |
| 5                             | $0.249 + 0.585 = 0.834$        |
| 10.3                          | $0.834 + 0.166 = 1$            |
### Cumulative Risk Profiles

**Third strategy:** “Counter $5\text{ billion}$ and if Texaco counter offers $3\text{ billion}$ accept this counteroffer of $3\text{ Billion}$”

<table>
<thead>
<tr>
<th>Outcome x ($\text{Billion}$)</th>
<th>Pr(Outcome $\mid D$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.15</td>
</tr>
<tr>
<td>3</td>
<td>0.33</td>
</tr>
<tr>
<td>5</td>
<td>0.42</td>
</tr>
<tr>
<td>10.3</td>
<td>0.1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Outcome x ($\text{Billion}$)</th>
<th>Pr(Outcome $\leq x \mid D$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.15</td>
</tr>
<tr>
<td>3</td>
<td>$0.15 + 0.33 = 0.48$</td>
</tr>
<tr>
<td>5</td>
<td>$0.48 + 0.42 = 0.90$</td>
</tr>
<tr>
<td>10.3</td>
<td>$0.90 + 0.10 = 1$</td>
</tr>
</tbody>
</table>
Deterministic Dominance

Original Tree

- Penzoil-Texaco
  - Accept $2 Billion
    - Decision 4.635
  - Counteroffer $5 Billion
    - TRUE

- Texaco Accept $5 Billion
  - Chance 4.635
    - High Award
      - 20% 10.3
    - Medium Award
      - 50% 5.0
    - Low Award
      - 30% 0.0

- Texaco Refuses Counteroffer
  - Chance 4.6
    - branch

- Texaco Counteroffers $3 Billion
  - Decision 4.560
    - branch

- FALSE
  - 0.000
  - 2.000

- TRUE
  - 0.000
  - 5.000

- 17%

- 33%

- 20%

- 10.3

- 0.169

- 0.150

- 0.100

- 0.066

- 0.000

- 0.000
Deterministic Dominance

Based on EMV analysis we still choose the alternative “Counteroffer $5 Billion”

Could we have made a decision here without an EMV analysis?
Deterministic Dominance

Formal Definition: Deterministic Dominance:
If the **worst outcome** of Alternative B is **at least as good** as that of the **best outcome** of Alternative A, then Alternative B **deterministically dominates** Alternative A.

- Deterministic dominance may also be concluded by drawing cumulative risk profiles and using the definition:

**Definition**: Range of a Cumulative Risk Profile = [L,U], where L= Smallest 0% point in Cumulative Risk Profile and U= Largest 100% point in Cumulative Risk Profile
Deterministic Dominance

- **Deterministic dominance via cumulative risk profiles:**
  - Step 1: Draw cumulative risk profiles in one graph
  - Step 2: Determine range for each risk profile
  - Step 3: If ranges are disjoint or their intersections contain a single point

Ranges 1 and 2 are disjoint. The Objective is Max Result, hence Green CRP deterministically dominates the Red one.
Stochastic Dominance: Example 1

Firm A: Original Tree

Decision 4.635

Accept $2 Billion

Decision 4.635

Counteroffer $5 Billion

Texaco Accept $5 Billion

Chance 4.635

High Award 10.3 0.01

Medium Award 5.0 0.249

Low Award 0.0 0.15

Texaco Refuses Counteroffer

Chance 4.6

High Award 10.3 0.1

Medium Award 5.0 0.249

Low Award 0.0 0.15

Texaco Counteroffers $3 Billion

Decision 4.560

High Award 10.3 0.066

Medium Award 5.0 0.166

Low Award 0.0 0.1

Firm A: Original Tree

Penzoil-Texaco

Accept $2 Billion

Decision 4.635

Counteroffer $5 Billion

 FALSE 0.000

2.000

TRUE 0.000

4.635

17% 0.169

5.0 0.169

20%

0.166

10.3 0.1

0.249

5.0 0.066

30%

0.066

10.3 0.066

0.166

5.0 0.166

30%

0.166

5.0 0.166
Stochastic Dominance: Example 1

Firm B: Modified Tree

[Diagram showing the decision tree for firm B with outcomes and probabilities]
Stochastic Dominance: Example 1

Based on EMV analysis we still choose the alternative “Firm B”

Could we have made a decision here without an EMV analysis?
Stochastic Dominance: Example 1

**Optimal** Cumulative risk profiles in “Firm A” Tree and “Firm B” Tree

![Cumulative Risk Profiles: Firm A and Firm B](image-url)
Stochastic Dominance: Example 1

Note that for all possible values of \( x \):
\[
\Pr(\text{Outcome} \leq x | \text{Firm B}) \leq \Pr(\text{Outcome} \leq x | \text{Firm A})
\]

or equivalently:
\[
\Pr(\text{Outcome} \geq x | \text{Firm B}) \geq \Pr(\text{Outcome} \geq x | \text{Firm A})
\]

Hence the chances of winning with Firm B are always better than that of Firm A.

