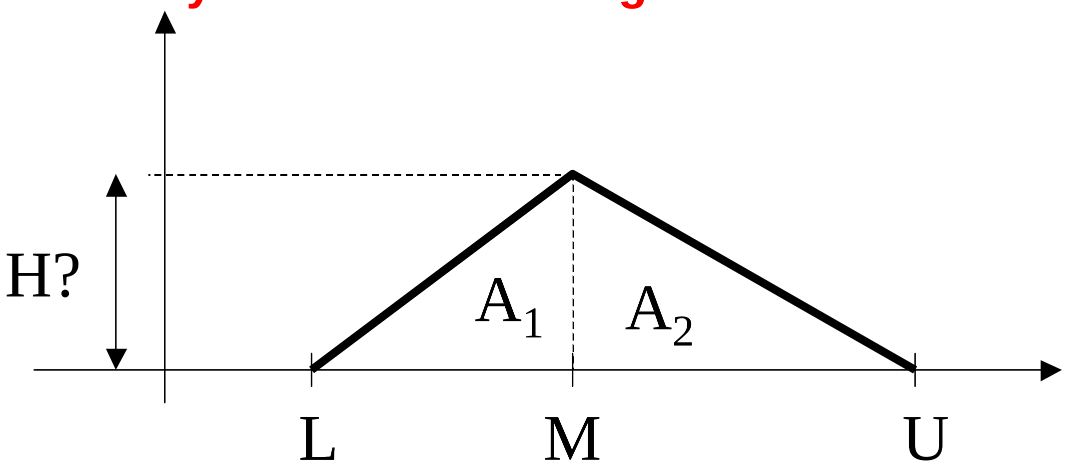
INTERMEZZO – TRIANGULAR DISTRIBUTION

Three Parameters:
Lower Bound: L
Upper Bound: U
Most Likely (or Best Guess): M

Density Function of Triangular Distribution



QUESTION:
What the value of the height h?

ANSWER:

Total Area under Density function should be one:

$$\frac{1}{2}(M-L)H+\frac{1}{2}(U-M)H=1 \Leftrightarrow$$

$$H\left\{\frac{1}{2}(M-L)+\frac{1}{2}(U-M)\right\}=1 \Leftrightarrow$$

$$H\left\{\frac{1}{2}(U-L)\right\}=1 \Leftrightarrow H=\frac{2}{U-L}$$

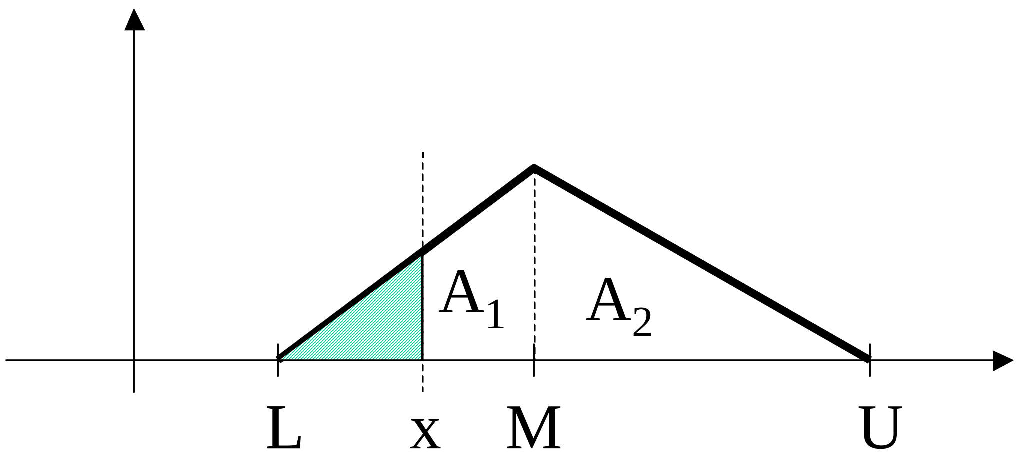
INTERESTING OBSERVATIONS:

- H does not depend on M
- $A_1 = \frac{1}{2}(M - L) \cdot \frac{2}{U - L} = \frac{M - L}{U - L}$ which equals the relative distance from the lower bound.
- $A_2 = \frac{1}{2}(U - M) \cdot \frac{2}{U - L} = \frac{U - M}{U - L}$ which equals the relative distance from the upper bound.

CUMULATIVE DISTRIBUTION FUNCTION OF TRIANG(L,M,U)

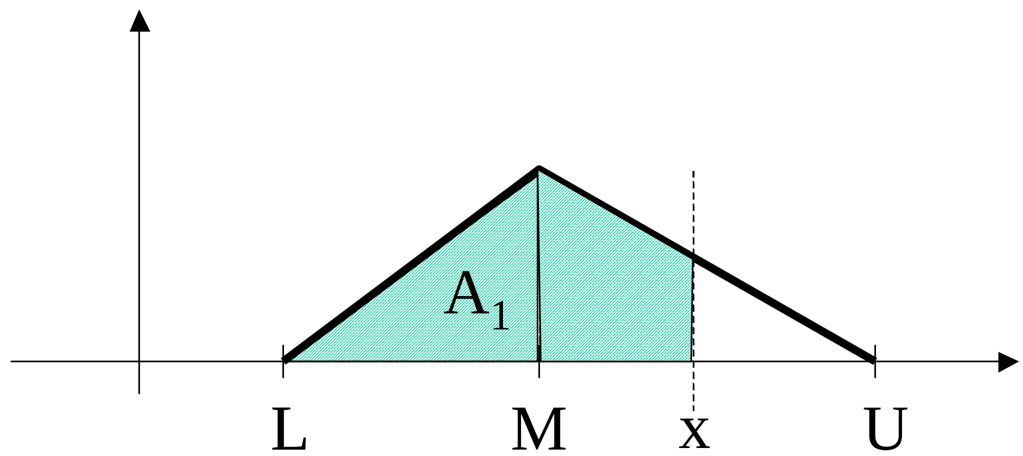
$$F(x) = \Pr(X \le x)$$

1. Suppose $L \le x \le M$



$$F(x) = \left(\frac{x-L}{M-L}\right)^2 \cdot A_1 = \left(\frac{x-L}{M-L}\right)^2 \cdot \frac{M-L}{U-L}$$

2. Suppose $M \leq x \leq U$



$$F(x) = 1 - \left(\frac{U-x}{U-M}\right)^2 \cdot A_2 = 1 - \left(\frac{U-x}{U-M}\right)^2 \cdot \frac{U-M}{U-L}$$

Hence

$$F(x)=\begin{cases}\left(\frac{x-L}{M-L}\right)^2\cdot\frac{M-L}{U-L} & L\le x\le M\\ 1-\left(\frac{U-x}{U-M}\right)^2\cdot\frac{U-M}{U-L} & M\le x\le U\end{cases}$$

- From the form of F(x), L ≤ X≤ U we can derive the expression for F^{-1}(y), 0 ≤ X ≤ 1.

$$F^{-1}(y)=\begin{cases}L+\sqrt{y\left(U-L\right)\left(M-L\right)} & 0\le y\le\frac{M-L}{U-L}\\ U-\sqrt{(1-y)\left(U-L\right)\left(U-M\right)} & \frac{M-L}{U-L}\le y\le 1\end{cases}$$

HOMEWORK: Derive the expression for F^{-1}(y). With the form of F^{-1}(y) we can sample from a Triang(L,M,U) distribution via a Uniform Random Variable on [0,1]