

EXTRA PROBLEM 6: SOLVING DECISION TREES

Read the following decision problem and answer the questions below.

A manufacturer produces items that have a probability p of being defective. These items are formed into batches of 150. Past experience indicates that some are of good quality (i.e. $p=0.05$) and others are of bad quality (i.e. $p=0.25$). Furthermore, 80% of the batches produced are of good quality and 20% of the batches are of bad quality. These items are then used in an assembly, and ultimately their quality is determined before the final assembly leaves the plant. The manufacturer can either screen each item in a batch and replace defective items at a total average cost of \$10 per item or use the items directly without screening. If the latter action is chosen, the cost of rework is ultimately \$100 per defective item. For these data, the costs per batch can be calculated as follows:

	$p = 0.05$	$p = 0.25$
Screen	\$1500	\$1500
Do not Screen	\$750	\$3750

Because screening requires scheduling of inspectors and equipment, the decision to screen or not screen must be made 2 days before the potential screening takes place. However, the manufactures may take one item taken from a batch and sent it to a laboratory, and the test results (defective or non-defective) can be reported before the screen/no-screen decision must be made. After the laboratory test, the tested item is returned to its batch. The cost of this initial inspection is \$125. Also note that the probability that a random sample item is defective is

$$0.8 * 0.05 + 0.2 * 0.25 = 0.09,$$

and the probability that an item in a lot is of good quality given a randomly sampled item is defective is 0.444 and the probability that an item in a lot is of good quality given a randomly sampled item is not defective is 0.835. The manufactures wants to minimize his cost.

A. Derive the cost figures in the table above. Clearly show your calculations

B. What Law of Probability was used in deriving. Provide an Explanation

$$\Pr(\text{Sampled Item is Defective}) = 0.09$$

C. Show that:

$$\Pr(\text{Good Quality}|\text{Sampled Item is Defective}) = 0.444$$

$$\Pr(\text{Good Quality}|\text{Sampled Item is Not Defective}) = 0.835$$

D. Calculate and show your calculations:

$\Pr(\text{Bad Quality} | \text{Sampled Item is Defective}) = ?$

$\Pr(\text{Bad Quality} | \text{Sampled Item is Not Defective}) = ?$

E. Model the decision problem in a decision tree and fill in ALL the details. Solve the tree using EMV and clearly show your calculations in the tree.

F. How many cumulative risk profiles can be drawn for the decision tree under E?
Provide an Explanation.

G. Solve the tree under E using EMV and clearly show your calculations in the tree under E

H. Describe in words the optimal decision strategy.

I. Draw the Cumulative Risk Profile for each alternative of the immediate decision, taking optimal decisions from thereon. What can you conclude with respect to dominance considerations? (Hint: You should be drawing 2 Cumulative Risk Profiles).

