LECTURE NOTES: EMGT 234

THE WORDS OF RISK ANALYSIS

SOURCE:

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1. INTRODUCTION

The Words of Risk Analysis have been,
and continue to be a problem

- Risk Analysis Committee to **"Define Risk"**, when Society of Risk Analysis was brand new, labored **4 years** and **gave up**.

Committee Recommendation:

Not to have a universal definition Risk. Let each author/risk analyst/risk manager define it **in its own way**.

BUT ONE HAS TO DEFINE IT!

2. PROBABILITY

- Risk Analysis **closely** connected to probability theory. Risk involves an uncertain event which likelihood is specified through through the use of probability.

However:

- Leading Scientist have argued about the meaning of the word "probability" for **at least hundred of years**. For some "probability thinking" has emerged as **a religion**.

- Three major meanings of probability (See Table 1).

TABLE 1: LINGUISTIC CHAOS

Traditional Meanings of Probability		
Statistician's (Frequency, Fraction)	**Bayesian (Probability)**	**Mathematical (Probability)**
Random	Belief	Formal Probability
Variability	"Personal" Probability	"Axiomatic" Probability
"Aleatory" Probability	Subjective probability	
"Objective" Probability	Uncertainty	
Stochastic Ontological	Confidence	
"In the world" Probability	Epsitimistic	
Reliability	Forensic Probability	
Chance	Plausibibility	
Risk	Credibility	
	"Evidence Based" probability	

New Theories		
Fuzzy theory (Fuzziness)	Possibility Theory	Demster Shafer (Relief)

- New recent theories: invented to fix alleged deficiencies in the traditional ideas e.g. fuzzy theory

Fuzzy Representation:

Let A and B be an event about which you are uncertain and your level of uncertainty is specified by a function/operator:

$$U(A) \in [0,1], U[B] \in [0,1]$$

Furthermore, let **boolean** operators **"AND"** and **"OR"** be defined on A and B and let U be defined on **" AND "** and on **" OR "**.

Then *U* is called a **fuzzy representation** if for some functions *G, H*, the following holds:

- $G, H : [0,1] \times [0,1] \to [0,1]$
- $U(A \cap B) = G(U(A), U(B))$
- $U(A \cup B) = H(U(A), U(B)).$

Popular representation of Zadeh;

$$U(A \cap B) = Min(U(A), U(B)),\, U(A \cup B) = Max(U(A), U(B)).$$

3. TWO COMMUNICATION THEOREMS

- Leading Scientists **in the field of probability** agree that their is nothing wrong with the traditional ideas for modeling uncertainty (e.g. Kaplan, Lindley, Cooke).

- But, multitude of viewpoints causes confusion and communication problems have emerged, big time.

The following theorems may prove useful in to take emotion out of heated arguments:

Theorem 1:

50% of the problems in the world result from people using the **same** words with **different** meanings.

Theorem 2:

The other 50% comes from people
using **different** words with the **same** meaning.

4. DEFINITION OF RISK

Figure: The Three Risk Questions

	Answer	Notation
1. What can happen? (What can go wrong?)	Fire / Explosion	s_i
2. How Likely is it? (What is it is frequency / probability?)	0.01%	l_i
3. What are the consequences? (What is the damage?)	$100,000, Two Injuries, Environmental problems, Embarrassment, reputations	x_i

Figure: Quantitative Definition of Risk

1. What can happen?
2. How Likely is that?
3. What are the consequences?

- "An" Answer: $< s_i , l_i , x_i >$
- Set of Answers: $\{< s_i , l_i , x_i >\}$
- Complete Set: $\{< s_i , l_i , x_i >\}_c$

$$R = \{< s_i , l_i , x_i >\}_c$$

Include s_0 = as-planned scenario

Question:
How could we extend this definition of risk to include the evaluation of risk reduction measure scenario's?

Note:
Risk is defined as **the complete set of triplets**. Risk is **not** a number, **nor** is a curve, **nor** a vector, etc.

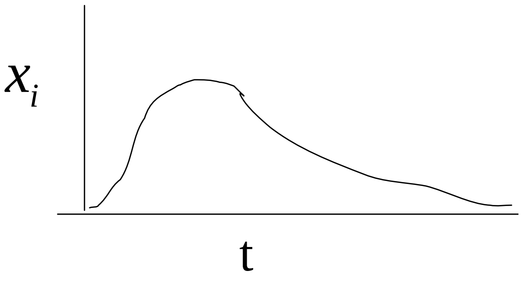
Damage Component of Risk

x_i can be a vector:

