LECTURE NOTES: EMGT 234

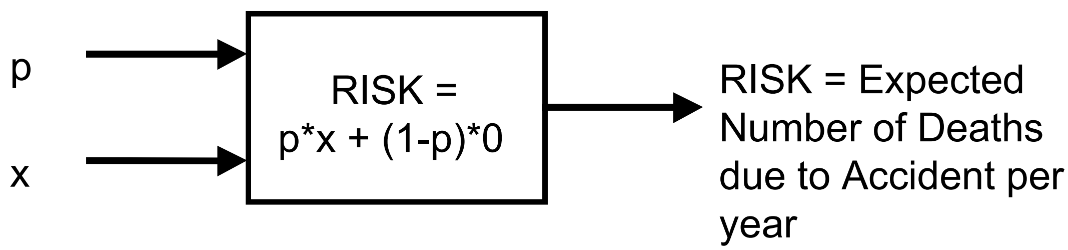
UNCERTAINTY MODELING

THE SIMPLEST RISK ANALYSIS MODEL

p = Pr(Accident during a Mission or Movement)
x =# (Deaths in a Mission Accident)
y =# (Deaths in case of no Mission Accident) = 0

Definition:
RISK = Expected Number of during the Mission

RISK = $p \cdot x + (1-p) \cdot y = p \cdot x$ = Probability * Impact



- **What Uncertainty Is Modeled?**

$$A = \begin{cases} a_0 & \text{No Accident during the Mission} \\ a_1 & \text{Accident during the Mission} \end{cases}$$

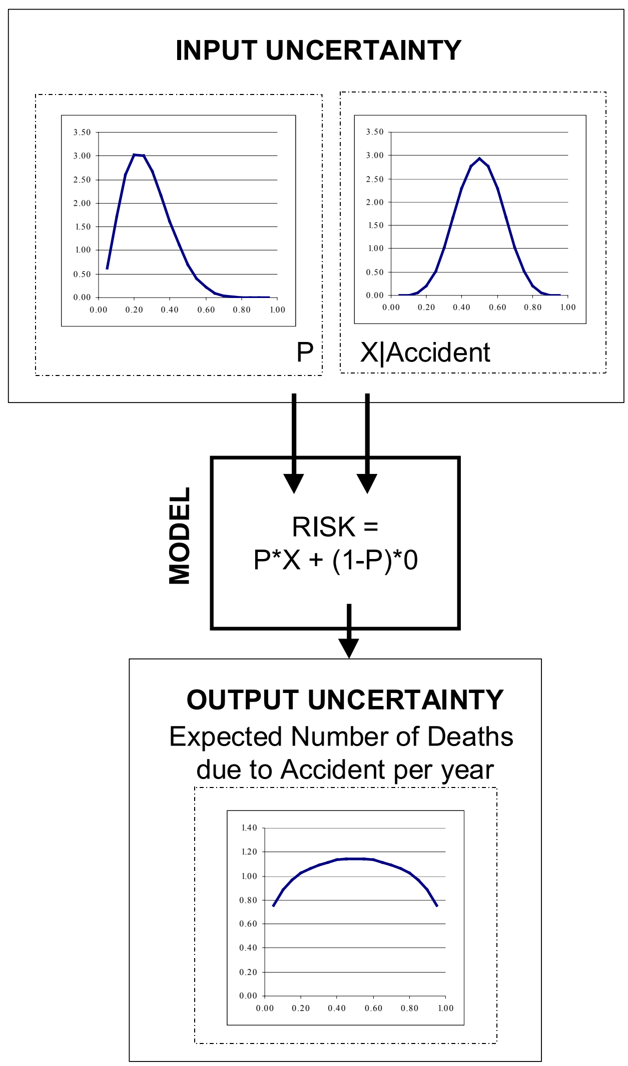
- **Uncertainty Model:**

$$\Pr(A = a_i) = \begin{cases} 1-p & \mathrm{i} = 0 \\ p & \mathrm{i} = 1 \end{cases}$$

- **Is This Satisfactory?**

Where are Additional Uncertainties located in the Model Above?

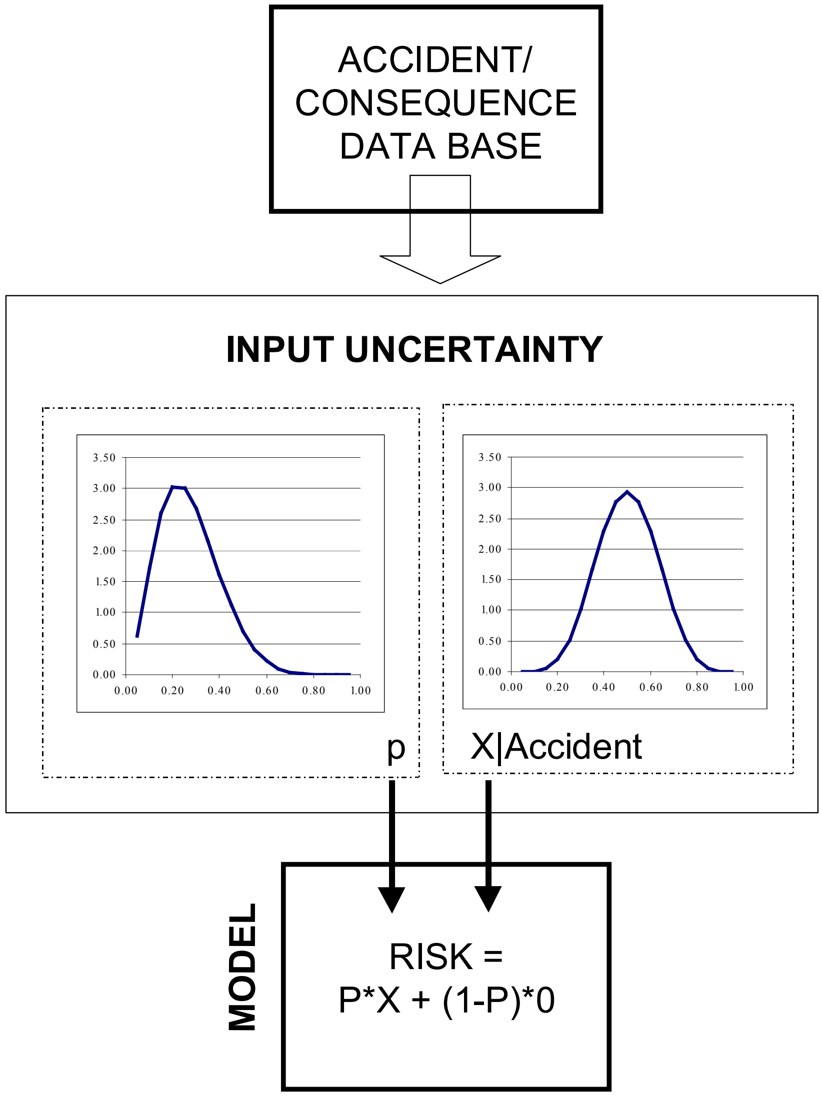
SOURCE 1: We are uncertain about the probability p $\longrightarrow$ P
SOURCE 2: We are uncertain about the consequence x $\longrightarrow$ X



Allows us to express Uncertainty in Output in terms of Credibility Intervals:

e.g. probability that Mission Mortality is between A and B is 95%.

How do we get Input Uncertainty?

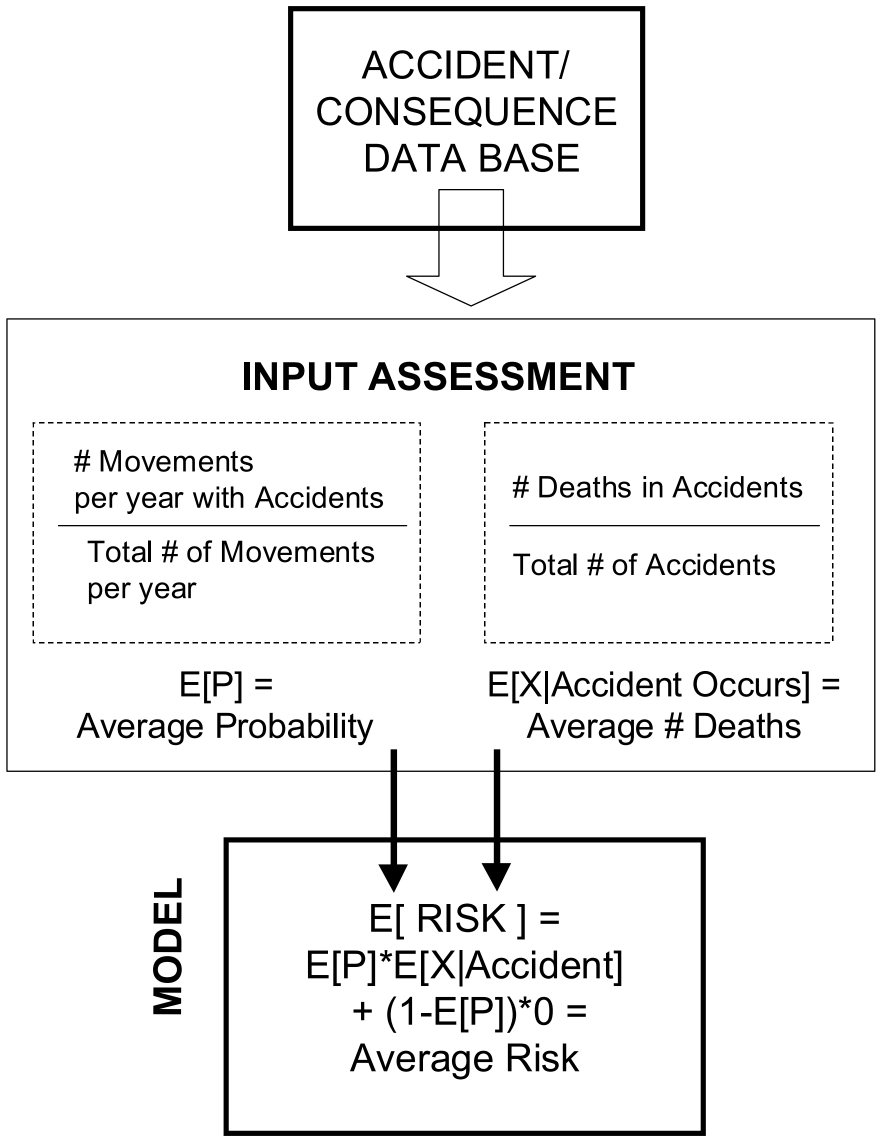


But:

- Accidents under study are typically **rare events,** often resulting in point estimates for probability and consequences with very large uncertainty bands

Uncertainty = Statistical Uncertainty
Because of focus on data-driven risk assessment.

