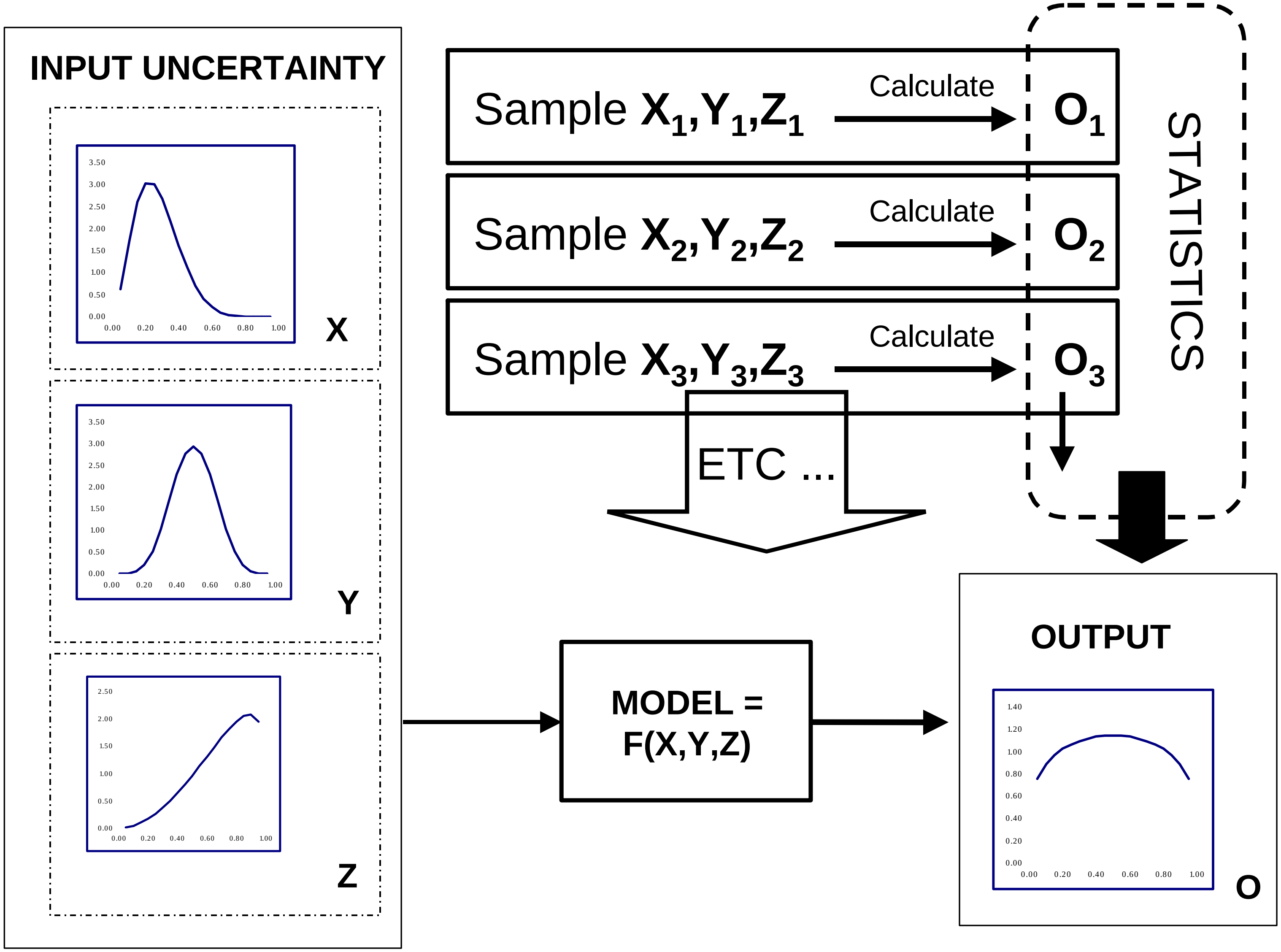
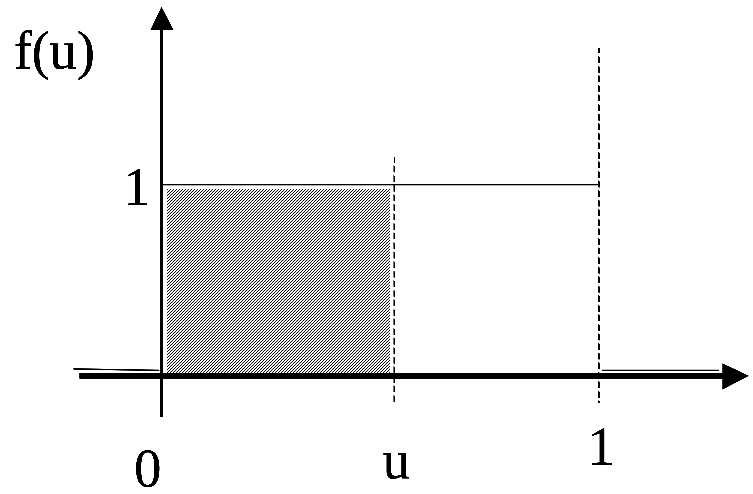
MONTE CARLO SIMULATION/INTEGRATION