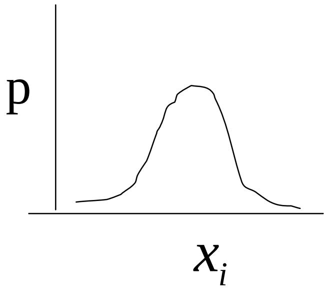
$$x_i = \begin{bmatrix} x_i^1 \\ x_i^2 \\ \vdots \\ x_i^n \end{bmatrix}$$

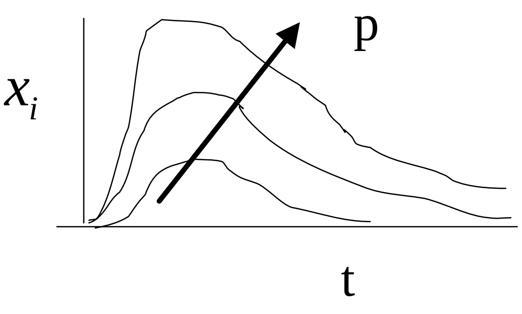
Damages to:
- People
- Property
- Environment
- Wildlife
- Reputation
- etc.

x_i can be time dependent:



x_i can be uncertain:





Likelihood Component of Risk

- Three formats which capture and quantify the intuitive idea of "likelihood":

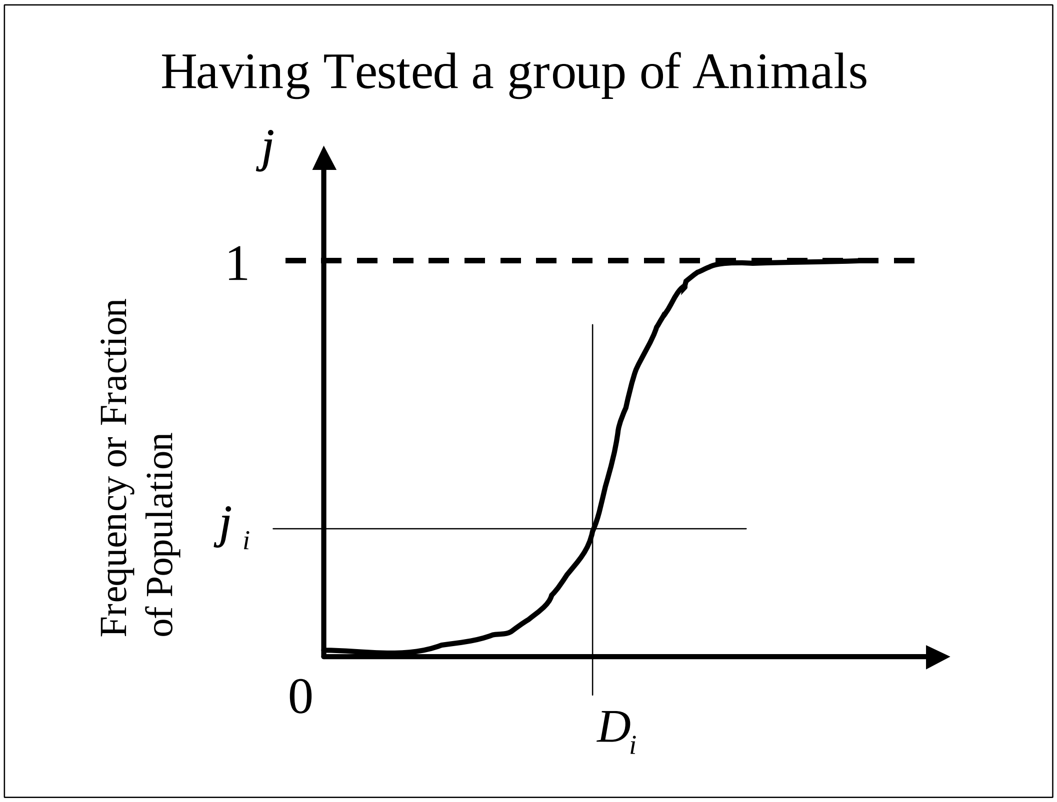
Format 1. (Frequency):
This applies when we have a **repetitive** situation, and we ask, "How frequently does scenario s_i occur?" In this case the likelihood is expressed as a frequency $l_i = j_i$ and risk becomes $R = \{< s_i, j_i, x_i >\}_c$

Format 2. (Probability):
When it is a "one shot" situation, like a mission to Mars, we want to quantify then our **degree of confidence** that the mission will succeed. In this case likelihood is expressed as a probability $l_i = p_i$ and the triplets become $R = \{< s_i, p_i, x_i >\}_c$

Format 3. (Probability of Frequency):
When we have a **repetitive** situation, or can image one as a thought experiment, so that the **frequency exists**, but since we haven't done the experiment we are **uncertain about the frequency** would be. We therefore express our state of knowledge about that frequency with a probability curve. We call this the "Probability of Frequency" format, $l_i = p(j_i)$;
; $R = \{< s_i, p_i(j_i), x_i >\}_c$

Format 3 is the most general and
Encapsulates both Format 2 and Format 1.

