## EXTRA PROBLEM 7: SENSITIVITY ANALYSIS



## A. Create a two-way sensitivity graph that shows optimal strategies for Liedtke for all possible values of $p$ and $q$

Strategy $A=$ Accept $\$ 2$ billion.
Strategy B = Counteroffer $\$ 5$ billion, then refuse if Texaco offers $\$ 3$ billion.
Strategy $\mathrm{C}=$ Counteroffer $\$ 5$ billion, then accept if Texaco offers $\$ 3$ billion.
$\operatorname{EMV}(\mathrm{A})=2$

$$
\begin{aligned}
\operatorname{EMV}(\mathrm{B}) & =0.17(5)+0.5[p 10.3+q 5+(1-p-q) 0]+0.33[p 10.3+q 5+(1-p-q) 0] \\
& =0.85+8.549 p+4.15 q .
\end{aligned}
$$

$\operatorname{EMV}(\mathrm{C})=0.17(5)+0.5[p 10.3+q 5+(1-p-q) 0]+0.33(3)$
$=1.85+5.15 p+2.5 q$.

## NOW CONSTRUCT THREE INEQUALITIES:

- $\operatorname{EMV}(\mathrm{A})>\operatorname{EMV}(\mathrm{B}) \Leftrightarrow$

$$
\begin{align*}
& 2>0.85+8.549 p+4.15 q \Leftrightarrow \\
& 0.135-0.485 q>p \tag{1}
\end{align*}
$$

- $\operatorname{EMV}(\mathrm{A})>\operatorname{EMV}(\mathrm{C}) \Leftrightarrow$

$$
2>1.85+5.15 p+2.5 q \Leftrightarrow
$$

$$
\begin{equation*}
0.03-0.485 q>p \tag{2}
\end{equation*}
$$

- $\operatorname{EMV}(\mathrm{B})>\operatorname{EMV}(\mathrm{C}) \Leftrightarrow$

$$
\begin{align*}
& 0.85+8.549 p+4.15 q>1.85+5.15 p+2.5 q \Leftrightarrow \\
& p>0.294-0.485 q . \tag{3}
\end{align*}
$$

Plot these three inequalities as lines on a graph with $p$ on the vertical axis and $q$ on the horizontal axis. Note that only the region below the line $p+q=1$ is feasible because $p+q$ must be less than or equal to one.


These three lines divide the graph into four separate regions, labeled I, II, III, and IV.


Inequality (3) divides regions I and II. For points above this line, $p>0.294-0.485 q$, and so $\operatorname{EMV}(B)>\operatorname{EMV}(C)$.

Inequality (1) divides regions II and III. For points above this line, $p>0.135-0.485 q$, and $\operatorname{EMV}(B)>\operatorname{EMV}(A)$. As a result of this, we know that $B$ is the preferred choice in region $I$ and that $C$ is the preferred choice in region II [where $E M V(C)>E M V(B)>E M V(A)]$.

Inequality (2) divides regions III and IV. For points above this line, $p>0.03-0.485 q$, and $\operatorname{EMV}(C)>\operatorname{EMV}(A)$. Thus, we now know that $C$ is the preferred choice in region III $[$ where $\mathrm{EMV}(\mathrm{C})>\mathrm{EMV}(\mathrm{A})$ and $\mathrm{EMV}(\mathrm{C})>\mathrm{EMV}(\mathrm{B})]$, and A is preferred in region IV .

Thus, we can redraw the graph, eliminating the line between regions II and III
B. If Liedtke thinks that $p$ must be at least 0.15 and $q$ must be more than 0.35 can he make the decision without further probability assessment.


## Conclusion:

The shaded area in the figure represents those points for which $p>0.15$ and $q>0.35$. Note that all of these points fall in the "Choose B" region. Thus, Liedtke should adopt strategy B: Counteroffer $\$ 5$ billion, then refuse if Texaco offers $\$ 3$ billion.