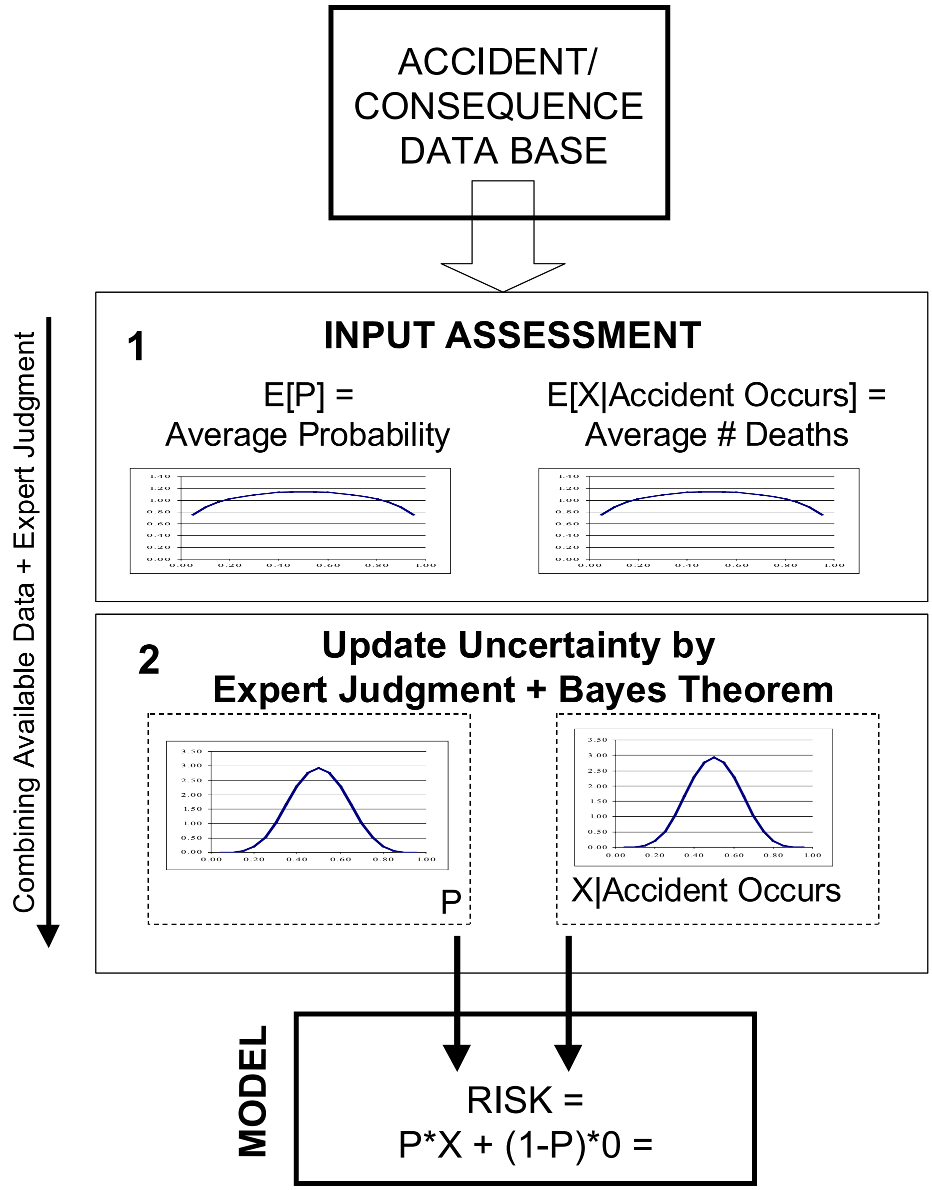
PRACTICAL LIMITATION OF DATA DRIVEN APPROACH FOR UNCERTAINTY



Conclusion:

- Uncertainty is often not analyzed due to limitations in availability of data + resource constraints.
- If uncertainty is analyzed based on data, uncertainty bands are too wide for meaningful interpretation.

HOW CAN WE DO BETTER?

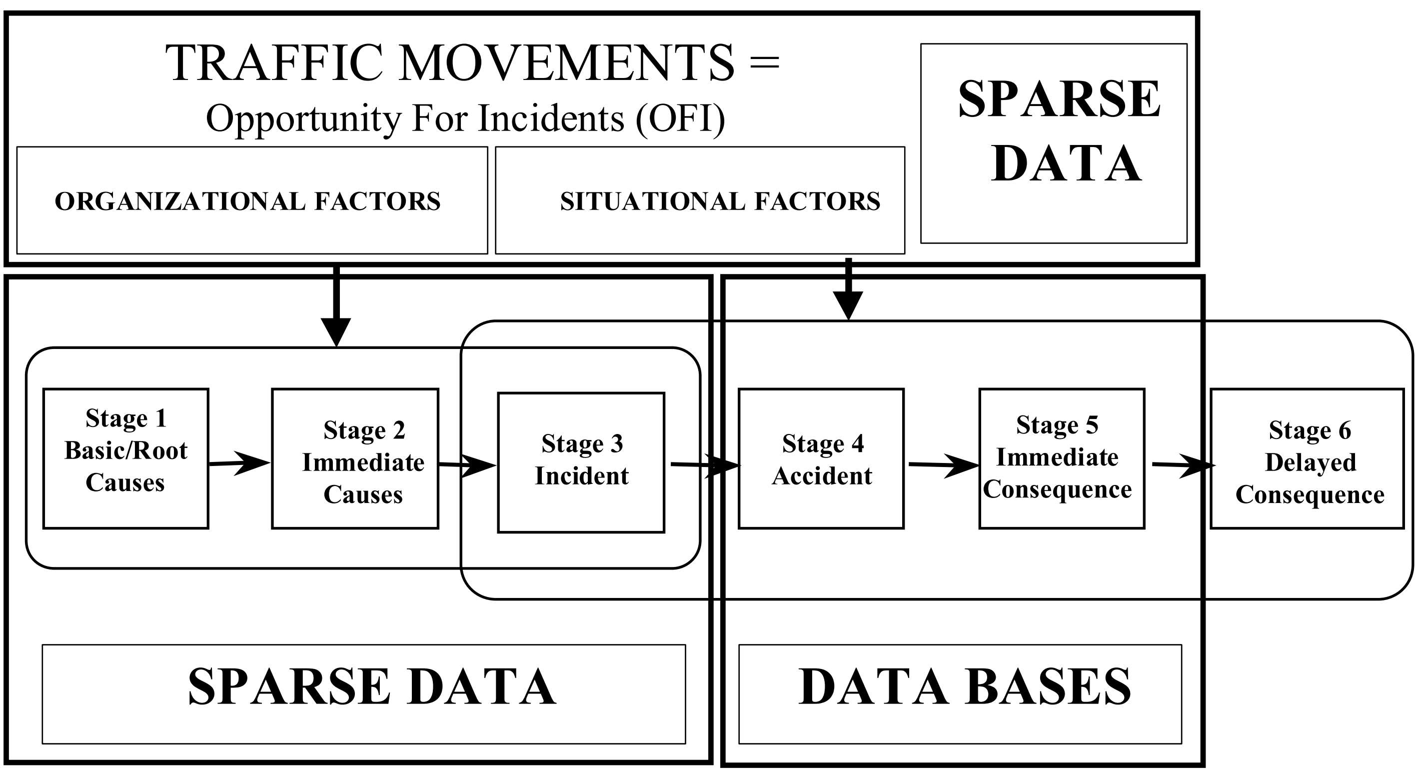


Approach above is advocated by Kaplan & Garrick

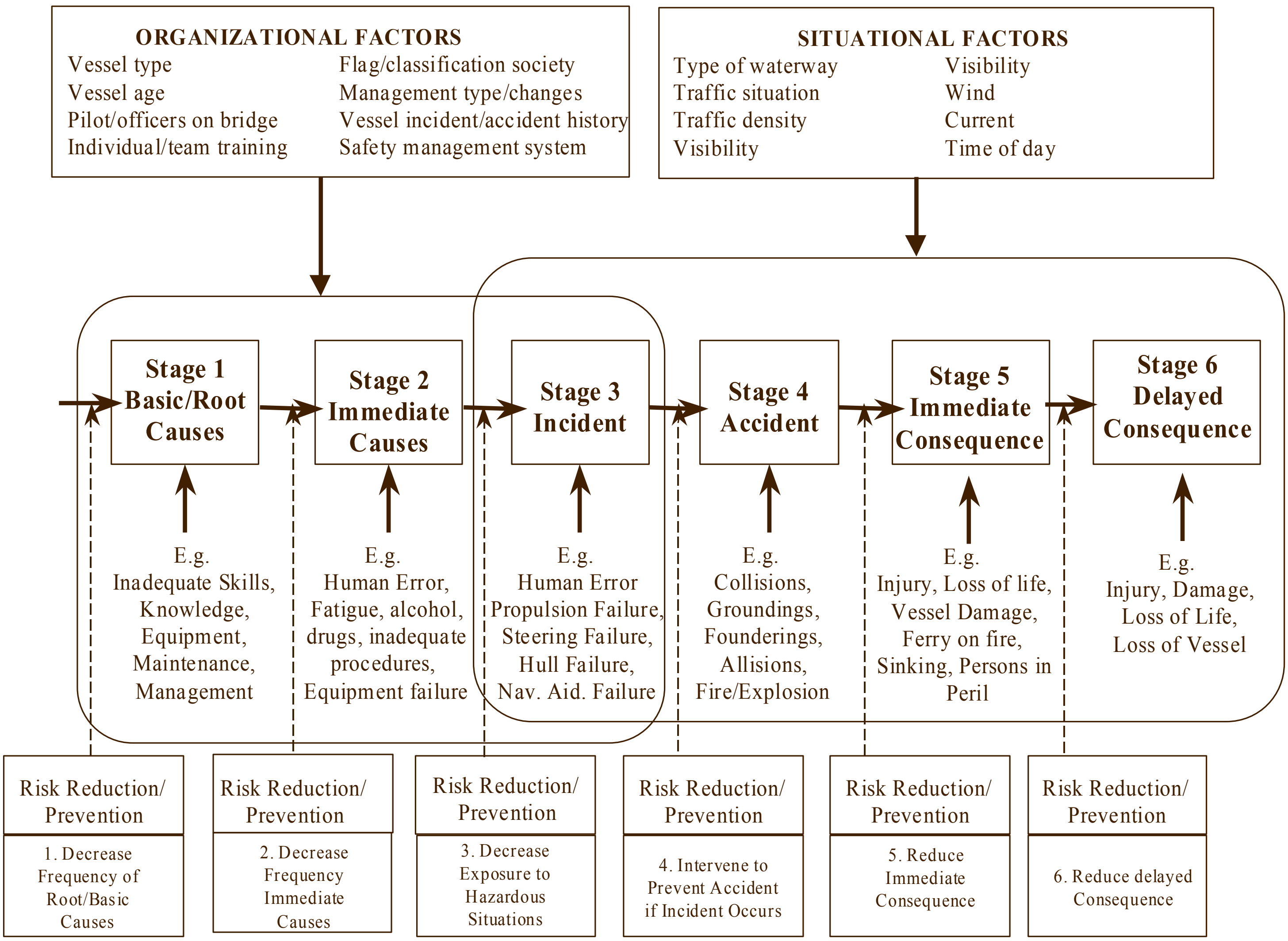
PRACTICAL LIMITATION OF DATA DRIVEN APPROACH FOR RISK MANAGEMENT

DILEMMA OF UNCERTAINTY MODELER
Client Always Wants More Detail:

Client prefers to have a very detailed "causal" probability model which explicitly models the effect of situational factors on the accident or consequence probability. E.g. the effect of visibility on the probability of an aircraft accident or the effect of guidance systems like GPS on the