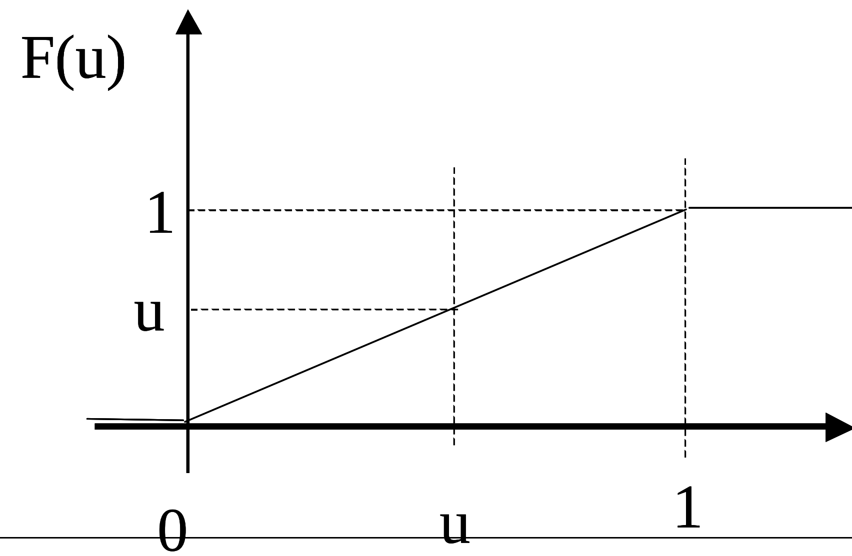
RANDOM VARIATE GENERATION: Let U be a Uniform Random Variable on [0,1] with density function f(u) and cumulative distribution function F(u).

