EXTRA PROBLEM 4:
SCREENING FOR COLORECTAL CANCER

The fecal occult blood test, widely used both in physicians’ offices and at home to screen patients for colon and rectal cancer, examines a patient’s stool sample for blood, a condition indicating that cancer may be present. A recent study funded by the National Cancer Institute found that of 15,000 people tested on an annual basis, 10% were found to have blood in their stools. These 10% underwent further testing, including colonoscopy, the insertion of an optical-fiber tube through the rectum in order to inspect the colon and rectum visually for direct indications of cancer. Only 2.5% of those (10%) having colonoscopy actually had cancer. Additional information in the study suggests that, of the patients who were tested, approximately 5 out of 1000 tested negative (no blood in stool) but eventually did develop cancer.
A. Create a probability table that shows the relationship between blood in a stool sample and colorectal cancer. Determine $P(\text{Cancer} | \text{Blood})$ and $P(\text{Cancer} | \text{No Blood})$.

Let $B = \{\text{Blood in Stool}\}$, Let $C = \{\text{Patient has Colorectal Cancer}\}$

“Of the people tested, 10% has blood in their stools”. Therefore:

$$P(B) = 0.10$$

“These 10% underwent colonoscopy, and 2.5% of those 10% actually had cancer”. Therefore:

$$P(C \mid B) = 0.025$$

We can thus derive:

$$P(B \cap C) = P(C \mid B) P(B) = 0.025 \times 0.10 = 0.0025$$
“Of the people tested, approximately 5 out of 1000 had no blood but did develop cancer”. Therefore:

\[ P(\overline{B} \cap C) = 0.005 \]

FILLING IN WHAT WE NOW:

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CALCULATE THE REMAINDER BY APPLYING LAW OF TOTAL PROBABILITY

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<td>$\overline{C}$</td>
<td>$P(B \cap \overline{C}) = 0.0975$</td>
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Calculation:

$$P(\overline{C} \mid \overline{B}) = \frac{P(\overline{B} \cap \overline{C})}{P(\overline{B})} = \frac{0.005}{0.90} = 0.00556$$
Note:

\[
\frac{P(C \mid B)}{P(B \cap C)} = \frac{0.025}{0.00556} = 4.456
\]

Hence, you are approximately 5 times more likely to attract cancer if you have blood in your stool than if you do not. However, the chances of having Cancer and Blood in your stool is still quite low (1 in 400).

\[
P(B \cap C) = 0.0025
\]

Note, that this probability is smaller than that of people having with no blood in their stool and having cancer (1 in 200)

\[
P(\overline{B} \cap C) = 0.005
\]

WHY?
The Study results have led some medical researchers to agree with the American’s Cancer Society’s long-standing recommendation that all US-residents over 50 years of age be tested annually. On the other hand, many researchers claim the cost of such screening, including the cost of follow-up testing on 10% of the populations, far exceeds its value. Assume that the test can be performed for as little as $10 per person, that colonoscopy cost $750 on average, and that about 60 million people in the United States are over age 50. Also assume that over these 13 years approximately a total of 60 million people participate in this policy.

B. What is the expected cost (including follow up colonoscopy) of implementing a policy of screening everyone over age 50? What is the expected number of people who must undergo colonoscopy? What is the
expected number of people who undergo colonoscopy only to find that they do not have cancer after all?

Annual Expected Cost = (60 Million)*$10 + (10%)*(60 Million)*$750 = 5.1 Billion 

Million = 10^6 Billion = 10^9

Annual Expected Number of People Undergoing Colonoscopy =
= (10%)*(60 Million) = 6 Million

Annual Expected Number of People undergoing Colonoscopy and do not have Cancer =

60Million * P(B \cap \overline{C}) = 60Million * 0.0975 = 5.85Million
Over 13 years of follow-up study, 0.6% (6 out of 1000) of those who were screened annually with the fecal blood test died from colon cancer anyway. Of those who were not screened, 0.9% (9 out of 1000) died of colon cancer during the same 13 years. Thus the screening procedure saves approximately 3 lives per 1000 every 13 years.

C. (3 Points) Use this information, along with your calculations from Questions A and B, to determine the expected cost of saving a life by implementing a policy requiring everyone over 50 to be screened every year.

Total cost over 13 years of screening = 13*5.1 Billion = 66.3 Billion
(6 out of 1000) of those who were screened annually died of cancer anyway
(9 out of 1000) of those who were not screened annually died of cancer.

**Conclusion:** we approximately save (3 out of 1000) = 0.003 = 0.3%

Expected Number of Lives Saved
over 13 years = 60 Million * 0.3% = 180000 Lives

Hence, Expected Cost per saved Life = 66.3 Billion/180000 = $368333

D. What is your conclusion? Do you think everyone over 50 should be
screened? From your personal point of view, informed now by your
calculations above, would the saved lives be worth the money spent and
the inconvenience, worry, discomfort and potential complications of
subjecting approximately 6 million people to each year to colonoscopy even though relatively few of them actually have detectable and curable cancer?

Answer may vary depending on personal views, but would have to address whether the inconvenience to 5.85 Million People and an Expected Cost of $368,333 per life saved is worth saving an expected number of 180,000 lives. Also, a full analysis would have to include the expected number of lives affected by complications due to the colonoscopy exam.