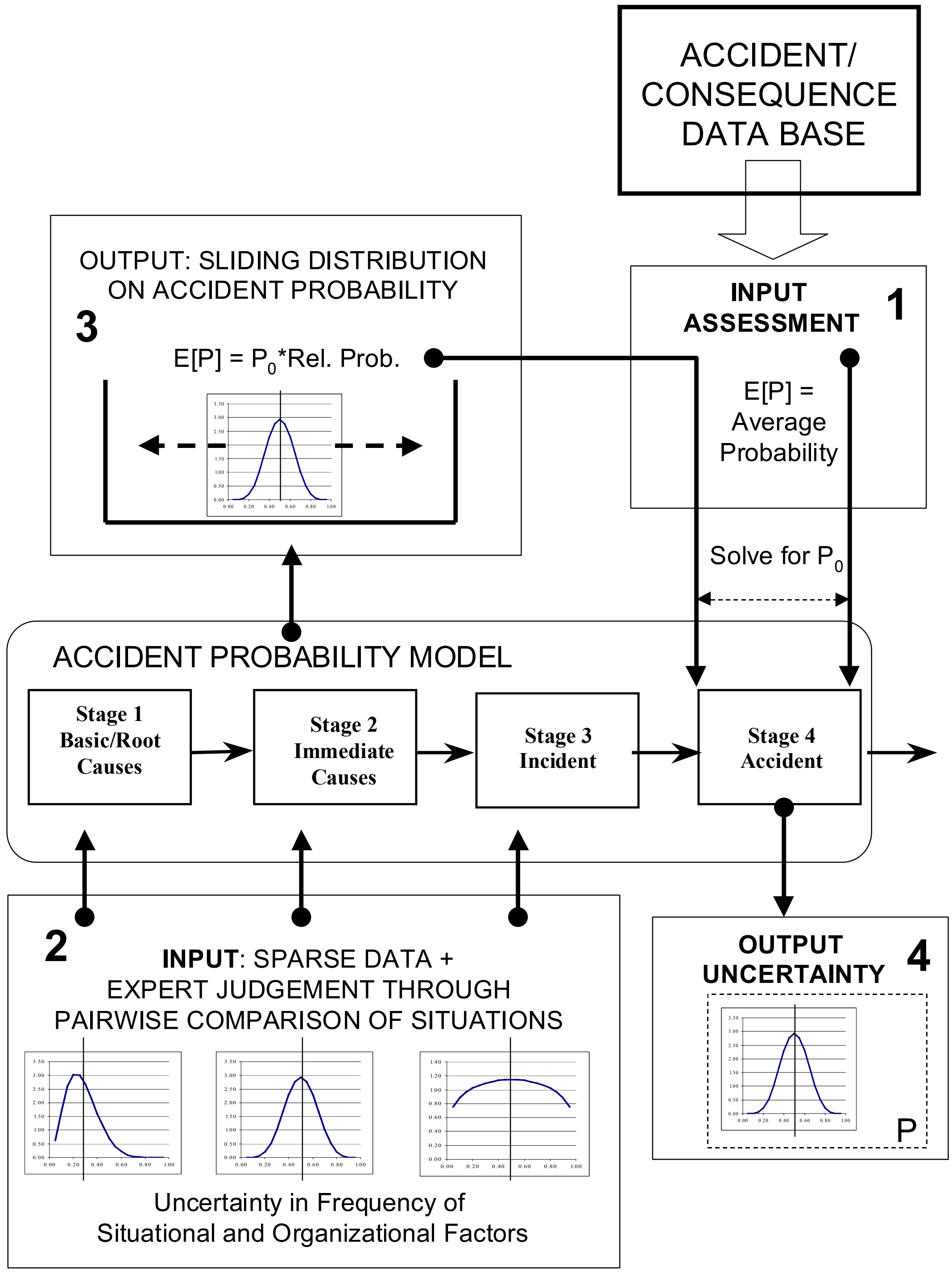
PRACTICAL LIMITATION OF DATA DRIVEN APPROACH FOR RISK MANAGMENT



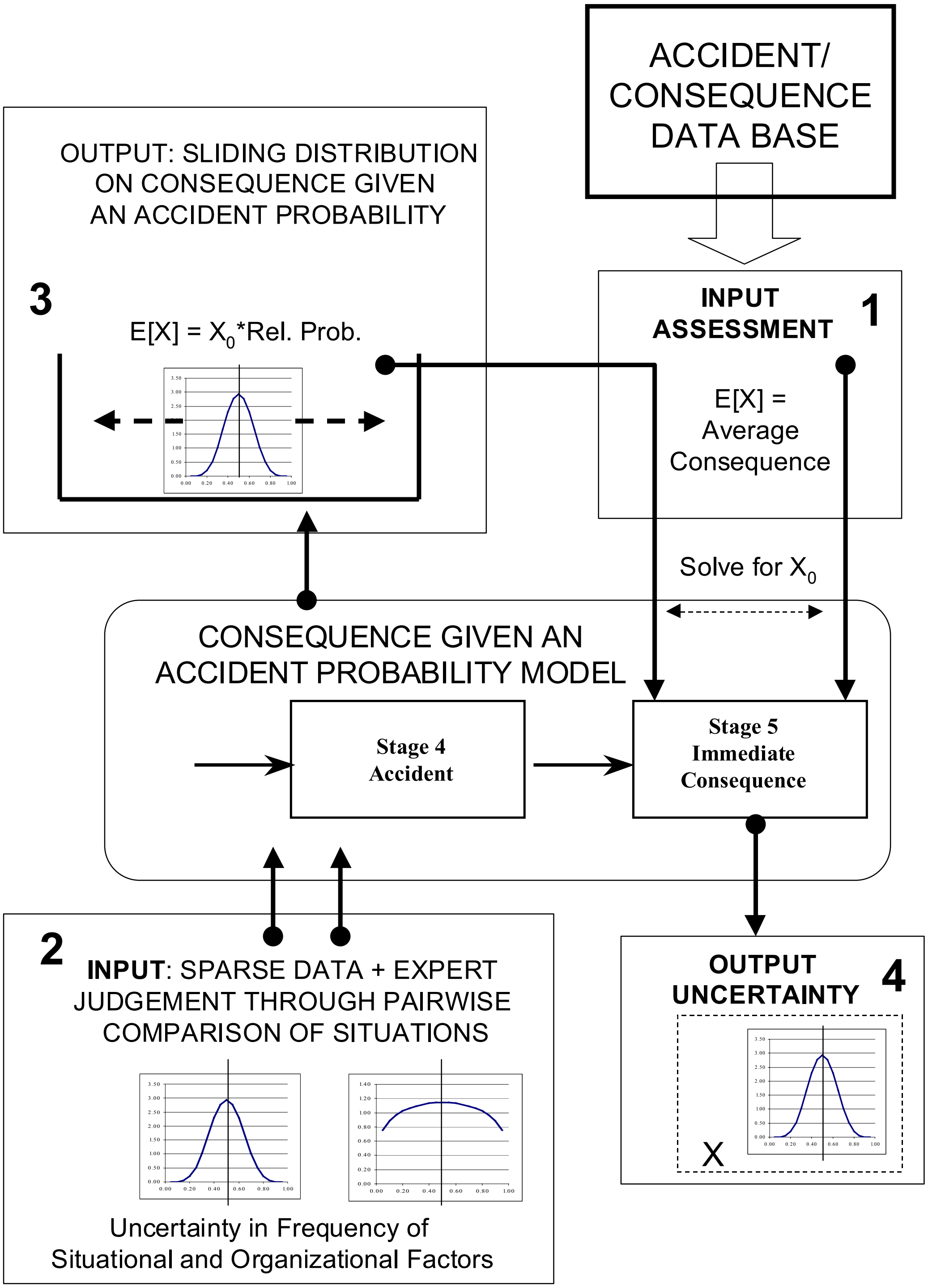
What to do?

Decompose Model in Smaller Components that may be estimated using Expert Judgment.

STEP 1: MODELING UNCERTAINTY IN ACCIDENT PROBABILITY

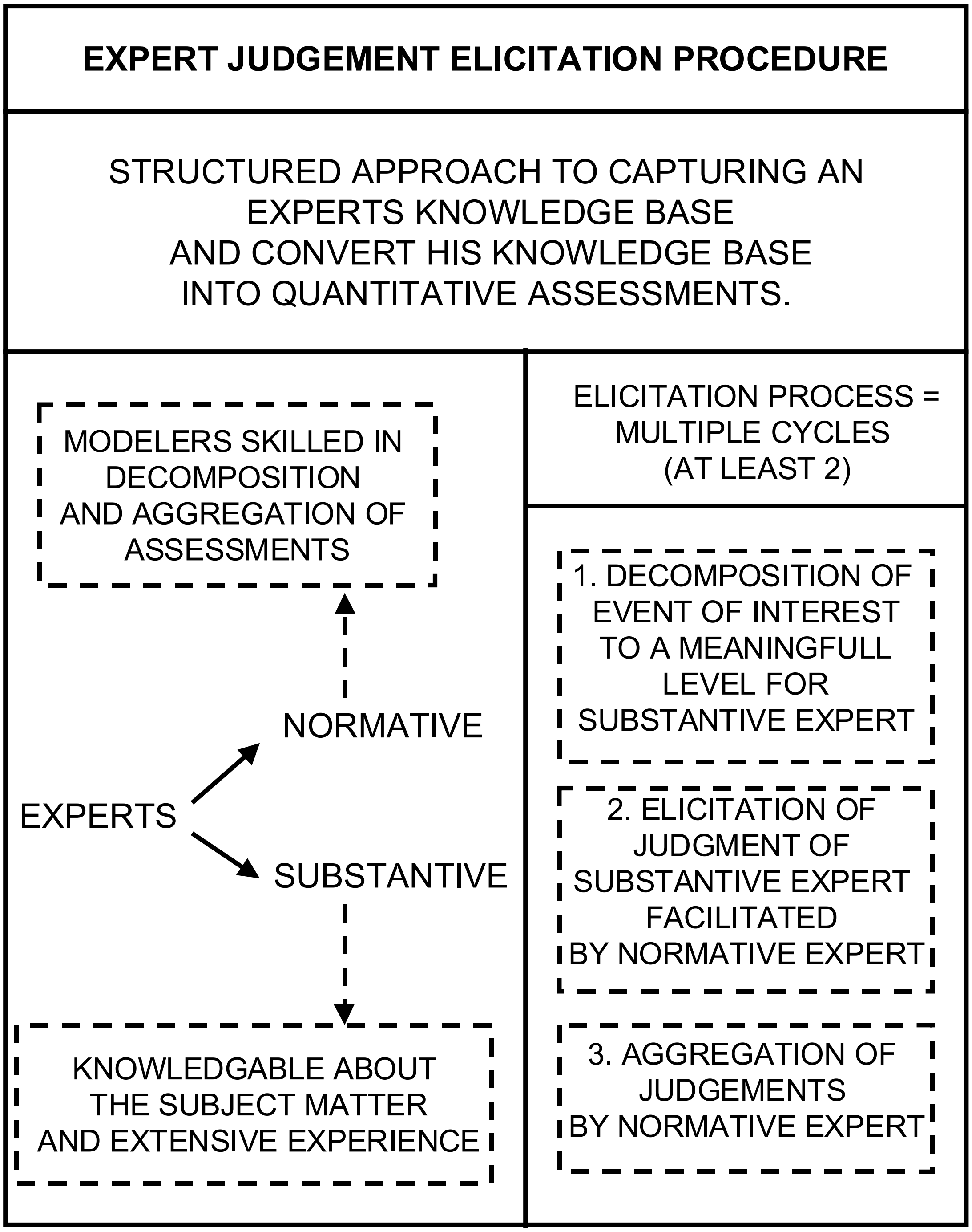


STEP 2: MODELING UNCERTAINTY IN CONSEQUENCE PROBABILITY GIVEN AN ACCIDENT



WHAT IS EXPERT JUDGMENT?

IT IS NOT!
A Group of Experts in
a Room deciding on Numbers.



EXAMPLE: THE DELPHI METHOD

- **Early 1950:** Developed by RAND Corporation as spin-off of an Air Force Research Rroject, "Project Delphi".
- **1963:** Wider audience due to 1963 RAND Study "Report on a long-range Forecasting study".

Probably, best known method to date of Eliciting and synthesizing expert judgment.

STEP 1:

Monitoring Team defines set of issues and selects sets of Respondents who are experts on the issues in question. Respondents do not know who other respondents are, and the responses are anonymous. Preliminary questionnaire is sent for comments, which are then used to establish a definitive questionnaire.

STEP 2:

Questionnaire is sent to respondents. Monitoring Team analyses the answers.

STEP 3:

The set of responses is sent back together with 25% lower and 25% upper responses. The respondents are asked if they wish to revise the initial predictions. Those who answered outside of the above range are asked to give arguments.

STEP 4:

The revised predictions are analyzed by the monitoring team and the outliers for arguments are summarized. GOTO STEP 2.

TYPICALLY THREE ROUNDS

EXAMPLE OF DELPHI QUESTIONNAIRE # 1

Questionnaire # 1

This is the first I a series of four questionnaires intended to demonstrate the use of the Delphi Technique in obtaining reasoned opinions from a group of respondents.

Each of the following six questions is concerned with developments in the United States with the next few decades.

In addition to giving your answer to each question, you are also being asked to rank the questions from 1 to 7. Here “1” means that in comparing your ability to answer this question with what you expect the ability of the other participants to be, you feel that you have the relatively best chance of coming closer to the truth than most of the others, while a “7” means that you regard that chance as relative least.

Rank		Answer*
☐	1. In your opinion, in what year will the median familiy income (in 1967 dollars) reach twice its present amount?	☐
☐	2. In what year will the percentage of electric automobiles along all automobile in use reach 50 percent?	☐
☐	3. In what year will the percentage of households that are equipped with computer consoles tied to a central computer and databank reach 50 percent?	☐
☐	4. By what year will the per-capita amount of personal cash transactions (in 1967 dollars) be reduced to one-tenth of what it is now?	☐
☐	5. In what year will power generation be thermonuclear fusion become comercially competitive with hydroelectric power?	☐
☐	6. By what year will it be possible by commercial carriers to get from New York's Time Square to San Fransisco's Union Suare in half the time that is now required to make that trip?	☐
☐	7. In what year will a man for the first time travel to the Moon, stay for at least 1 month, and return to earth?	☐

* "Never" is alson an acceptable answer

Please also answer the following question, and give your name (this is for identification purposes during the exercise only; no opinions will be attributed to a particular person).

Check One:

☐ I would like

☐ I am willing but no anxious

☐ I would prefer not

participate in the three remaining qustionnaires

Name (block letter please):
……………………………………………

Source: Helmer (1968)

CRITIQUE ON DELPHI METHOD:

(Sackman's Delphi Critique (1975))

Methodological

- Questions are vague are often so vague that it would be impossible to determine when, if ever, they occurred.
- Furthermore, the respondents are not treated equally.
- Many dropouts. No Explanation for # of dropouts given or researched, nor are effects assessed on eventual assessment. Does Delphi convergence because of boredom in stead of consensus.
- Sackman argues that experts and non experts produce comparable results.