5. DOSE REPONSE EXAMPLE:



Having Tested a group of Animals

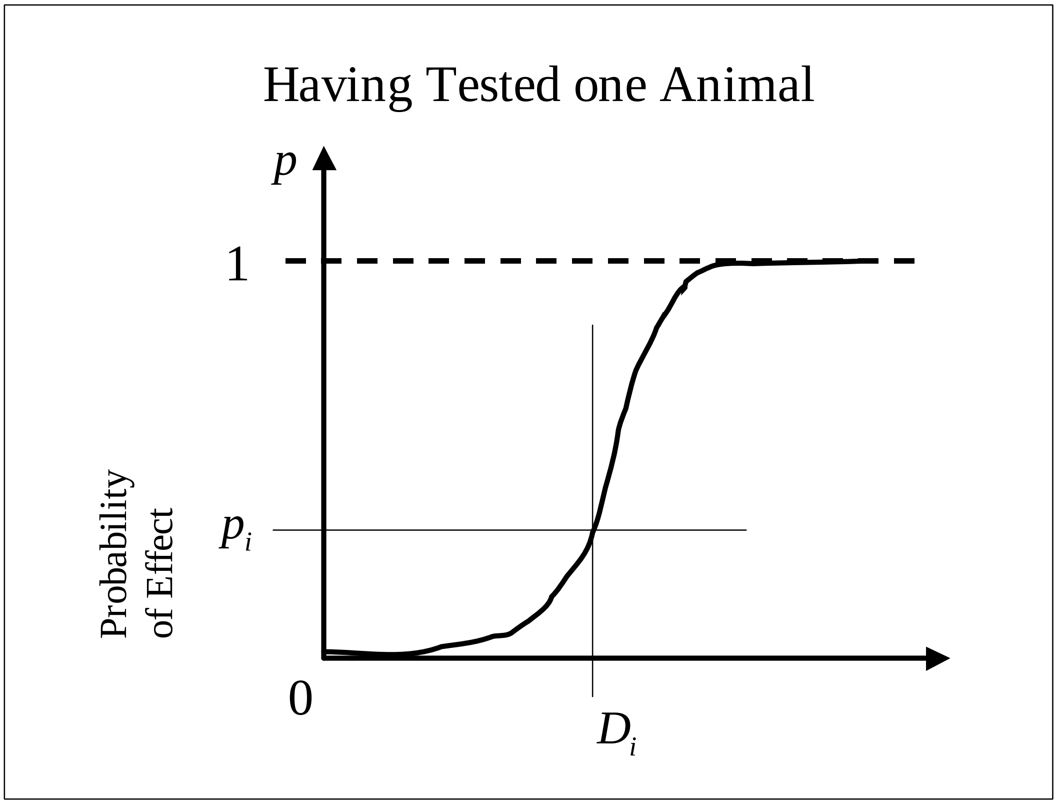
S_o = Animal stays healthy

S_i = Animal gets sick after receiving dose D_i

What can happen?
How likely is that?
What are the consequences?

Coincides with **Format 1: Frequency Format**

Dose Reponse Example (Continued):


Having Tested one Animal

S_o = Animal stays healthy

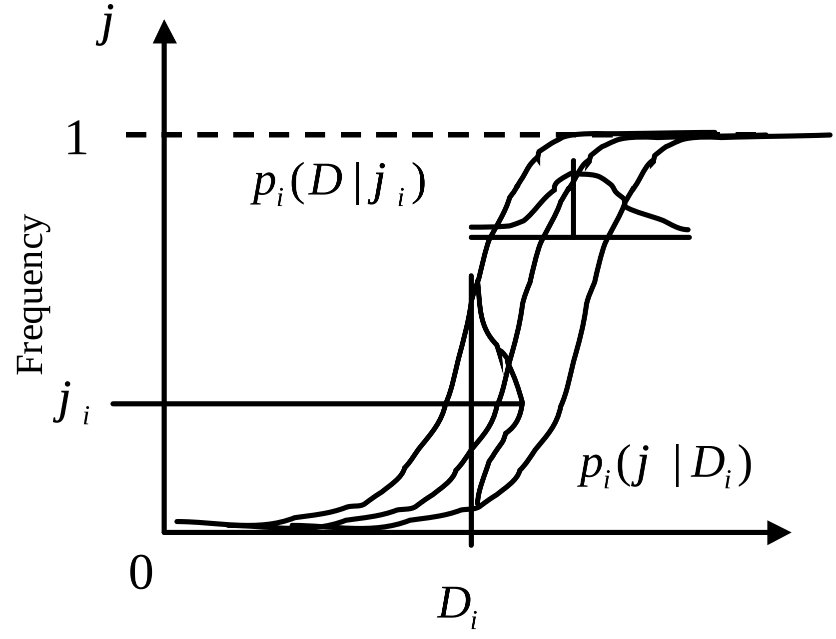
S_i = Animal gets sick after receiving dose D_i

What can happen?
How likely is that?
What are the consequences?

