EXTRA PROBLEM 10: THEORETICAL PROBABILITY MODELS

In bottle production, bubbles that appear in the glass are considered defects. Any bottle that has more than two bubbles is classified as "nonconforming" and is sent to recycling. Suppose that the number of bubbles in a bottle is Poisson distributed with an average of 1.1 bubbles per bottle. Bubbles occur independently from one another. In answering the questions below use the attached probability tables.

A. What is the probability that a randomly chosen bottle is nonconforming?

Define the random variable X such that:

$$X \equiv Number\, of\, bubbles\, in\, a\, bottle$$

Then from the text we have:

$$Pr(X = k) = \frac{m^k}{k!}e^{-m}, k = 0, 1, 2, \ldots, \text{ where } m = 1.1$$

Hence,

$$Pr(Random\,bottle\,is\;\;non\text{-}conforming) = Pr(X > 2) =$$

$$1 - Pr(X \leq 2) = 1 - 0.90\;\; = 0.10$$ **(See, Table Page 702)**

B. Bottles are packed in cases of 12. An inspector chooses one bottle from each case. If it is nonconforming, she inspects the entire case, replacing nonconforming bottles with good ones. This process is called rectification. If the chosen bottle conforms (has two or fewer bubbles), then she passes the case. In total, 20 cases are produced. What is the probability that at least 18 of them pass?

Define the random variable Y such that:

$$Y \equiv Number\,of\,cases\,(out\,of\,20)\,that\,pass$$

Define: $"Succes" = A\,case\,passes$

Then: $p \equiv Pr("Succes") = Pr(X \leq 2) = 0.90$ **(See, Question A)**

And: $Y \sim Binomial(N, p)$, **where** $N = 20$ **and** $p = 0.90$

We need to calculate: $Pr(Y \geq 18 | N = 20, p = 0.90)$.

But, cumulative binomial probability tables (Starting at page 688, only go up to a "success" probability of 0.50. **Hence, we need to use the dual binomial random variable of** Y**, that is**

$$Z \equiv 20 - Y = Number\,of\,cases\,(out\,of\,20)\,that\,do\,not\,pass$$

And: $Z \sim Binomial(N, q)$, **where** $N = 20$ **and** $q = 1 - p = 0.10$

Hence:

$$Pr(Y \geq 18|N=20, p=0.90) = Pr(20-Y \leq 20-18|N=20, p=0.90)$$

$$= Pr(Z \leq 2|N=20, q=0.10) = 0.677$$ **(see Table page 694)**

C. What is the expected number of noncomforming bottles in the 20 cases after they have been inspected and rectified using the scheme described in part B.

Define: $A = Number\, of\, non\text{-}conforming\, bottles\, in\, a\, case\, that\, passes.$

Then:

$A \sim Binomial(M, p)$, **where** $M = 12 - 1 = 11$ **and** $p = 0.10$

Because the case passed and hence the randomly selected bottle conformed.

Hence: $E[A] = 11 \cdot 0.10 = 1.1$

Define:

$$B = Number\, of\, non-conforming\, bottles\, in\, a\, case\, that\, did\, not\, pass$$

Then: $B \equiv 0$

Because the entire case is rectified.

Define:

$$C = Number\, of\, non-conforming\, bottles\, in\, 20\, cases\, following\, the\, above\, rectification\, policy.$$

Then: $C = Y \cdot A + Z \cdot B = Y \cdot A$ **(Because** $B \equiv 0$**)**

and: $$E[C] = E[Y] \cdot E[A]$$

Noting that: $E[Y] = 20 \cdot 0.9 \; = 18$ **(Average Number of Cases that pass)**

We have:

$$E[C] = 18 \cdot 1.1 = 19.8$$

(Average number of non-conforming bottles in 20 cases following the above rectification policy).