Comparison to other Methods

(Delbecq, Van de Ven, and Gusstafson, 1975);

- **Method 1:** "nominal group technique"; participants confront each other directly in a controlled environment.
- **Method 2:** "no interaction model”; initial assessments are simply aggregated mathematically.

Results:

- Nominal group technique superior to the others,
- Delphi worst of the three.

EXPERT JUDGMENT ELICITATION PRINCIPLES

(Source: Experts in Uncertainty, Roger M. Cooke)

1. Reproducibility:
It must be possible for Scientific peers to review and if necessary **reproduce** all calculations. This entails that the calculational model must be fully specified and the ingredient data must be made available.

2. Accountability:
The source of Expert Judgment must be identified.

3. Empirical Control:
Expert probability assessment must in principle be susceptible to empirical control.

4. Neutrality:
The method for combining/evaluating expert judgements should encourage experts to state true opinions.

5. Fairness:
All Experts are treated equally, prior to processing the results of observation

PRACTICAL EXPERT JUDGMENT ELICITATION GUIDELINES

1. **The questions must be clear**
2. **Prepare an attractive format for the questions and graphic format for the answers**
3. **Perform a dry run**
4. **An Analyst must be present during the elicitation**
5. **Prepare a brief explanation of the elicitation format, and of the model for processing the responses.**
6. **Avoid Coaching**
7. **The elicitation session should not exceed 1 hour.**

ELICITATION PROCEDURES

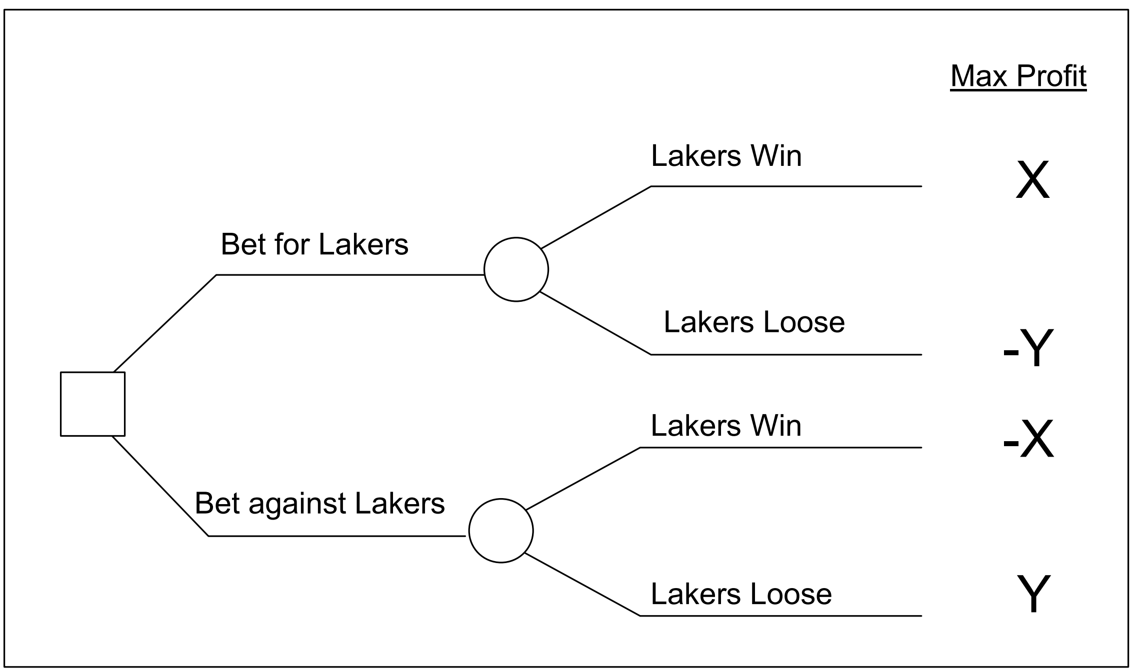
- **Direct Procedures:** Ask for Probabilities\Measures of Central Tendency\Measures of Variability
- **Indirect Procedures:** Use Betting Strategies; Pairwise Comparisons

Example Betting Strategies: Indifference Expected payoffs are the same

1. Betting Stategies

Event: Lakers winning the NBA title this season

STEP 1: Offer a person to choose between following the following bets, where X=100, Y=0.



STEP 2: Offer a person to choose between following the following bets, where X=0, Y=100. (Consistency Check)

STEP 3: Offer a person to choose between following the following bets, where X=100, Y=50.

STEP 4: Offer a person to choose between following the following bets, where X=50, Y=100. (Consistency Check)

Continue until point of indifference has been reached.

Assumption:

When a person is indifferent between bets the expected payoffs from the bets must be the same.

Thus:

$$X*Pr(LW) - Y*Pr(LL) = -X*Pr(LW) + Y*Pr(LL) \Leftrightarrow$$

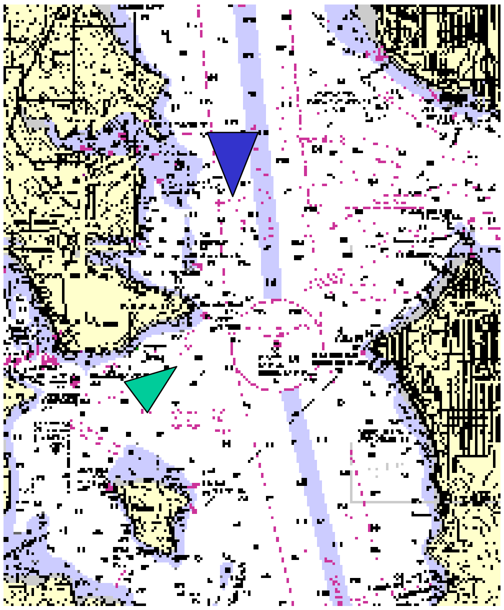
$$2*X*Pr(LW) - 2*Y*(1- Pr(LW)) = 0 \Leftrightarrow$$

$$Pr(LW) = \frac{Y}{X+Y}.$$

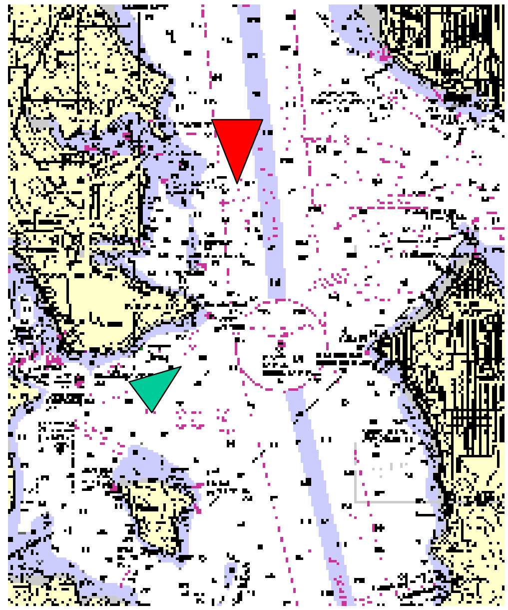
Example: X=100, Y=50 $\Rightarrow$ Pr(LW)= $\frac{2}{3} \approx 66.66\%$

2. Pairwise Comparisons of Situations

Issaquah class ferry on the Bremerton to Seattle route in a crossing situation within 15 minutes, no other vessels around, good visibility, negligible wind.



Other vessel is a navy vessel



Other vessel is a product tanker

Question: 1 89

Situation 1	Attribute	Situation 2
Issaquah	**Ferry Class**	-
SEA-BRE(A)	**Ferry Route**	-
Navy	**1st Interacting Vessel**	Product Tanker
Crossing	**Traffic Scenario 1st Vessel**	-
0.5 – 5 miles	**Traffic Proximity 1st Vessel**	-
No Vessel	**2nd Interacting Vessel**	-
No Vessel	**Traffic Scenario 2nd Vessel**	-
No Vessel	**Traffic Proximity 2nd Vessel**	-
> 0.5 Miles	**Visibility**	-
Along Ferry	**Wind Direction**	-
0	**Wind Speed**	-
	Likelihood of Collision Avoidance	
	9 8 7 6 5 4 3 2 1 2 3 4 5 6 7 8 9	
Situation 1 is worse	**<===================X====================>**	**Situation 2 is worse**

9: **VERY MUCH MORE LIKELY** to result in a collision.
7: **MUCH MORE LIKELY** to result in a collision.
5: **MODERATELY LIKELY** to result in a collision.
3: **SOMEWHAT MORE LIKELY** to result in a collision.
1: **EQUALY LIKELY** to result in a collision.

Underlying Model for Pairwise Comparison Questionnaire

1 $\text{Traffic Scenario 1} = \underline{X}^1 \xleftrightarrow{\text{Paired Comparison}} \text{Traffic Scenario 2} = \underline{X}^2$

$\underline{Y}(\underline{X}) = \text{Vector including 2 - way interactions}$

2 $$\Pr(\text{Accident } \mid \text{Propulsion Failure, } \underline{X}^1) = P_0 e^{\underline{\beta}^T \underline{Y}(\underline{X}^1)}$$

3 $$\frac{\Pr(\text{Accident} \mid \text{Prop. Failure}, \underline{X}^1)}{\Pr(\text{Accident} \mid \text{Prop. Failure}, \underline{X}^2)} = \frac{P_0 e^{\underline{\beta}^T \underline{Y}(\underline{X}^1)}}{P_0 e^{\underline{\beta}^T \underline{Y}(\underline{X}2)}} = e^{\underline{\beta}^T \left(\underline{Y}(\underline{X}^1) - \underline{Y}(\underline{X}^2) \right)}$$

4 $$LN\left\{ \frac{\Pr(\text{Accident} \mid \text{Prop. Failure}, \underline{X}^1)}{\Pr(\text{Accident} \mid \text{Prop. Failure}, \underline{X}^2)} \right\} = \underline{\beta}^T \left(\underline{Y}(\underline{X}^1) - \underline{Y}(\underline{X}^2) \right)$$

Calibration to Accident Data

1. Calibrating the constructed scale on which experts responded e.g. fix the ratio of collisions of Washington State Ferries with Washington State Ferries and NON-WSF Vessels.

2. Calibrating to convert relative probabilities to absolute probabilities by solving for P_0 e.g. fix the total number of expected collisions over a given time period.

3. Pairwise Comparisons of Attributes

- Some variables do not have a natural attributes scale e.g. the combination of ferry class & ferry routes =(High Speed Ferry on the Seatle Bremerton Run, Super Class Ferry on the Seattle Bainbridge run).

26 Combinations of Ferry Class & Ferry Route $\Rightarrow$ 325 Paired Comparisons
Too Many Questions

Solution
• Compare Ferry Classes $\Rightarrow$ R(Ferry Class) • Compare Ferry Routes $\Rightarrow$ R(Ferry Route) • Ask relative importance W of Ferry Class to Ferry Route Rank = W*R(Ferry Class) + (1-W)*R(Ferry Route)

Question Format

Issaquah	<--	=	-->	Jumbo Mark II	?

If you think collision avoidance is equally likely for Issaquah as for Jumbo Mark II you answer:

Issaquah	<--	= X	-->	Jumbo Mark II	?

If you think collision avoidance is more likely for Issaquah as for Jumbo Mark II you answer:

Issaquah	<-- X	=	-->	Jumbo Mark II	?

If you think collision avoidance is more likely for Issaquah as for Jumbo Mark II you answer:

Issaquah	<--	=	--> X	Jumbo Mark II	?