Conclusion: Firm B stochastically dominates Firm A
Stochastic Dominance: Example 2

Firm A: Original Tree

Making Hard Decisions
R. T. Clemen, T. Reilly

Chapter 4 – Making Choices
Lecture Notes by: J.R. van Dorp and T.A. Mazzuchi
http://www.seas.gwu.edu/~dorpjr/

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Stochastic Dominance: Example 2

Firm C: Modified Tree

- Texaco Accepts $5 Billion
- Texaco Refuses Counteroffer
- Texaco Counteroffers $3 Billion
- Accept $2 Billion
- Counteroffer $5 Billion
Stochastic Dominance: Example 2

Based on EMV analysis we still choose the alternative “Firm C”

Could we have made a decision here without an EMV analysis?
Stochastic Dominance: Example 2

Optimal Cumulative risk profiles in “Firm A” Tree and “Firm C” Tree

Cumulative Risk Profiles: Firm A and Firm C

Pr(Outcome \leq x)

Outcome ($Billion)
Stochastic Dominance: Example 2

Note that for all possible values of x:
\[ \Pr(\text{Outcome} \leq x| \text{Firm C}) \leq \Pr(\text{Outcome} \leq x| \text{Firm A}) \]

or equivalently:
\[ \Pr(\text{Outcome} \geq x| \text{Firm C}) \geq \Pr(\text{Outcome} \geq x| \text{Firm A}) \]

Hence the chances of winning with Firm C are always better than that of Firm A.

Conclusion: Firm C stochastically dominates Firm A
Commonality CRP plots:

- Cumulative risk profiles in both plots do not cross

- The CRP that is toward the “right and below” stochastically dominates

- The objective in both plots is to **Maximize the Result**

- What if the objective is **Minimize the Result**?
Making Decisions with Multiple Objectives

• Two Objectives:

Making Money
(Measured in $)

Having Fun
(Measured on Constructed attribute scale, see page 138): Best(5), Good(4), Middle(3), Bad(2), Worst (1)
Making Decisions with Multiple Objectives

For each job, the consequences are:
- Salary
- Fun Level

Forest Job:
- 5 hours (0.10) → $2600.00, Fun Level 5
- 4 hours (0.25) → $2600.00, Fun Level 4
- 3 hours (0.40) → $2600.00, Fun Level 3
- 2 hours (0.20) → $2600.00, Fun Level 2
- 1 hour (0.05) → $2600.00, Fun Level 1

In-Town Job:
- 40 hours (0.35) → $2730.00, Fun Level 3
- 34 hours (0.50) → $2320.50, Fun Level 3
- 30 hours (0.15) → $2047.50, Fun Level 3

Salary Fun Level: 5, 4, 3, 2, 1

Fun Level: 5, 4, 3, 2, 1
Analysis Salary Objective

Conclusion:

- Forest Job preferred Over In-Town job
- CRP’s cross. Hence, No Stochastic Dominance
### Fun Level Objective

#### Conclusion:
- Forest Job preferred Over In-Town job
- CRP’s cross. Hence, No Stochastic Dominance

<table>
<thead>
<tr>
<th>Outcome</th>
<th>Fun Level</th>
<th>Prob</th>
<th>Fun Level*Prob</th>
<th>Prob</th>
<th>Fun Level*Prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 -BEST</td>
<td>100.00%</td>
<td>0.10</td>
<td>10.0%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 -GOOD</td>
<td>90.00%</td>
<td>0.25</td>
<td>22.5%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 - MIDDLE</td>
<td>60.00%</td>
<td>0.40</td>
<td>24.0%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 - BAD</td>
<td>25.00%</td>
<td>0.20</td>
<td>5.0%</td>
<td>1.00</td>
<td>60.00%</td>
</tr>
<tr>
<td>1 - WORST</td>
<td>0.00%</td>
<td>0.05</td>
<td>0.0%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ E[\text{Fun Level}] = 61.5\% \quad \text{E[Fun Level]} = 60.00\% \]
Multiple Objective Analysis

- It is clear from both objective analyses that the Forest-Job is the strongly preferred, although neither Stochastic nor Deterministic Dominance can be observed in them.