Coincides with **Format 2: Probability Format**

Dose Reponse Example (Continued):

Having Tested a group of Animals and infering about an entire population of Animals



S_o = Animal stays healthy

S_i = Animal gets sick after receiving dose D_i

What can happen?
How likely is that?
What are the consequences?

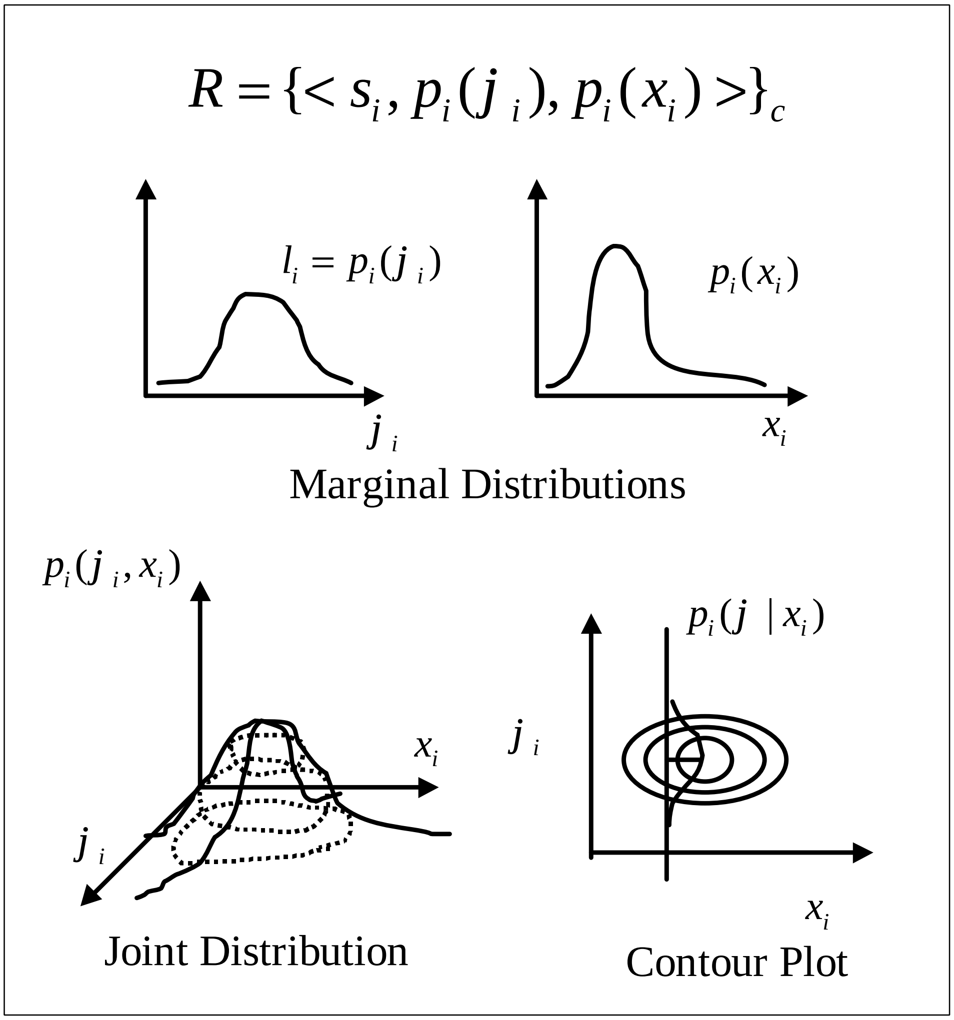
Coincides with **Format 3: Probability of Frequency Format**

Definition of Risk follows as:

$$R = \{< s_i , p_i (j_{\ i}), p_i (x_i) >\}_c$$

Scenario **Likelihood** **Consequence**

Figure: Graphical portrayal of Risk



Could you critisize the figure caption?

Triple Risk Definition has been successfully applied in:

- engineering risk,
- programmatic risk,
- strategic risk,
- environmental risk,

etc.