If you cannot answer the question, you answer:

Issaquah	<--	=	-->	Jumbo Mark II	? X

9 Ferry Classes $\Rightarrow$ 36 Paired Comparisons

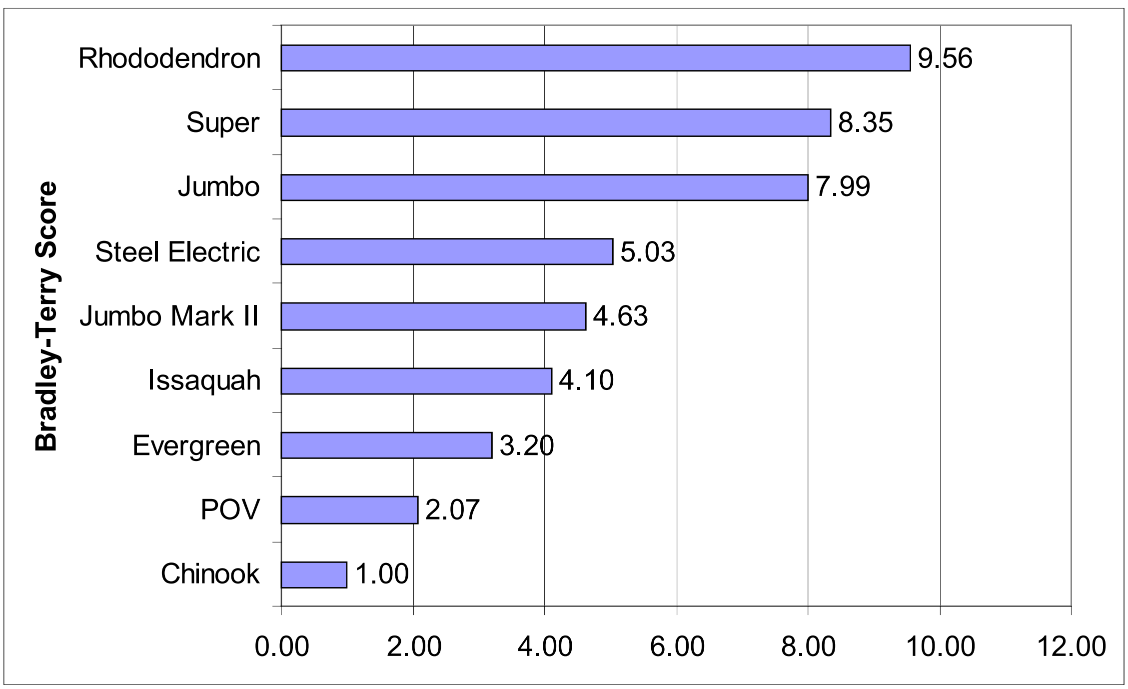
Perform Bradley Terry Pairwise Comparison Analysis to:

1. Test for Preference Structure of individual expert by counting circular triads.

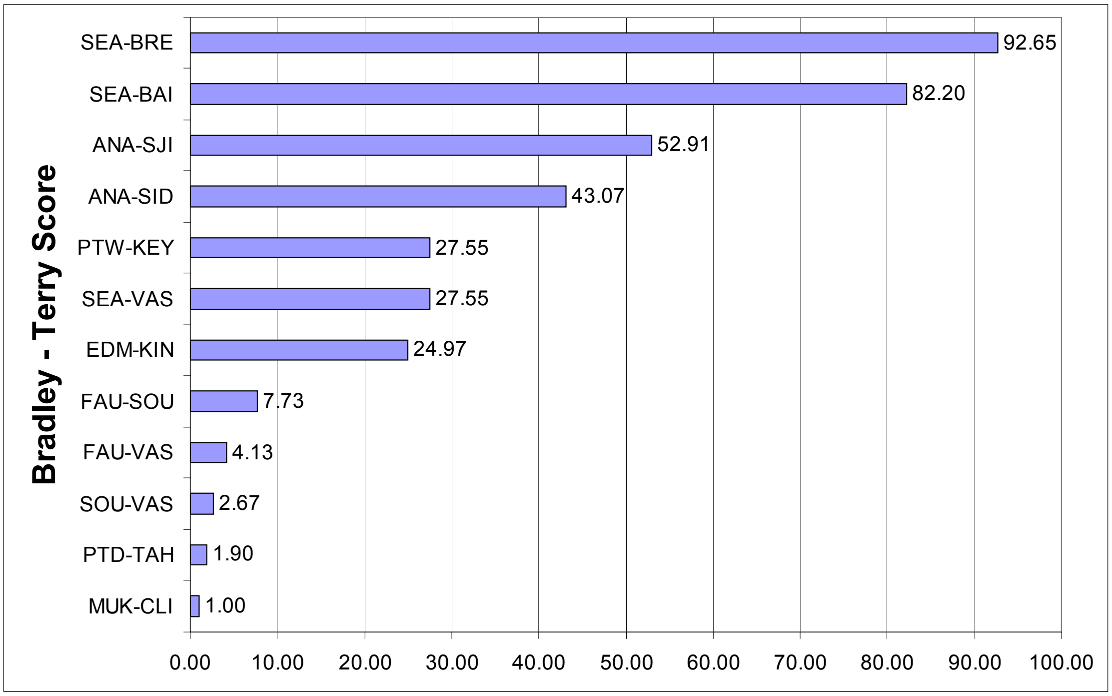
Circular Triad: A is better than B, B is better than C, C is better than A.

2. Test for Agreement between experts as a group.

Results Attribute Scale for Ferry Class



Results Attribute Scale for Ferry Route

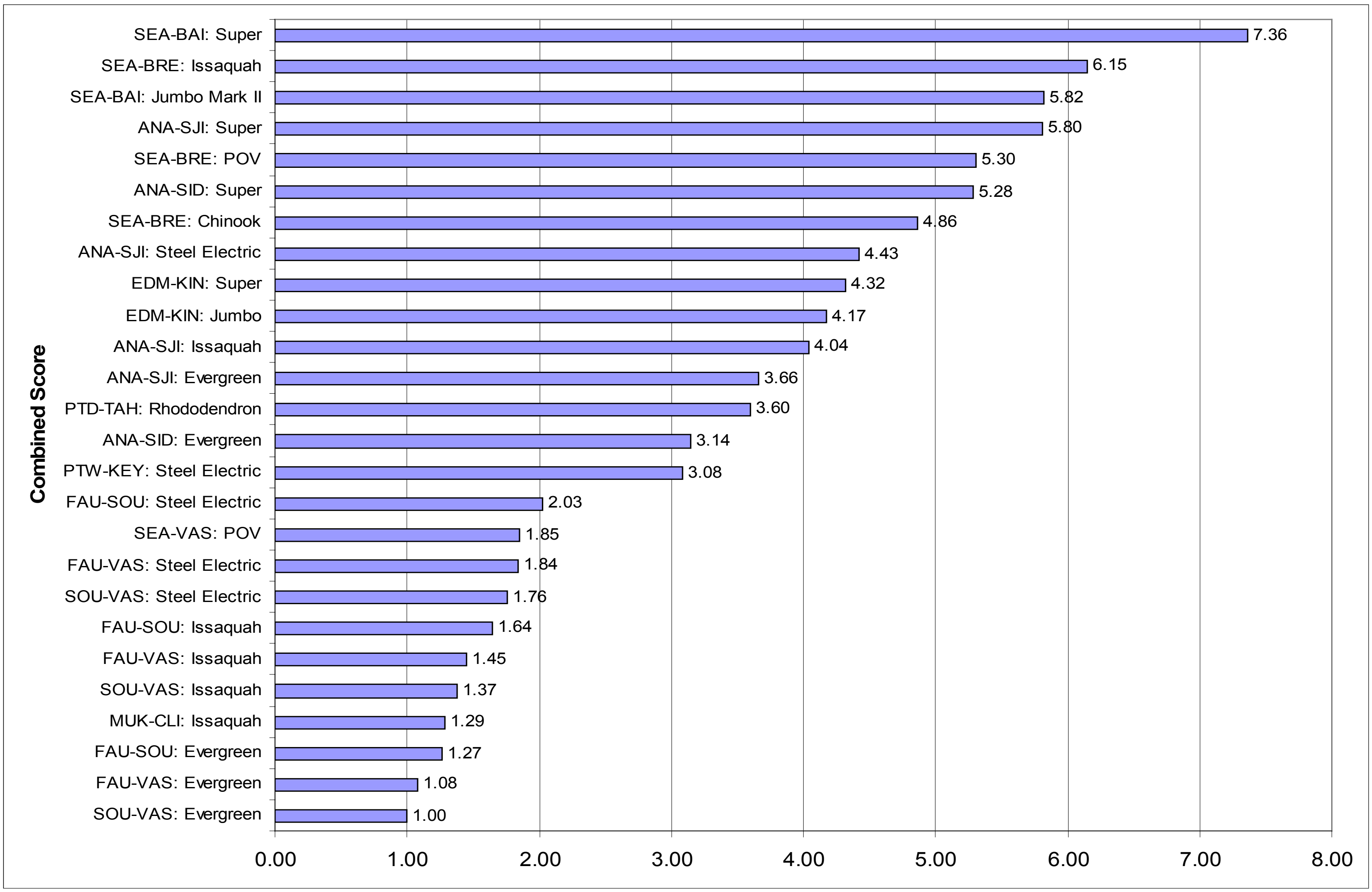


Using Swing Weights Elicitation Method (EMGT 269)

Ferry Class Weight = 0.42

Ferry Route Weight = 0.59

Results Attribute Scale for Ferry Class & Ferry Route Combination



LAWS OF PROBABILITY

A = an event with possible outcomes $A_1, \cdots, A_n$;

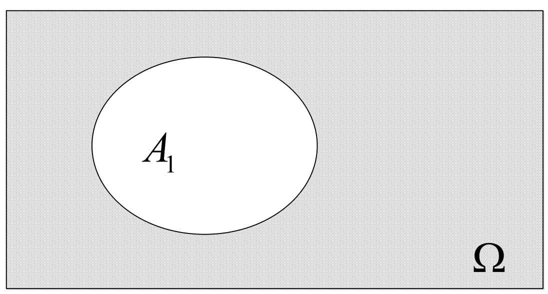
Ω = Total Event

Venn Diagrams:

Ratio of **area of the event** and the **area of the total rectangle** can be interpreted as the probability of the event

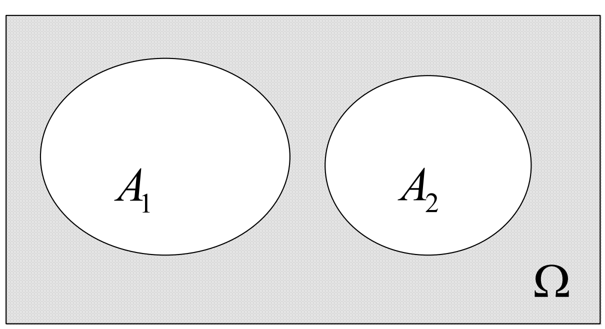
- Probabilities must lie between 0 and 1:

$$0 \leq \Pr(A_1) \leq 1, \forall A_1 \subset \Omega$$



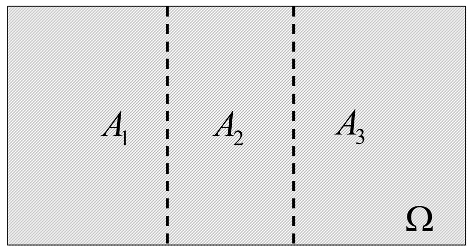
- Probabilities must add up:

$$A_1 \cap A_2 = \phi \Rightarrow \Pr(A_1 \cup A_2) = \Pr(A_1) + \Pr(A_2)$$



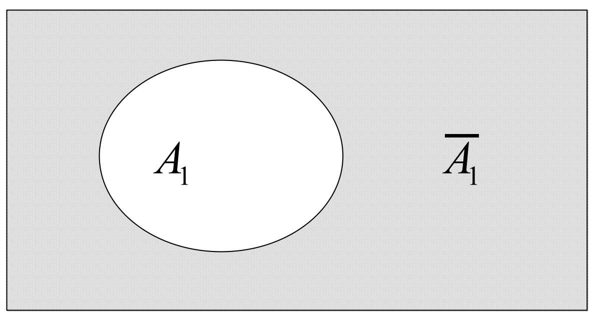
- Total Probability Must Equal 1:

$$\left(A_i \cap A_j = \phi, \forall i \neq j \wedge \bigcup_{i=1}^{3} A_i = \Omega\right) \Rightarrow \Pr\left(\bigcup_{i=1}^{3} A_i\right) = 1$$



