EXTRA PROBLEM 3:
PROBABILITY CALCULUS

$$P(A) = 0.68\ ,\ P(B \mid A) = 0.30\ ,\ P(B \mid \overline{A}) = 0.02$$

1. **Find** $P(\overline{A})$, $P(A \cap B)$, $P(\overline{A} \cap B)$

$$P(\overline{A}) = 1 - P(A) = 1 - 0.68 = 0.32$$

$$P(A \cap B) = P(B \mid A)P(A) = 0.30 * 0.68 = 0.2040$$

$$P(\overline{A} \cap B) = P(B \mid \overline{A})P(\overline{A}) = 0.02 * 0.32 = 0.0064$$

Next, summarize what we know in a probability table.

	A	$\overline{A}$	
B	$P(A \cap B) = 0.2040$	$P(\overline{A} \cap B) = 0.0064$	$P(B) = ?$
$\overline{B}$	$P(A \cap \overline{B}) = ?$	$P(\overline{A} \cap \overline{B}) = ?$	$P(\overline{B}) = ?$
	$P(A) = 0.68$	$P(\overline{A}) = 0.32$	1

2. Use your results under 1 and complete the following probability table

	A	$\overline{A}$	
B	$P(A \cap B) = 0.2040$	$P(\overline{A} \cap B) = 0.0064$	$P(B) = 0.2104$
$\overline{B}$	$P(A \cap \overline{B}) = 0.4760$	$P(\overline{A} \cap \overline{B}) = 0.3136$	$P(\overline{B}) = 0.7896$
	$P(A) = 0.68$	$P(\overline{A}) = 0.32$	1

3. Now use the table to find the following:

$$P(\overline{B}\,|\,A),\ P(\overline{B}\,|\,\overline{A}),\ P(A\,|\,B),$$
$$P(\overline{A}\,|\,B),\ P(A\,|\,\overline{B}),\ P(\overline{A}\,|\,\overline{B}).$$

Note: Always normalize using the probability of the conditioning argument.

$$P(\overline{B}\,|\,A)=\frac{P(A\cap\overline{B})}{P(A)}=\frac{0.4760}{0.68}=0.7000$$

$$P(\overline{B}\,|\,\overline{A})=\frac{P(\overline{A}\cap\overline{B})}{P(\overline{A})}=\frac{0.3136}{0.32}=0.98$$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{0.2040}{0.2104} = 0.9696$$

$$P(\overline{A} \mid B) = \frac{P(\overline{A} \cap B)}{P(B)} = \frac{0.0064}{0.2104} = 0.0304$$

$$P(A \mid \overline{B}) = \frac{P(A \cap \overline{B})}{P(\overline{B})} = \frac{0.4760}{0.7896} = 0.6028$$

$$P(\overline{A} \mid \overline{B}) = \frac{P(\overline{A} \cap \overline{B})}{P(\overline{B})} = \frac{0.3136}{0.7896} = 0.3972$$