6. BAYES THEOREM

"SPEAKING THE TRUTH" ARGUMENT

1. The truth is that we are uncertain.
2. Speaking the truth means that we express our uncertainty.
3. Probability is the language of uncertainty.
4. Express our assessments in terms of probability curves

Question:
How do we get these curves?

Answer:
From **evidence**, or better "absence of evidence". The more evidence you acquire the less uncertain you become and in theory one can arrive at complete certainty.

Question:

If we start out with a level evidence, culminated into a level of uncertainty about an assessment through a probability distribution (=uncertainty distribution) and new evidence becomes available, how do we revise our uncertainty?

In other words, how do we learn from evidence?

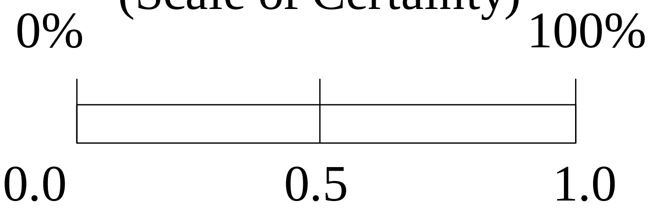
Answer:

Through **logical reasoning**, i.e. by taking full use of the **mathematical language of uncertainty**. If we have a level of uncertainty (i.e. a probability distribution), we can mathematically derive how that uncertainty would change if we were to obtain evidence, in general. If we then in fact observe **specific evidence** it only make sense to revise our uncertainty **accordingly**.

HOW?
BAYES THEOREM.

WHAT IS BAYES THEOREM?

(Scale of Certainty)



$$\Pr(A) + \Pr(\overline{A}) = 1$$

$$\Pr(A \cap E) = \Pr(E)\Pr(A \mid E)$$

$$\Pr(A \cap E) = \Pr(A)\Pr(E \mid A)$$

Therefore:

$$\Pr(E)\Pr(A \mid E) = \Pr(A)\Pr(E \mid A)$$

Bayes Theorem: $\Pr(A \mid E) = \Pr(A)\left[\frac{\Pr(E \mid A)}{\Pr(E)}\right]$

Posterior Prior Correction Factor

Example: Learning through Evidence

B = {Killer in a Murder Case}, B $\in$ {B_1, $B_{2,}$ B_3},
B_1 = Hunter, B_2 = Near Sighted Man, B_3 = Sharp Shooter

- After **interrogations, interviews with witnesses**, we are able to establish the following **prior distribution.**

  $Pr(B= B_1)=0.2$, $Pr(B= B_2)=0.7$, $Pr(B= B_3)=0.1$.

- Evidence A becomes available, being that the victim was shot from 2000 ft. We establish the **following probability model.**

  $Pr(A|B_1)=0.7$, $Pr(A|B_2)=0.1$, $Pr(A|B_3)=0.9$.

- We update our prior distribution using the evidence into a **posterior distribution** using Bayes Theorem.

$$Pr(A) = Pr(A|B_1)Pr(B_1)+ Pr(A|B_2)Pr(B_2)+ Pr(A|B_3)Pr(B_3)$$
$$= 0.7 * 0.2+0.1 * 0.7+0.9 * 0.1=0.30$$

$$\Pr(B_1 \mid A) = \frac{\Pr(A|B_1)\Pr(B_1)}{\Pr(A)} = \frac{0.7 \cdot 0.2}{0.3} = 0.47$$
$$\Pr(B_2 \mid A) = \frac{\Pr(A|B_2)\Pr(B_2)}{\Pr(A)} = \frac{0.1 \cdot 0.7}{0.3} = 0.23$$
$$\Pr(B_3 \mid A) = \frac{\Pr(A|B_3)\Pr(B_3)}{\Pr(A)} = \frac{0.9 \cdot 0.1}{0.3} = 0.30$$

Conclusion:

Refocus investigation on Hunter and Sharp shooter.

What has Bayes Theorem to do with Logical Reasoning?

$$\Pr(A \mid B) = \Pr(A)\left[\frac{\Pr(B \mid A)}{\Pr(B)}\right]$$

LOGIC	BAYES THEOREM
1. MODUS PONENS (Syllogism of Aristotle) • Statement: If A occurs, B occurs • Now: A Occurs •Conclusion: B Occurs	$\Pr(B \mid A) = 1.0$ $\Pr(A) = 1.0 = \Pr(A \mid B)$ Calculation: $\Pr(B) = 1.0$
2. MODUS TOLENS (Syllogism of Aristotle) • Statement: If A occurs, B occurs • Now: B does not occur •Conclusion: A did not occur	$\Pr(B \mid A) = 1.0$ $\Pr(B) = 0.0$ Calculation: $\Pr(A) = 0.0$
3.PLAUSIBLE REASONING • Statement: If A occurs, B occurs • Suppose B Occurs •Conclusion: A is more likely	$\Pr(B \mid A) = 1.0$ Calculation: $\Pr(A \mid B) \geq \Pr(A)$
4.PLAUSIBLE REASONING • Statement: B is unlikely, except when A occurred. • Suppose B Occurs •Conclusion: A is much more likely	$\left\{\begin{array}{l}\Pr(B) \text{ is small} \\ \Pr(B \mid A) \text{ is sizable}\end{array}\right.$ Calculation: $\Pr(A \mid B) >> \Pr(A)$