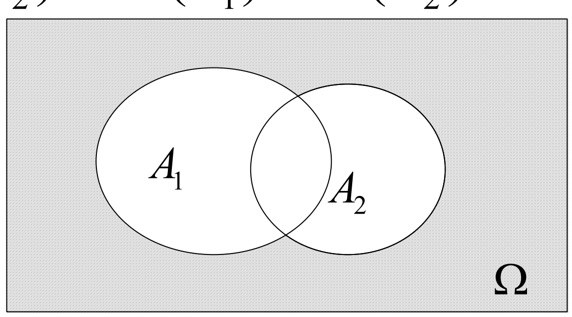
- Complement Rule:

$$\Pr(\overline{A_1}) = 1 - \Pr(A_1)$$

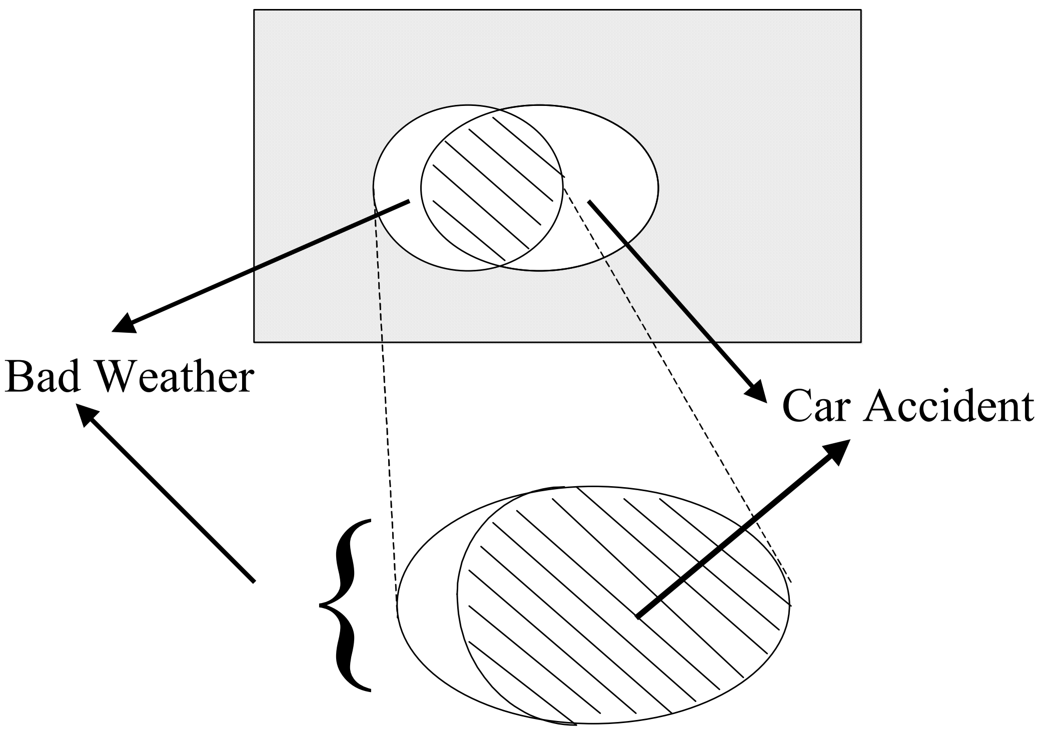


- Probability of union of two events that can happen at the same time

$$\Pr(A_1 \cup A_2) = \Pr(A_1) + \Pr(A_2) - \Pr(A_1 \cap A_2)$$



Conditional Probability:



$$\Pr(\text{Carr Accident} \mid \text{Bad Weather}) = \frac{\Pr(\text{Car Accident} \cap \text{Bad Weather})}{\Pr(\text{Bad Weather})}$$

Informally: Conditioning on an event coincides with reducing the total event to **the conditioning event**

- Multiplicative Rule:

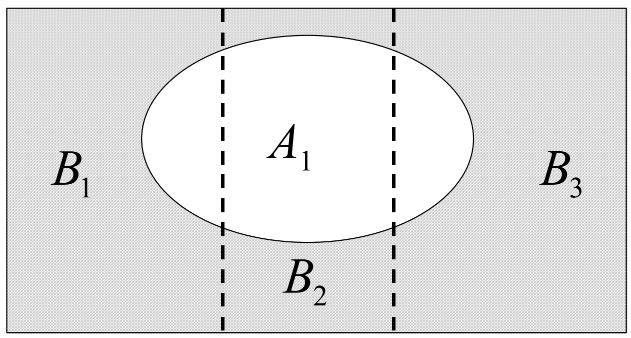
$$\Pr(A_1 \cap B_1) = \Pr(B_1 \mid A_1) * \Pr(A_1) = \Pr(A_1 \mid B_1) * \Pr(B_1)$$

Law of Total Probability:

$B_1, \cdots, B_3$ mutually exclusive, collectively exhaustive:

$$\Pr(A_1) = \Pr(A_1 \cap B_1) + \Pr(A_1 \cap B_2) + \Pr(A_1 \cap B_3) \Leftrightarrow$$

$$\Pr(A_1) = \Pr(A_1 \mid B_1)\Pr(B_1) + \Pr(A_1 \mid B_2)\Pr(B_2) + \Pr(A_1 \mid B_3)\Pr(B_3)$$



Example Law of Total Probability:

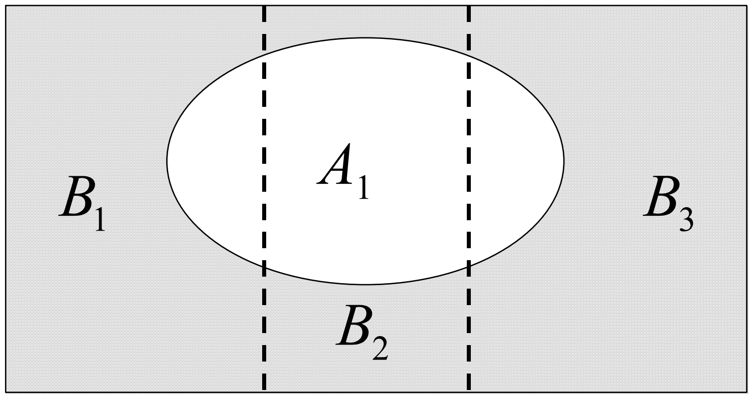
Pr(Hospital) =
Pr(Hospital|Car Accident) Pr(Car Accident)+
Pr(Hospital|No Car Accident)Pr(No Car Accident)

- Also Referred To As LOEC:

Law Of Extension of Conversation

Bayes Theorem

$B_1, \cdots, B_3$ mutually exclusive, collectively exhaustive:



$$1.\ \Pr(A_1 \cap B_j) = \Pr(B_j \mid A_1)\Pr(A_1) = \Pr(A_1 \mid B_j)\Pr(B_j)$$

$$2.\ \Pr(B_j \mid A_1) = \frac{\Pr(A_1 \mid B_j)\Pr(B_j)}{\Pr(A_1)}$$

$$3.\ \Pr(A_1) = \Pr(A_1 \mid B_1)\Pr(B_1) + \Pr(A_1 \mid B_2)\Pr(B_2) + \Pr(A_1 \mid B_3)\Pr(B_3)$$

$$4.\ \Pr(B_j \mid A_1) = \frac{\Pr(A_1 \mid B_j)\Pr(B_j)}{\Pr(A_1 \mid B_1)\Pr(B_1) + \Pr(A_1 \mid B_2)\Pr(B_2) + \Pr(A_1 \mid B_3)\Pr(B_3)}$$

Example: Game Show

Suppose we have a game show host and you. There are three doors and one of them contains a prize. The game show host knows the door containing the prize but of course does not convey this information to you. He asks you to pick a door. You picked door 1 and are walking up to door 1 to open it when the game show host screams: STOP. You stop and the game show host shows door 3 which appears to be empty. Next, the game show asks.

"DO YOU WANT TO SWITCH TO DOOR 2?"
WHAT SHOULD YOU DO?

Assumption 1: The game show host will never show the door with the prize.
Assumption 2: The game show will never show the door that you picked.

- D_i ={Prize is behind door i }, i=1,…,3
- H_i ={Host shows door i containing no prize after you selected Door 1}, i=1,…,3

Initially: $\Pr(D_i) = \frac{1}{3}$

1. $\Pr(H_3) = \sum_{i=1}^{3} \Pr(H_3 \mid D_i)\Pr(D_i) = \frac{1}{2} * \frac{1}{3} + 1 * \frac{1}{3} + 0 * \frac{1}{3} = \frac{1}{2}$

2. $\Pr(D_1 \mid H_3) = \frac{\Pr(H_3 \mid D_1)\Pr(D_1)}{\Pr(H_3)} = \frac{\frac{1}{2} * \frac{1}{3}}{\frac{1}{2}} = \frac{1}{3}$

3. $\Pr(D_2 \mid H_3) = 1 - \Pr(D_1 \mid H_3) = 1 - \frac{1}{3} = \frac{2}{3}$.

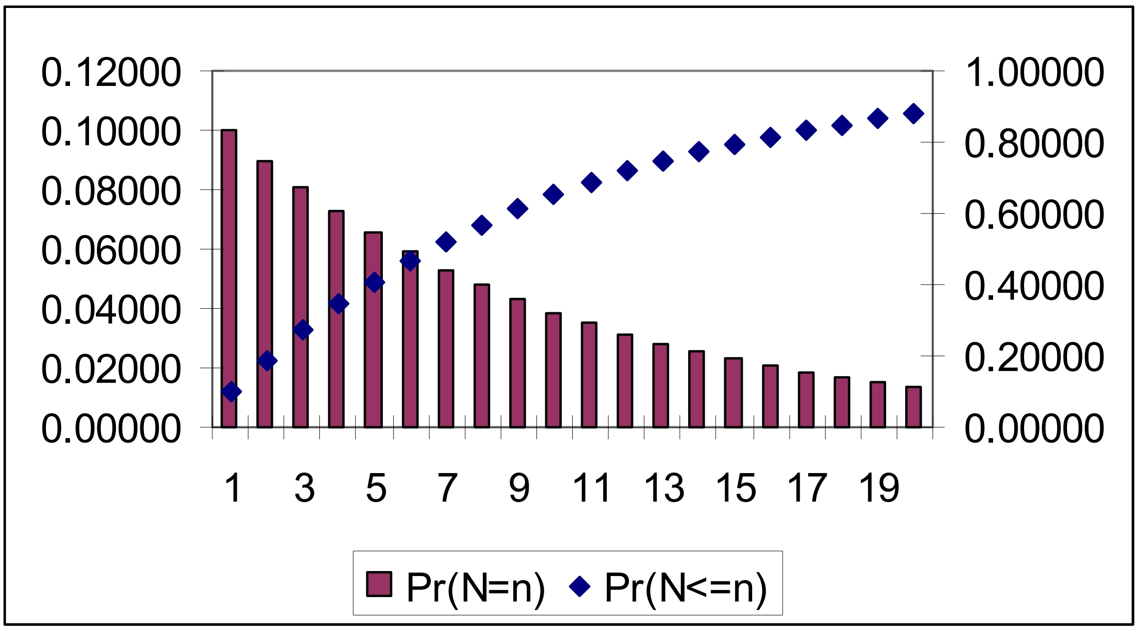
So Yes, you should switch!

RANDOM VARIABLES

Mathematical tool to shorten description
of a complex real event.

Discrete Random Variable:

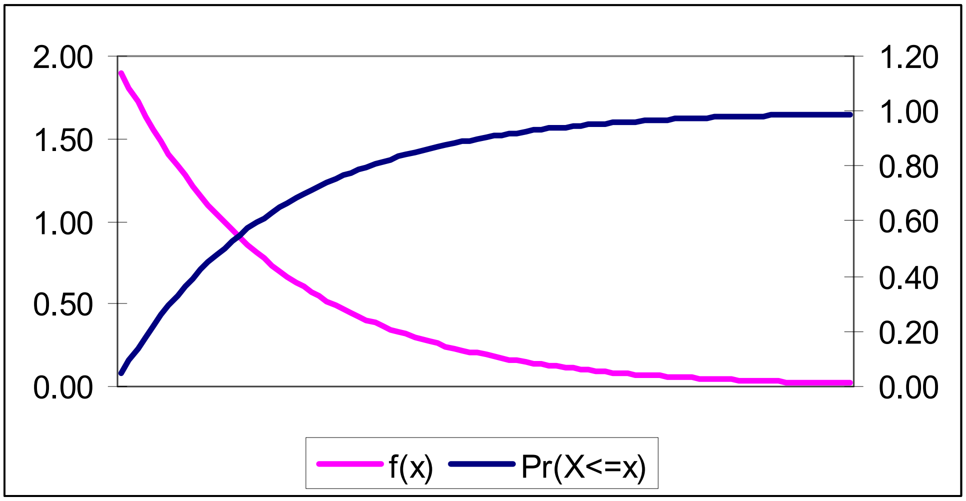
X=# of Car Accidents on Inner loop
of the Capital Beltway, X=0,1,2,3,....