$$f(u)=\begin{cases}0 & u<0\\ 1 & 0\le u\le 1\\ 0 & 0\end{cases}$$



$$F(u)=\Pr(U\le u)=$$

$$=\begin{cases}0 & u<0\\ u & 0\le u\le 1\\ 1 & 1\end{cases}$$



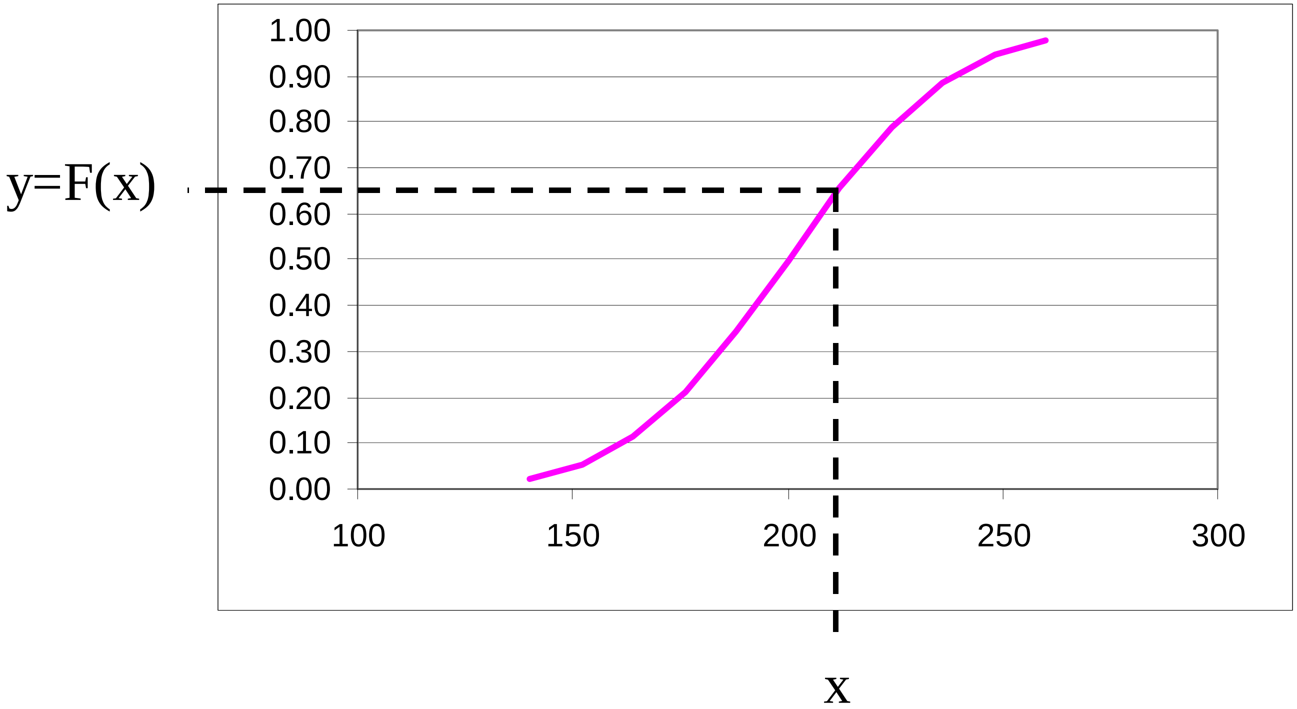
CONTINUOUS RANDOM NUMBER GENERATION

Suppose X is a CONTINUOUS random variable with cumulative distribution function F

$$\Pr(X \leq x) = F(x)$$

A random Variable Z would **also** have cumulative distribution function F if:

$$\Pr(Z \leq z) = F(z)$$



CONCLUSION:

CDF is Continuous Strictly Increasing Function. Therefore F(x) has a well defined inverse function:

$$F^{-1}(y)=x$$

THEOREM :

Let X be a continuous random variable with cdf F(x).

Let U be a uniform random variable on [0,1].

