MONTE CARLO SIMULATION/INTEGRATION

INPUT UNCERTAINTY

X

Y

Z

MODEL = F(X,Y,Z)

Calculate

Sample X₁,Y₁,Z₁ → O₁

Calculate

Sample X₂,Y₂,Z₂ → O₂

Calculate

Sample X₃,Y₃,Z₃ → O₃

ETC ...

OUTPUT

Lecture Notes by Instructor: Dr. J. Rene van Dorp
RANDOM VARIATE GENERATION: Let $U$ be a Uniform Random Variable on $[0,1]$ with density function $f(u)$ and cumulative distribution function $F(u)$.

$$f(u) = \begin{cases} 
0 & u < 0 \\
1 & 0 \leq u \leq 1 \\
0 & 0 
\end{cases}$$

$$F(u) = \Pr(U \leq u) = \begin{cases} 
0 & u < 0 \\
u & 0 \leq u \leq 1 \\
1 & 1 
\end{cases}$$
CONTINUOUS RANDOM NUMBER GENERATION

Suppose $X$ is a CONTINUOUS random variable with cumulative distribution function $F$

$$\Pr(X \leq x) = F(x)$$

A random Variable $Z$ would also have cumulative distribution function $F$ if:

$$\Pr(Z \leq z) = F(z)$$
CONCLUSION:
CDF is Continuous Strictly Increasing Function. Therefore $F(x)$ has a well defined inverse function:

$$F^{-1}(y) = x$$
THEOREM :

Let $X$ be a continuous random variable with cdf $F(x)$.

Let $U$ be a uniform random variable on $[0,1]$.

Let $Z$ be the random variable, such that

$$Z = F^{-1}(U)$$

$Z$ is a continuous random variable with cdf $F(z)$. 
STEP 1: Sample Realization $u$ from Uniform Random Variable $U$

STEP 2: Calculate realization $x=F^{-1}(u)$ from Random Variable

$x=F^{-1}(u)$
EXAMPLE:
Sampling from an Exponential Distribution with Mean 4

Density: \( f(x \mid \lambda) = \lambda \exp(-\lambda x), \lambda > 0 \)

Cumulative Distribution Function: \( F(x \mid \lambda) = 1 - \exp(-\lambda x), \lambda > 0 \)

Inverse Distribution Function: \( F^{-1}(u \mid \lambda) = -\frac{\ln(1-u)}{\lambda}, \lambda > 0 \)

Average Interarrival time 4.000 Min
Lambda 0.250
Random U 0.100
Realization Arrival Time 0.421 Min

1. Using DATATABLE and RANDOM NUMBER GENERATOR in EXCEL we can create a Sample of the Continuous Distribution.
2. A RANDOM NUMBER GENERATOR generates realizations of a uniform random variable on [0,1]