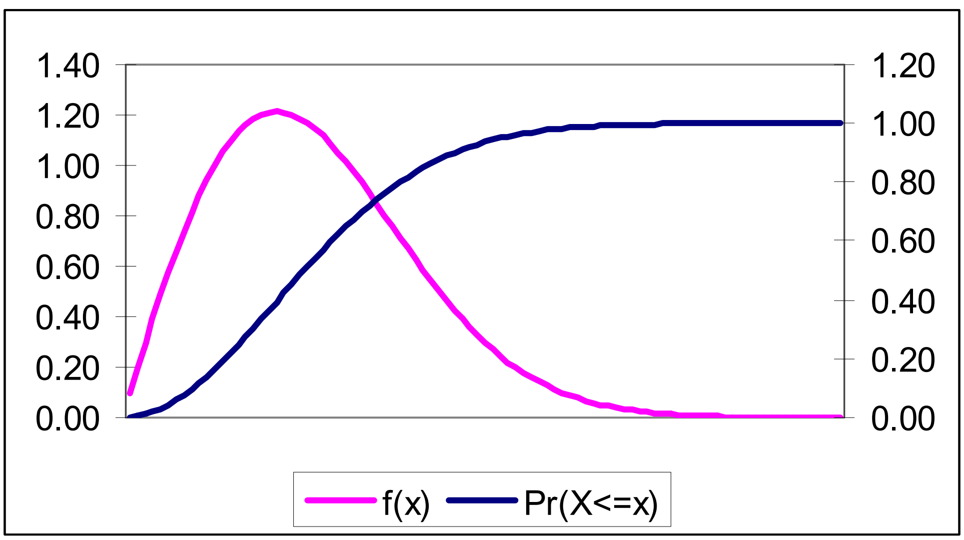
Continuous Random Variable:

X= Failure Time of a Pressure Relief Valve under continuous pressure, X=[0, ∞}

Exponential Life Time Distribution:

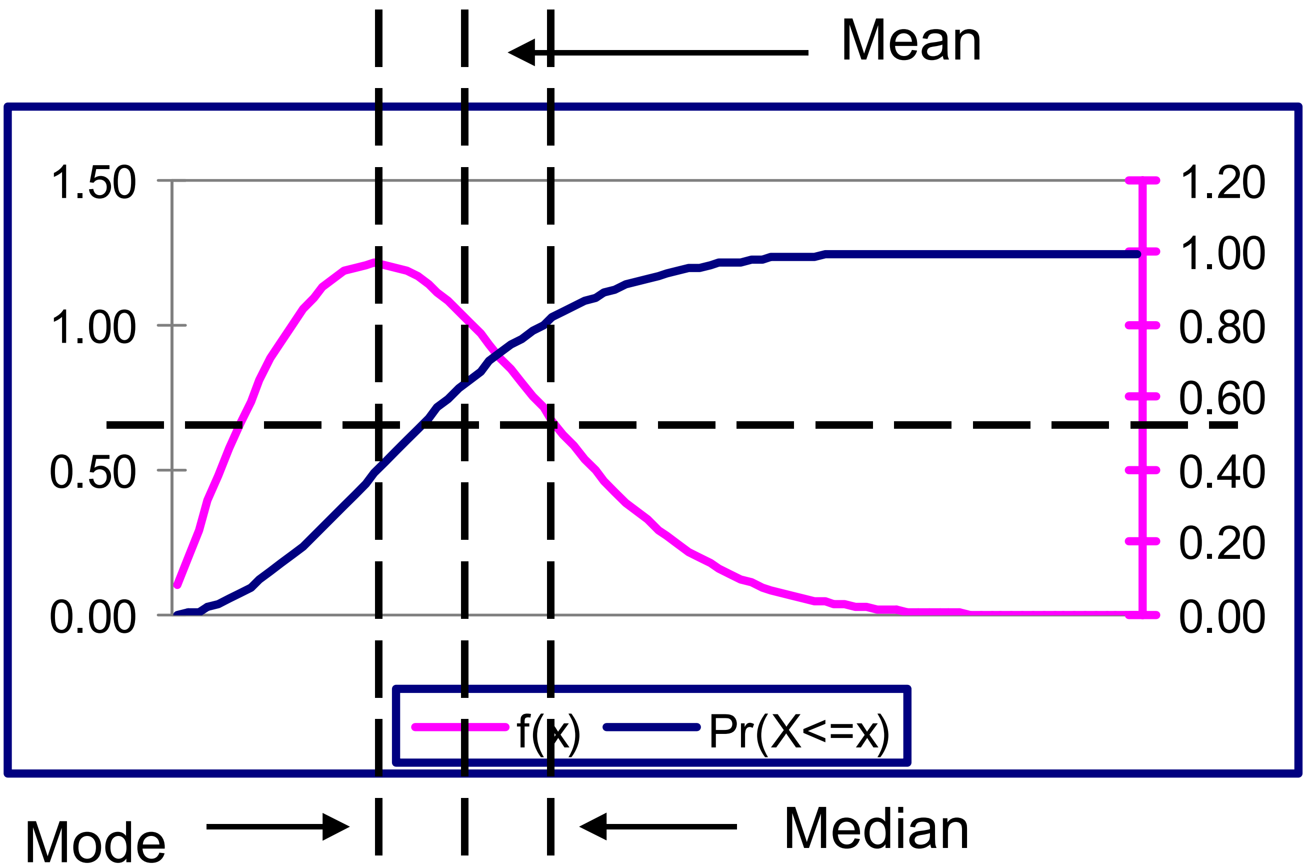


Weibull Life Time Distribution:



$$\text{Failure Rate} = \Pr(t < T < t + \Delta t \mid T > t) = \frac{\Pr(t < T < t + \Delta t)}{\Pr(T > t)}$$

MEASURES OF CENTRAL TENDENCY



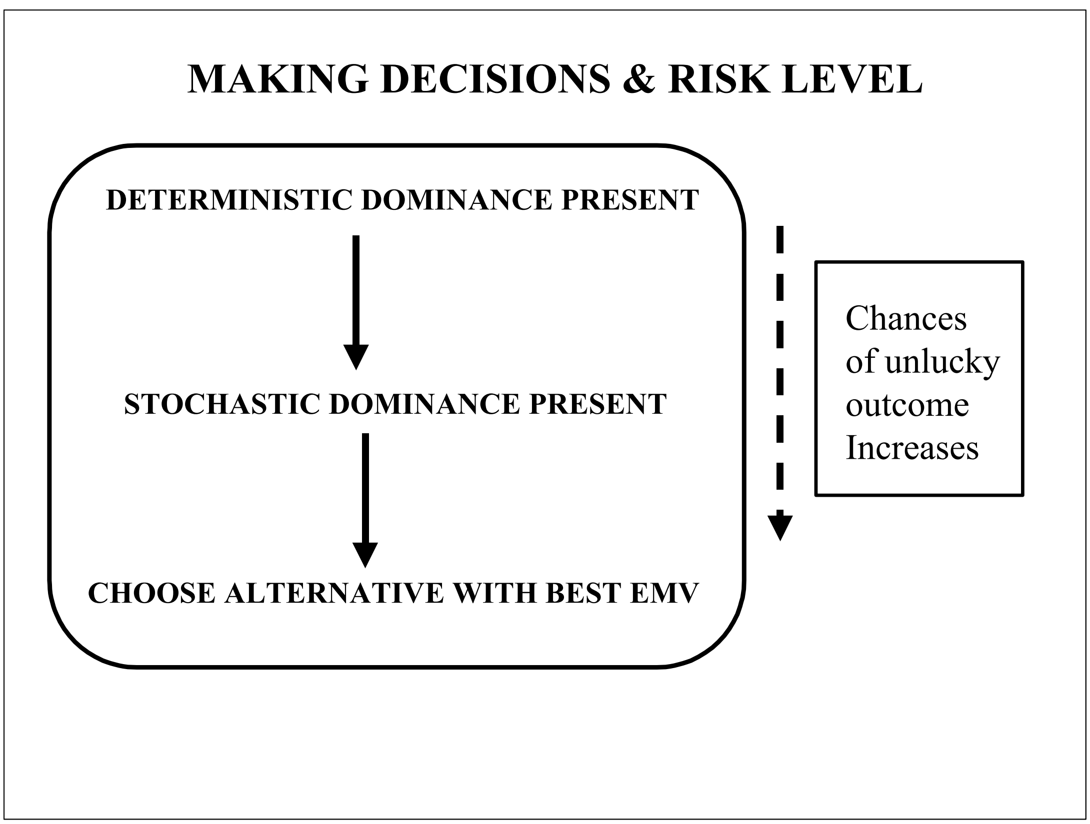
SKEWED TO LEFT : Mode < Mean < Median
SKEWED TO RIGHT : Mode > Mean > Median
SYMMETRIC : ?

DOMINANCE AND MAKING DECISIONS UNDER UNCERTAINTY

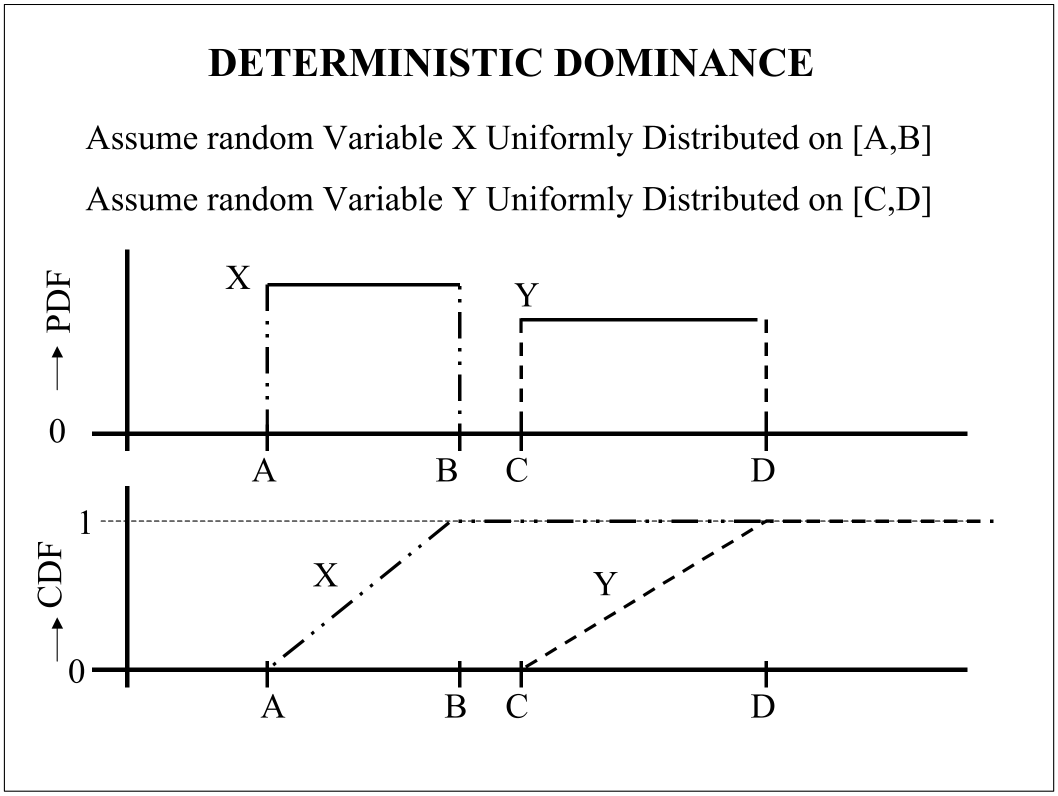
Suppose you have to choose between two lottery tickets and the only information you have is that the expected pay-off of the first lottery ticket is lower than the second. Which one would you choose?

You picked your ticket and the lotteries are played and you learn your outcome. Is your pay-off higher than the pay-off of the first lottery-ticket?

Conclusion: There is a chance of an **unlucky outcome.** In other words there is no dominance (=deterministic dominance.



SITUATION 1:
You are given more information about both lotteries. The pay-off X of lottery 1 falls in the range from [A,B]. The pay-off from lottery 2 falls in the range from [C,D].



Which one would you choose?

You picked your ticket and the lotteries are played and you learn your outcome. Is your pay-off higher than the pay-off of the first lottery-ticket?

Conclusion: There is a no chance of an **unlucky outcome.** In other words there is dominance (=deterministic dominance).