7. OBJECTIVE VS. SUBJECTIVE PROBABILITY CONTROVERSY

Bayesians:

1. Probability = "Confidence".
2. Confidence is a state of mind
3. State of mind is personal.
4. Probability is subjective.

Classical Statistical School:

1. This is "unscientific"
2. Dismissed Bayesian thinking
3. Dismissed the use of expert judgment.

Bayesian reacted and dispute is still on going.

- Dispute is a result of miscommunication due to the **personal dimensions** of the word "subjective", "confidence", and "belief".
- Better to use the words "Plausibility" or "Credibility" which are **properties of evidence**, not of the person.

True Bayesian uses probability in that sense,
dictated by evidence through Bayes Theorem,
no personality, no "opinion".

"**Probability Theory** is an **extension of logic**, which describes the **inductive reasoning** of an idealized being who represents degrees of plausibility by real numbers. The numerical value of any probability (A/B) will in general depend not only on A, or B, but also on the entire background of other propositions that this being is taken into account. A **probability** is **"subjective"** in the sense that it describes **a state of knowledge** rather than any property of the "real" world; but it is completely **"objective"** in the sense that it **is independent of the personality of the user**; two beings faced with the same total background of knowledge must assign the same probabilities"

- E.T. Jaynes.

- To classical statisticans Bayes Theorem is just another law of probability, but not very usefull.

- To a Bayesian, this is not just another theorem, its the fundamental law governing the evaluation of evidence.

- To an extreme Bayesian it is the very definition of logical, rational thinking.

8. EVIDENCE-BASED DECISION MAKING

"Objectifying" the **"Subjective probability"** resolves:

- Resolves historical controversies
- Puts Risk Analysis on solid conceptual foundation.
- Opens the way to "evidence based risk assessment".

"Let the Evidence Speak", not the opinions, personalitities, moods, politics, positions, special interests, or whishful thinking!

Guideline for dealing with experts:

- Never asks for his opinion, always ask for his experience, his information, or his evidence.
- Collect all information from all experts and work with the group over this list to arrive at the "consensus body of evidence".

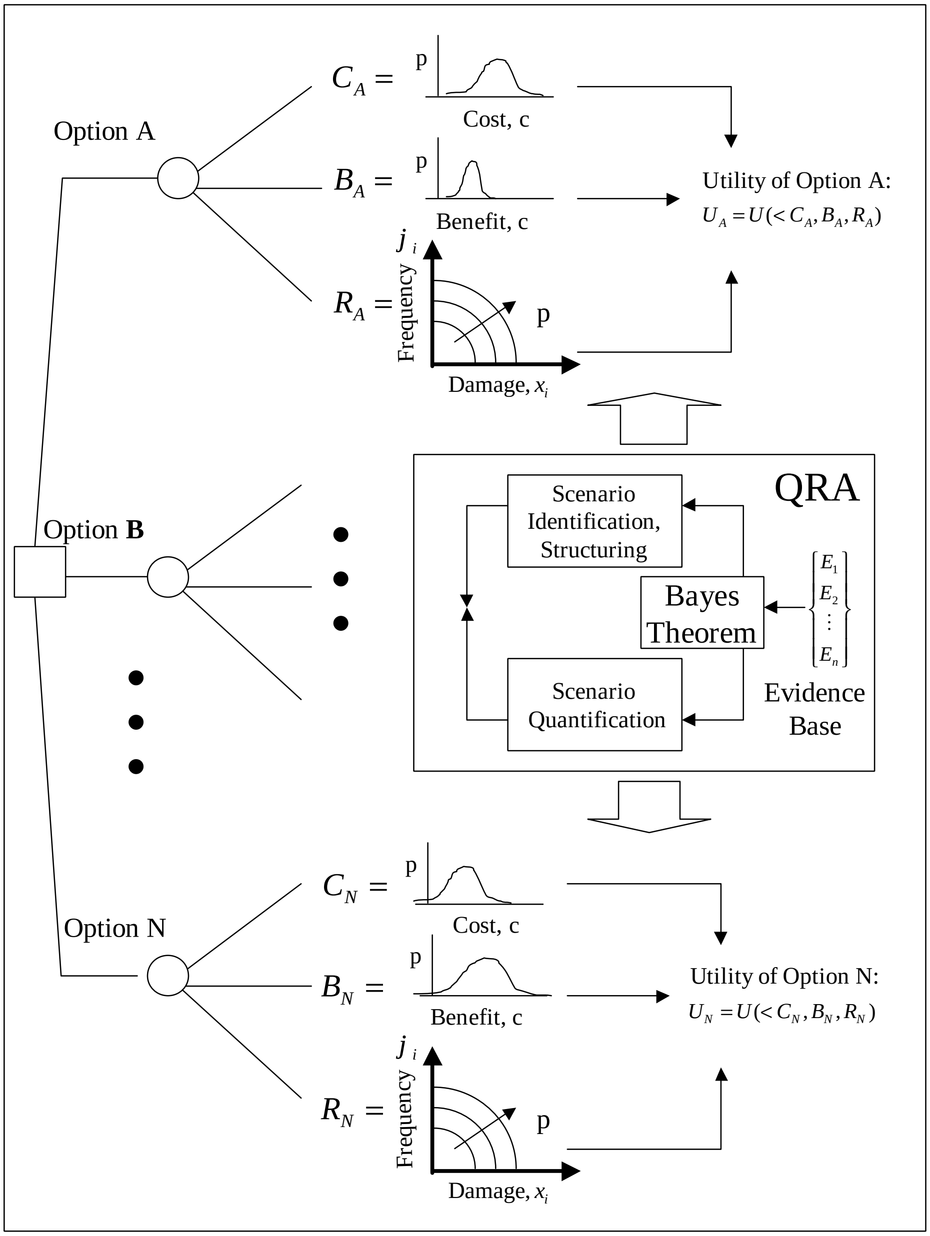
Role of Quantitative Risk Analysis (QRA):