- Careful as you are in your decisions you decide to trade-off the salary objective and having fun objective in a multiple objective analysis.

- Before trade-off analysis can be conducted both objectives have to be measured on a “comparable” scale.
Multiple Objective Analysis: Construct 0-1 Scale

• **Having Fun Objective** already has a 0-1 scale:
  
  Transformed to 0-1 scale or 0%-100% scale

  Set Best=100%, Worst=0%, Determine intermediate values

  **Having Fun objective:**
  Best(100%), Good(90%), Middle(60%), Bad(25%), Worst (0%)

• Construct 0-1 scale for **Salary Objective**:

  $2730.00=100%, \$2047.50=0%$

  Intermediate dollar amount $X$:

  \[
  \frac{X - \$2047.50}{\$2730 - \$2047.50} \cdot 100\%
  \]
Multiple Objective Analysis: Assess Trade-Off

\[ k_s = \text{weight for salary} \quad k_f = \text{weight for fun} \]

\[ k_s + k_f = 1 \]

Using Expert Judgment:
Going from worst to best in salary objective is 1.5 times more important than going from worst to best in having fun objective. Hence: \( k_s = 1.5 \cdot k_f \)

\[
\begin{align*}
\begin{cases}
    k_s + k_f = 1 \\
    k_s = 1.5 \cdot k_f
\end{cases} & \iff \begin{cases}
    1.5 \cdot k_f + k_f = 1 \\
    k_s = 1.5 \cdot k_f
\end{cases} \iff \begin{cases}
    k_f = \frac{1}{2.5} = \frac{2}{5} \\
    k_s = 3 \cdot \frac{3}{5} = \frac{3}{5}
\end{cases}
\end{align*}
\]
### Multiple Objective Analysis: Convert Scales

#### Consequences
- Salary (0.6)
- Fun Level (0.4)

#### Fun Level
- 5 (0.10)
  - 81% 100%
- 4 (0.25)
  - 81% 90%
- 3 (0.40)
  - 81% 60%
- 2 (0.20)
  - 81% 25%
- 1 (0.05)
  - 81% 0%

#### # hours per week
- In-Town Job
  - 40 hours (0.35)
    - 100% 60%
  - 34 hours (0.50)
    - 40% 60%
  - 30 hours (0.15)
    - 0% 60%

- Forest Job
  - 81% 100%

### Multiple Objective Analysis
- Salary (0.6)
- Fun Level (0.4)

#### Options
- Forest Job
- In-Town Job

#### Criteria
- # hours per week
- Fun level

#### Weights
- Salary: 0.6
- Fun Level: 0.4
Multiple Objective Analysis: Combine Objectives

---

**Forest Job**

- **Fun level**
  - 5 (0.10) 88.6%
  - 4 (0.25) 84.6%
  - 3 (0.40) 72.6%
  - 2 (0.20) 58.6%
  - 1 (0.05) 48.6%

- **# hours per week**
  - 40 hours (0.35) 84.0%
  - 34 hours (0.50) 48.0%
  - 30 hours (0.15) 24.0%

**In-Town Job**

- 34 hours (0.50) 48.0%

**Total Score**

- 88.6%
- 84.6%
- 72.6%
- 58.6%
- 48.6%
- 84.0%
- 48.0%
- 24.0%
Analysis Overall Satisfaction

<table>
<thead>
<tr>
<th>Overall Satisfaction</th>
<th>Prob</th>
<th>OS*Prob</th>
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</thead>
<tbody>
<tr>
<td>88.57%</td>
<td>0.10</td>
<td>8.9%</td>
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<td>84.57%</td>
<td>0.25</td>
<td>21.1%</td>
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<tr>
<td>72.57%</td>
<td>0.40</td>
<td>29.0%</td>
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<tr>
<td>58.57%</td>
<td>0.20</td>
<td>11.7%</td>
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<td>48.57%</td>
<td>0.05</td>
<td>2.4%</td>
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E[OS] = 73.2%

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<th>Prob</th>
<th>OS*Prob</th>
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<tr>
<td>84.00%</td>
<td>0.35</td>
<td>29.40%</td>
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<td>48.00%</td>
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<tr>
<td>24.00%</td>
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<td>3.60%</td>
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E[OS] = 57.00%

**Conclusion:**
- Forest Job preferred Over In-Town job
- CRP’s do not cross. Hence, **Stochastic Dominance present.**