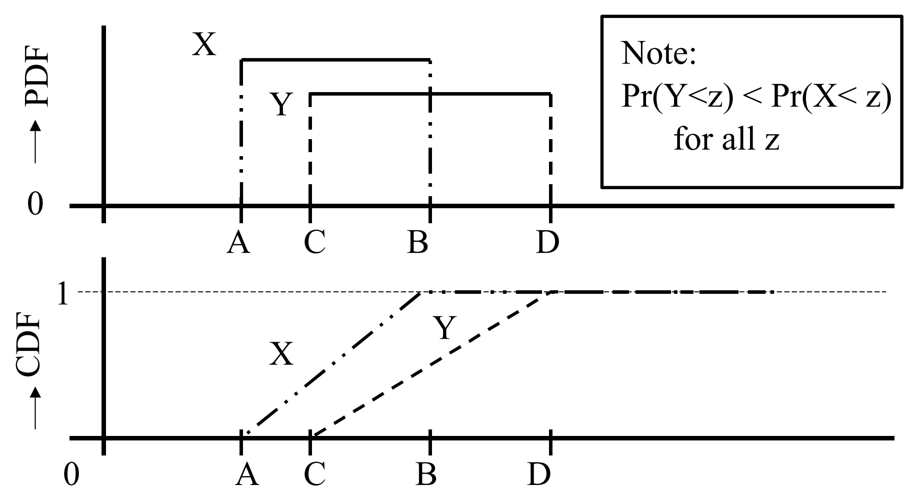
SITUATION 2:

You are given more information about both lotteries. The pay-off X of lottery 1 falls in the range from [A,B]. The pay-off from lottery 2 falls in the range from [C,D].

STOCHASTIC DOMINANCE

Assume random Variable X Uniformly Distributed on [A,B]

Assume random Variable Y Uniformly Distributed on [C,D]

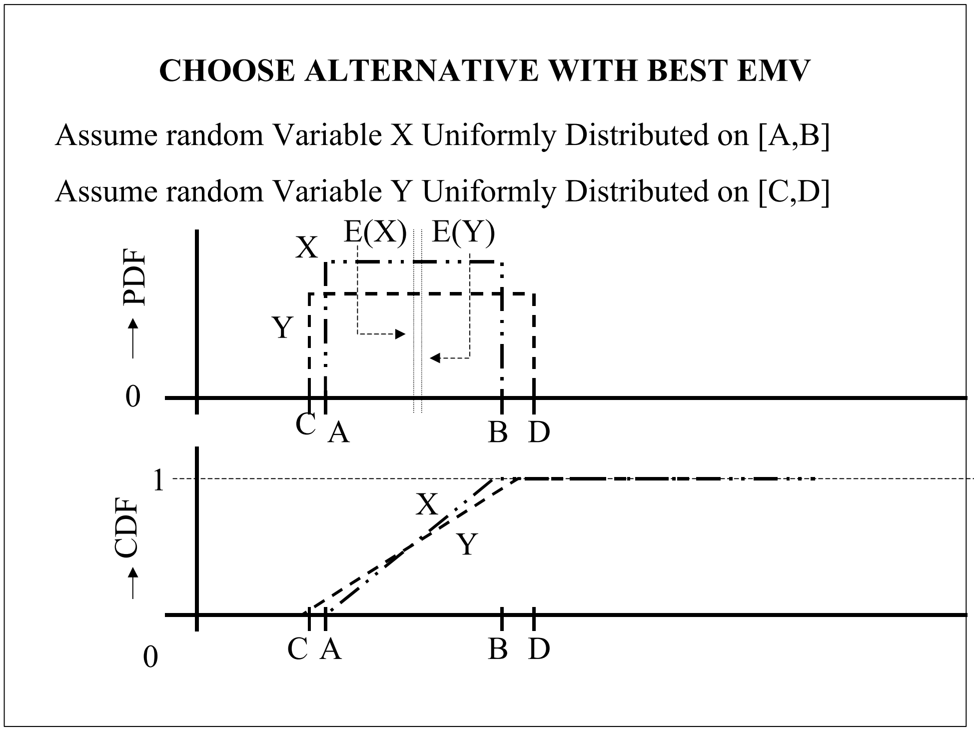


You picked your ticket and the lotteries are played and you learn your outcome. Is your pay-off higher than the pay-off of the first lottery-ticket?

Conclusion: There is a a chance of an **unlucky outcome.** In this case there is stochastic dominance, but no deterministic dominance.

SITUATION 3:

You are given more information about both lotteries. The pay-off X of lottery 1 falls in the range from [A,B]. The pay-off from lottery 2 falls in the range from [C,D].

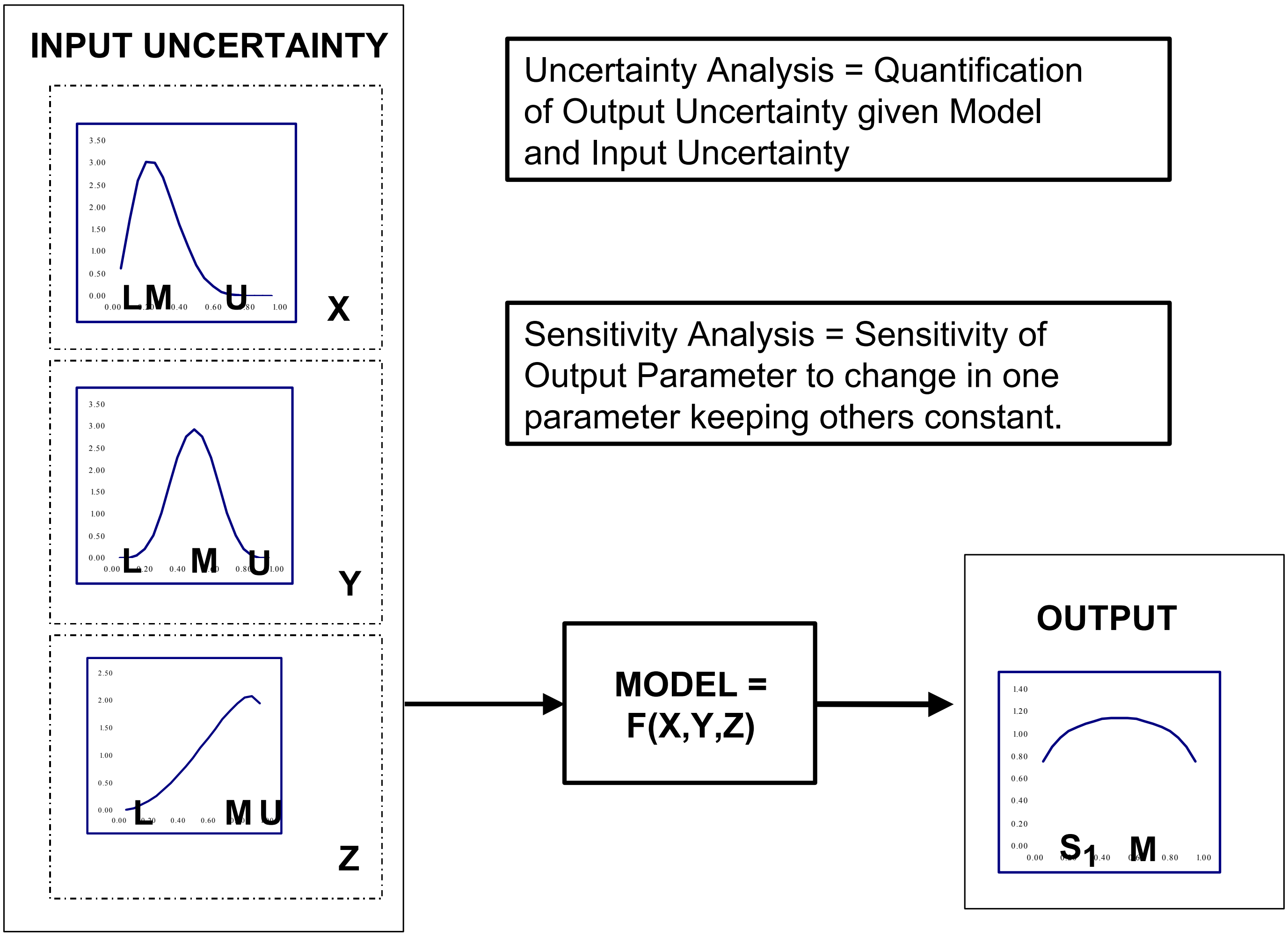


You picked your ticket and the lotteries are played and you learn your outcome. Is your pay-off higher than the pay-off of the first lottery ticket?

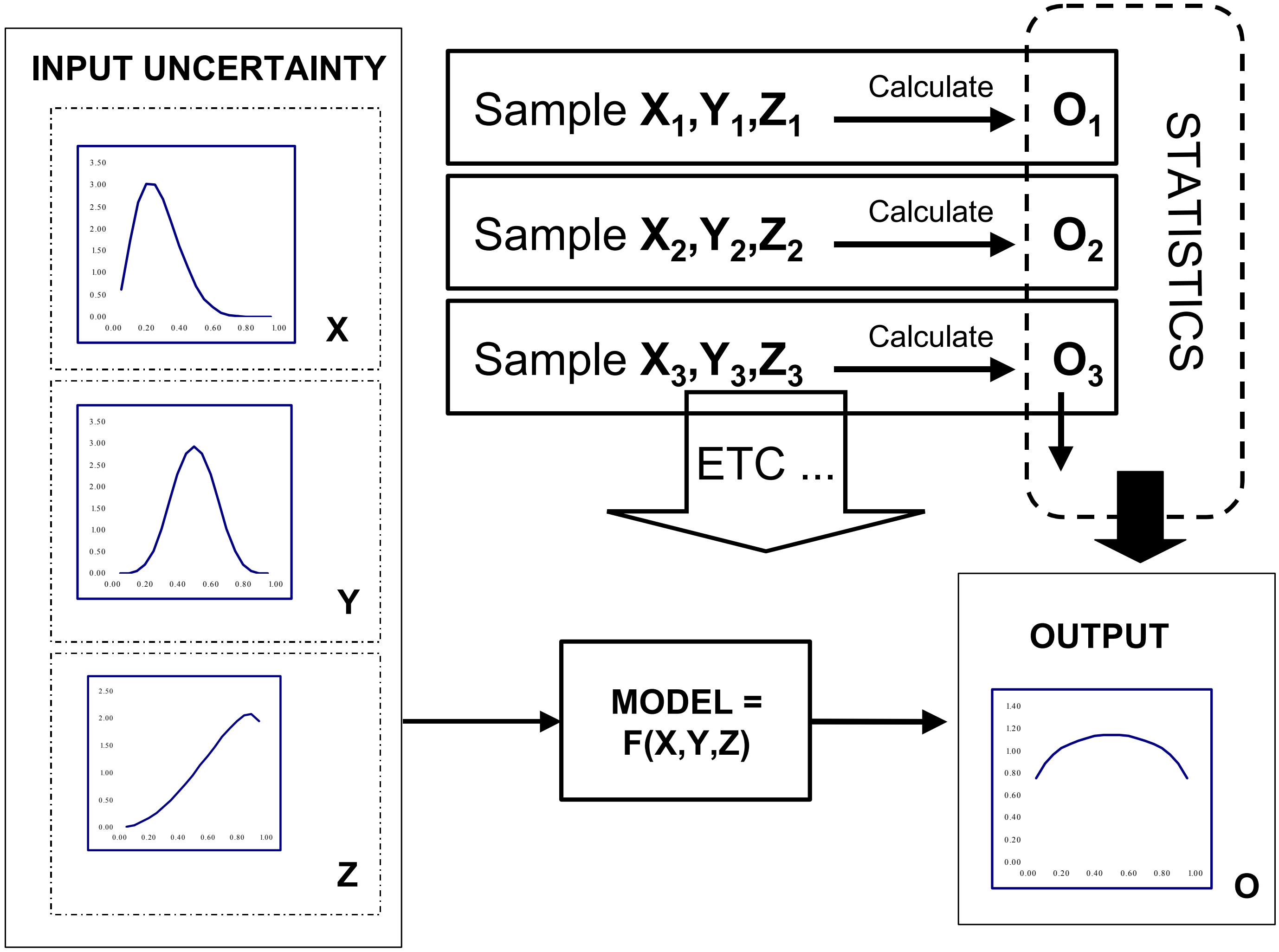
UNCERTAINTY ANALYSIS VERSUS SENSITIVITY ANALYSIS

Uncertainty Analysis = Quantification of Output Uncertainty given Model and Input Uncertainty

Sensitivity Analysis = Sensitivity of Output Parameter to change in one parameter keeping others constant.



MONTE CARLO SIMULATION



REGRESSION ANALYSIS