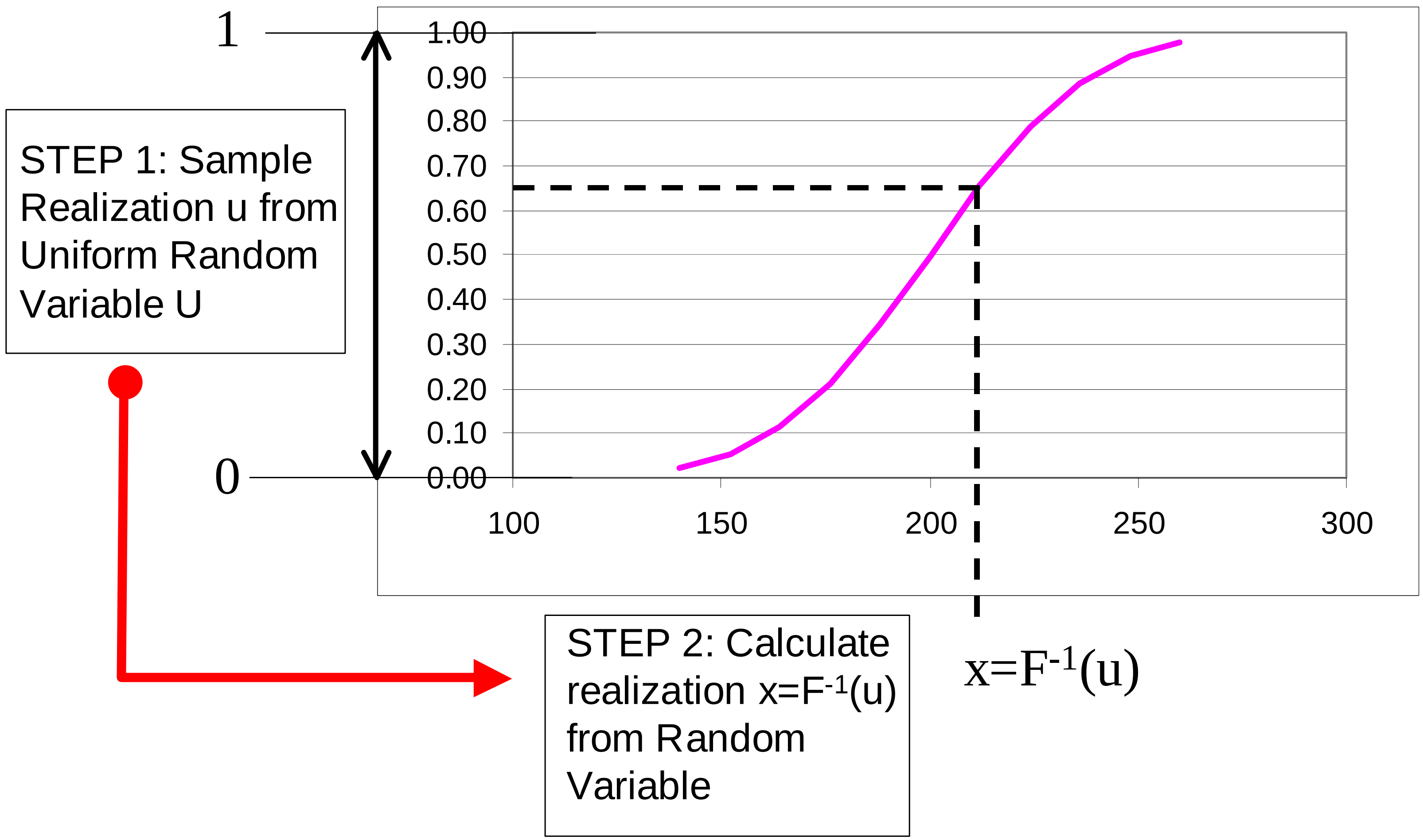
Let Z be the random variable, such that

$$Z = F^{-1}(U)$$

↓

Z is a continuous random variable with cdf F(z).

SAMPLING ALGORITHM



EXAMPLE:
Sampling from an Exponential Distribution with Mean 4

Density: $f(x \mid \lambda) = \lambda \exp(-\lambda x), \lambda > 0$

Cumulative Distribution Function: $F(x \mid \lambda) = 1 - \exp(-\lambda x), \lambda > 0$

Inverse Distribution Function: $F^{-1}(u \mid \lambda) = -\frac{\ln(1-u)}{\lambda}, \lambda > 0$

Average Interarrival time	**4.000**	Min
Lambda	**0.250**	
Random U	**0.100**	
Realization Arrival Time	**0.421**	Min

1. Using DATATABLE and RANDOM NUMBER GENERATOR in EXCEL we can create a Sample of the Continuous Distribution.
2. A RANDOM NUMBER GENERATOR generates realizations of a uniform random variable on [0,1]