EXTRA PROBLEM 1:
MONTHLY LOAN PAYMENT

Suppose you are planning to buy a new car. The car costs $25000, you have been approved for financing the loan with a 5 year term through the car dealer and the annual interest rate on the loan is 5% - compounded monthly.

A. Using the following notation

B = Loan Balance (i.e. in description above B=$25000)
R = Annual Interest Rate (i.e. in description above R=0.05)
MP = Monthly Payment

derive the monthly payment MP as a function of B and R by setting the net present value of the cash-flow of monthly payments of size MP for the term of the loan equal to the loan amount B.

Hint: Use the relationship

$$\sum_{j=1}^{n} x^{j} = \frac{x^{n+1} - x}{x - 1}$$

ANSWER:

Derive the monthly payment MP as a function of B and R by setting the net present value of the cash-flow of monthly payments of size MP for the term of the loan equal to the loan amount B. Hence,

$$B = \sum_{j=1}^{n} \frac{MP}{(1+R/12)^{j}} = MP \cdot \sum_{j=1}^{n} \left(\frac{1}{1+R/12} \right)^{j}$$

Next, use hint:

$$B = MP \cdot \sum_{j=1}^{n} \left(\frac{12}{12+R} \right)^{j} = MP \cdot \frac{\left(\frac{12}{12+R} \right)^{n+1} - \frac{12}{12+R}}{\frac{12}{12+R} - 1}$$

Thus,

$$MP = B \cdot \frac{\frac{12}{12+R} - 1}{\left(\frac{12}{12+R}\right)^{n+1} - \frac{12}{12+R}} = B \cdot \frac{\frac{12}{12+R} - 1}{\frac{12}{12+R}\left(\frac{12}{12+R}\right)^{n} - \frac{12}{12+R}}$$

Finally, simplify by multiplying numerator and denominator by (12+R):

$$MP = B \cdot \frac{12 - 12 - R}{12\left(\frac{12}{12+R}\right)^{n} - 12} = \frac{B}{12} \cdot \frac{R}{1 - \left(\frac{12}{12+R}\right)^{n}}$$

B. Calculate MP for the case study description above using the relationship you derived under A.

B= $20000
R= 0.04
n = 48

$$MP = \frac{20000}{12} \cdot \frac{0.04}{\left(\frac{12}{12+0.04}\right)^{48} - 1} = \$451.58$$

B= $25000
R= 0.05
n = 60

$$MP = \frac{25000}{12} \cdot \frac{0.05}{\left(\frac{12}{12+0.05}\right)^{60} - 1} = \$471.78$$