- A decision is selecting **one** choice out of a set of options.
- An option is characterized by its **Cost**, its **Benefit**, and its **Risk.**
- Cost, Benefit, and Risk are uncertain and uncertainty needs to be expressed as probability curves = **Role of QRA**

Role of Regulation:

- Evaluation of different option require a trade-off of Cost-Benefit-Risk (=Cost Benefit Analysis)
- Trade-off involves making value judgements (=Utitility Theory).
- Regulators are supposed to represent the value judgements of the public. (=Tough Job).

The Anatomy of a Decision – the role of QRA and Bayes Theorem



- Regulators often set a level of **acceptable risk** without performing the decision analysis.
- The question is not: "How much is acceptable".
- The question is: "What is the best decision option".

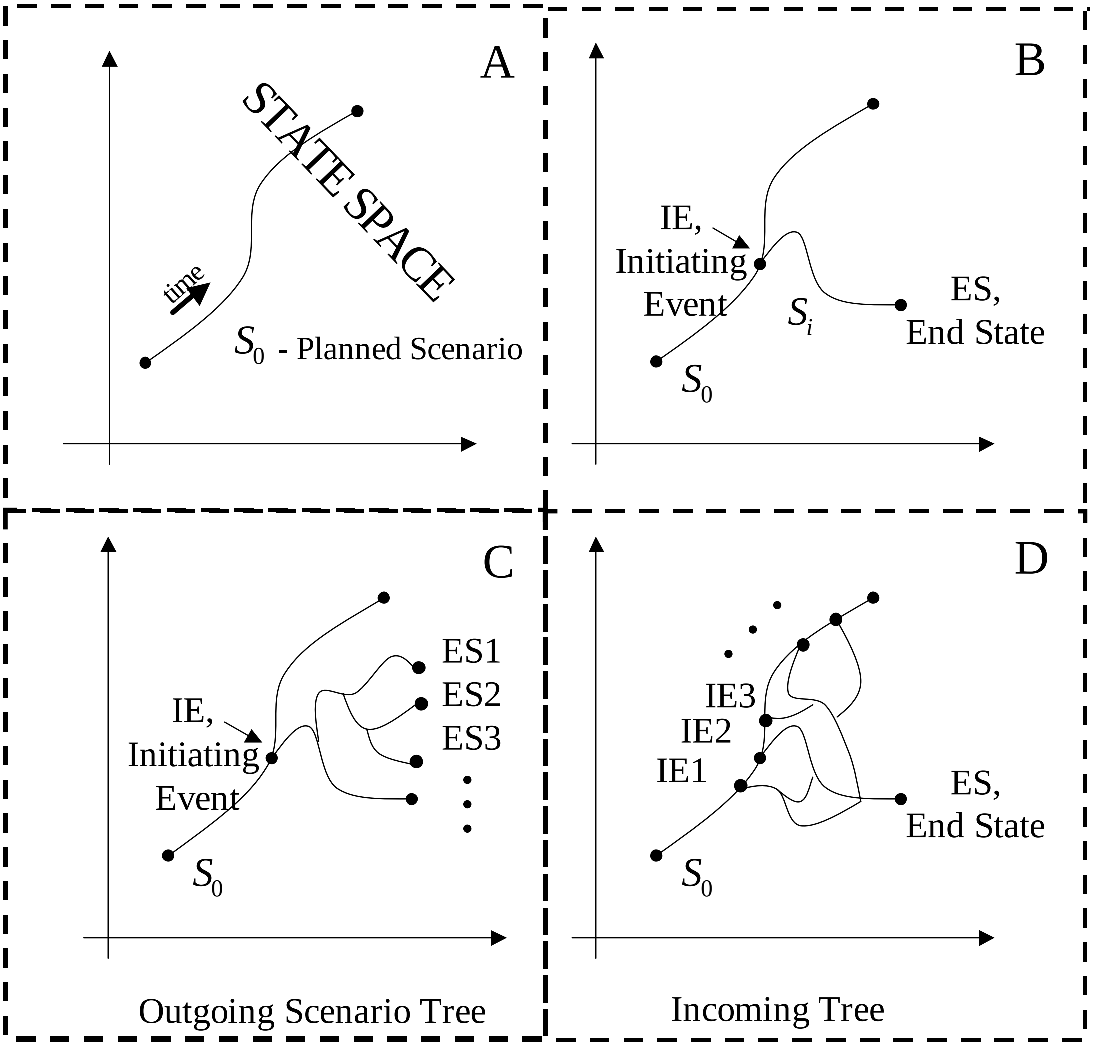
9. FINDING THE SCENARIO'S

Risk Analysis Is As Much An **Art** As A **Science**

Expressing out uncertainty in Damage and Frequency is considered **the Science Part**. What is the **Art Part?**

- Concentrated on second and third term of the **RISK TRIPLET, i.e. Likelihood and Consequence**

Art Part: Establishing The Scenarios?



Method 1: Event Trees – Inductive Approach

Find Initiating events and draw the outgoing tree **from** each.

Method 2: Fault Trees – Deductive Appoach

Find the end states and draw the incoming tree **to** each .

Interesting Russian Approach: Instead of asking "What can go wrong?" ask, "If I want to make something go wrong, how would I do it?".

10. SUMMARIZING

- We do risk assessment because we have decisions to make
- To make decisions we need three things; a set of **options**, **outcomes** with these options and a **value judgement.**
- **Role of QRA** is quantitatively evaluate the outcomes
