EXTRA PROBLEM 1:
MONTHLY LOAN PAYMENT

Suppose you are planning to buy a new car. The car costs $25000, you have been approved for financing the loan with a 5 year term through the car dealer and the annual interest rate on the loan is 5% - compounded monthly.

A. Using the following notation

B = Loan Balance (i.e. in description above B=$25000)
R = Annual Interest Rate (i.e. in description above R=0.05)
MP = Monthly Payment

derive the monthly payment MP as a function of B and R by setting the net present value of the cash-flow of monthly payments of size MP for the term of the loan equal to the loan amount B.

**Hint:** Use the relationship

$$
\sum_{j=1}^{n} x^j = \frac{x^{n+1} - x}{x - 1}
$$
ANSWER:

Derive the monthly payment $MP$ as a function of $B$ and $R$ by setting the net present value of the cash-flow of monthly payments of size $MP$ for the term of the loan equal to the loan amount $B$. Hence,

$$B = \sum_{j=1}^{n} \frac{MP}{(1 + R/12)^j} = MP \cdot \sum_{j=1}^{n} \left( \frac{1}{1 + R/12} \right)^j$$

Next, use hint:

$$B = MP \cdot \sum_{j=1}^{n} \left( \frac{12}{12 + R} \right)^j = MP \cdot \left( \frac{12}{12 + R} \right)^{n+1} - \frac{12}{12 + R}$$
Thus,

\[ MP = B \cdot \frac{12}{12 + R} - 1 \]

\[ \left( \frac{12}{12 + R} \right)^{n+1} - \frac{12}{12 + R} \]

\[ = B \cdot \frac{12}{12 + R} - 1 \]

\[ \frac{12}{12 + R} \left( \frac{12}{12 + R} \right)^{n} - \frac{12}{12 + R} \]

Finally, simplify by multiplying numerator and denominator by \((12+R)\):

\[ MP = B \cdot \frac{12 - 12 - R}{12 \left( \frac{12}{12 + R} \right)^{n} - 12} \]

\[ = B \cdot \frac{R}{12} \left( 1 - \left( \frac{12}{12 + R} \right)^{n} \right) \]
B. Calculate MP for the case study description above using the relationship you derived under A.

\[ B = \$20000 \]
\[ R = 0.04 \]
\[ n = 48 \]

\[
MP = \frac{20000}{12} \cdot \frac{0.04}{\left(\frac{12}{12 + 0.04}\right)^{48}} - 1 = \$451.58
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\[ B = \$25000 \]
\[ R = 0.05 \]
\[ n = 60 \]

\[
MP = \frac{25000}{12} \cdot \frac{0.05}{\left(\frac{12}{12 + 0.05}\right)^{60}} - 1 = \$471.78
\]