- Since the Truth is that we are Uncertain, outcomes should be expressed in terms of uncertainty/probability curves
- For these curves to be **worthy of trust** their establishment should be based on the **entire body of evidence** available.
- Decision Analysis needs a set of options. You need stakeholder representation to establish this set of options (= risk reduction measures)
- Decision Analysis recommends the "optimal option".

WE ARE NOT DONE!

Recommendation needs to be **accepted** and **implemented.**

Requires **Building of Trust** and
Risk Communication Skills

ANOTHER APPLICATION OF BAYES THEOREM

Game Show Example:

Suppose we have a game show host and you. There are **three doors** and **one of them contains a prize**. The game show host knows the door containing the prize but of course does not convey this information to you. He asks you to pick a door. **You picked door 1** and are walking up to door 1 to open it when the game show host screams: **STOP**. You stop and the game show host shows door 3 which appears to be empty. Next, the game show asks.

"DO YOU WANT TO SWITCH TO DOOR 2?"

WHAT SHOULD YOU DO?

Assumption 1: The game show host will never show the door with the prize.
Assumption 2: The game show will never show the door that you picked.

- D_i ={Prize is behind door i }, i=1,... ,3
- H_i ={Host shows door i containing no prize after you selected Door 1}, i=1,... ,3

Initially: $\Pr(D_i)=\frac{1}{3}$

1. $\Pr(H_3)=\sum_{i=1}^{3}\Pr(H_3\mid D_i)\Pr(D_i)=\frac{1}{2}*\frac{1}{3}+1*\frac{1}{3}+0*\frac{1}{3}=\frac{1}{2}$

2. $\Pr(D_1\mid H_3)=\frac{\Pr(H_3\mid D_1)\Pr(D_1)}{\Pr(H_3)}=\frac{\frac{1}{2}*\frac{1}{3}}{\frac{1}{2}}=\frac{1}{3}$

3. $\Pr(D_2 \mid H_3) = 1 - \Pr(D_1 \mid H_3) = 1 - \frac{1}{3} = \frac{2}{3}$.

So Yes, you should switch!