# What do Coin Tosses and Decision Making under Uncertainty, have in common?

**J. Rene van Dorp (GW)**

Presentation EMSE 1001 November 12, 2021

Presented by: J. Rene van Dorp

# About J. René van Dorp

**Some Pages from my Faculty Page.**

**EMSE**
Engineering Management and Systems Engineering

**Dr. Johan René van Dorp**
Professor

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background | research areas | projects | publications | students | courses

***Interests:***

**Probabilistic Risk Assessment**

**Reliability Analysis**

**Monte Carlo Analysis**

**Distribution Theory**

**Financial Engineering**

**EMSE**
Engineering Management and Systems Engineering

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***Faculty Profile:***

**Education**

**Experience**

**Honors & Awards**

**Resume**

**<< Home Page**

***Introduction:***

I received degrees from the Delft University of Technology (The Netherlands) and The George Washington University (GWU). Thomas A. Mazzuchi acted as my dissertation advisor at GWU. My education focused on several disciplines of operations research such as probabilistic risk analysis, reliability analysis, computer science and mathematical control and policy modeling.

I joined GWU's the Engineering Management faculty as a visiting assistant professor from September 1997 to August 1999. In September 1999, I started a tenure track position as an Assistant Professor in the Engineering Management and Systems Engineering Department (EMSE) at GWU. Effective September 2004 I was promoted to Associate Professor and effective September 2008 I was promoted to Professor. I teach on average two courses per semester. I have been awarded a courtesy appointment in the Decision Sciences department of the GWU School of Business since November of 2010.

# Undergraduate Courses taught by J. René van Dorp

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*Teaching:*

**Undergraduate**

**Graduate**

**<< Courses Intro**

**<< Home Page**

## *Undergraduate-level:*

I serve as lead professor of the Introductory Course on Probability and Statistics (ApSc 3115) and am teaching in two subject matters (EMSE 3760 and EMSE 4765) at the undergraduate level. I have taught the EMSE 4755 course and occassionallly provide guest lectures in EMSE1001. Below you can link to short course descriptions.

- **ApSc 3115** - Engineering Analysis III (Introductory Course on Probability and Statistics)
- **EMSE 1001** - Guest Lectures on Simulation and Decision Analysis
- **EMSE 3760** - Discrete System Simulation
- **EMSE 4755** - Quality Control and Acceptance Sampling
- **EMSE 4765** - Data Analysis for Engineers and Scientists

http://www2.seas.gwu.edu/~dorpjr/index.html

# OUTLINE

1. Coin Tosses
2. Decision Making under Uncertainty
3. Decision Trees
4. Elements of Decision Analysis

1. Imagine we have a coin and we flip it repeatedly


2. When heads turns up you "win" when tails turns up you "lose"

Suppose we flip the coin four times, how many times do you expect to win?

2 times

Suppose we flip the coin ten times, how many times do you expect to win?

5 times

WHAT ASSUMPTION(S) DID YOU MAKE?

**Conclusion:** you made **reasonable assumptions** –

1. The coin has two different sides
2. When flipping it, each side turns up 50% of the time "on average".

Would it have made sense to assume
the coin had only one face
i.e. both sides show heads (or tails)?

No

Assuming both sides show heads or tails
is equivalent to making
a **worst case** or **best case** assumption.

Suppose you actually flip the “fair” coin ten times

How many times will “heads” actually turn up?

**Answer could vary from 0 to 10 times, for example,**

First ten times : 3 times heads turns up

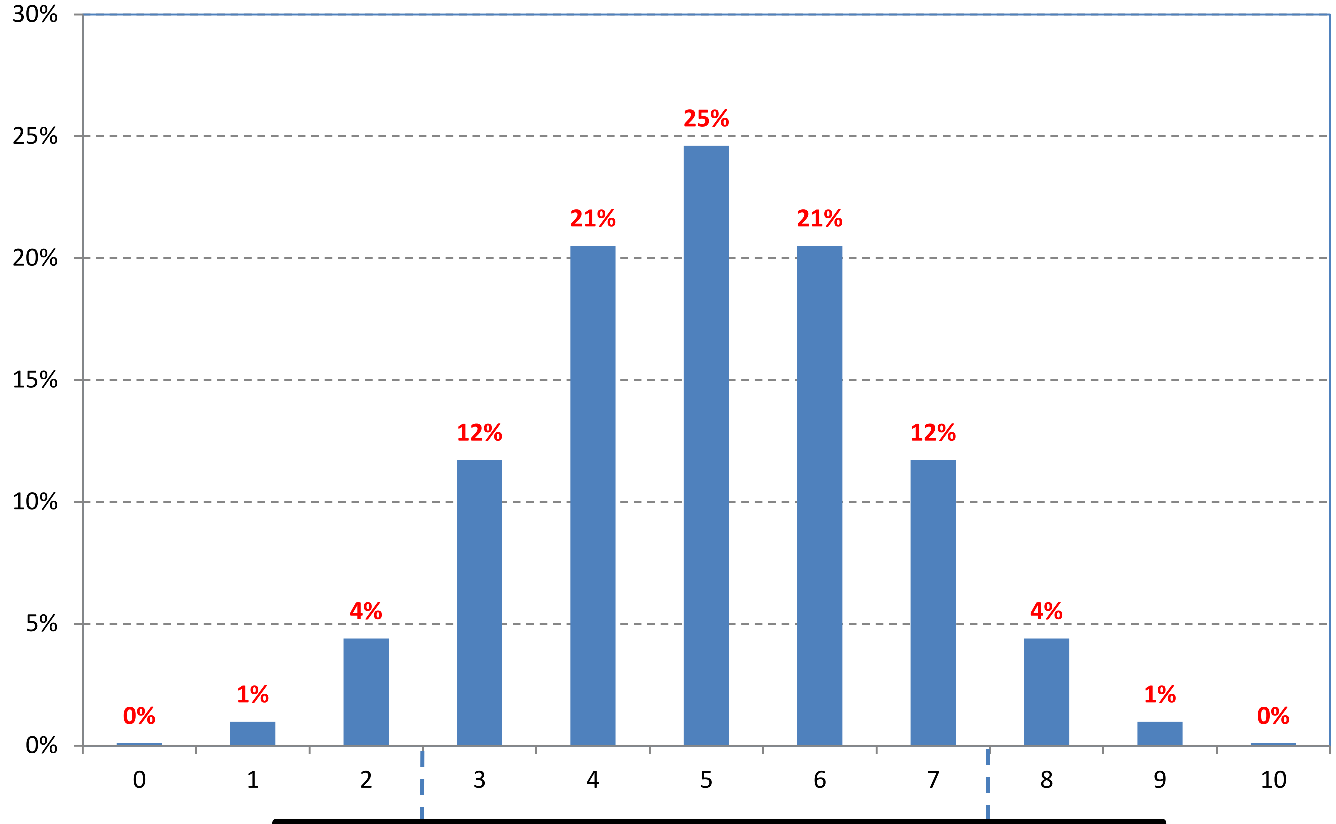
Second ten times : 7 times heads turns up

Third ten times : 6 times heads turns up

Fourth ten times : 4 times heads turns up

etc.

**We say “on average” 5 out of ten times heads turns up**




Approximately 90% of ten throw series will have 3, 4, 5, 6 or 7 times heads turn up

**Conclusion:** While we expect 5 times heads to turn up, the actual number is uncertain!

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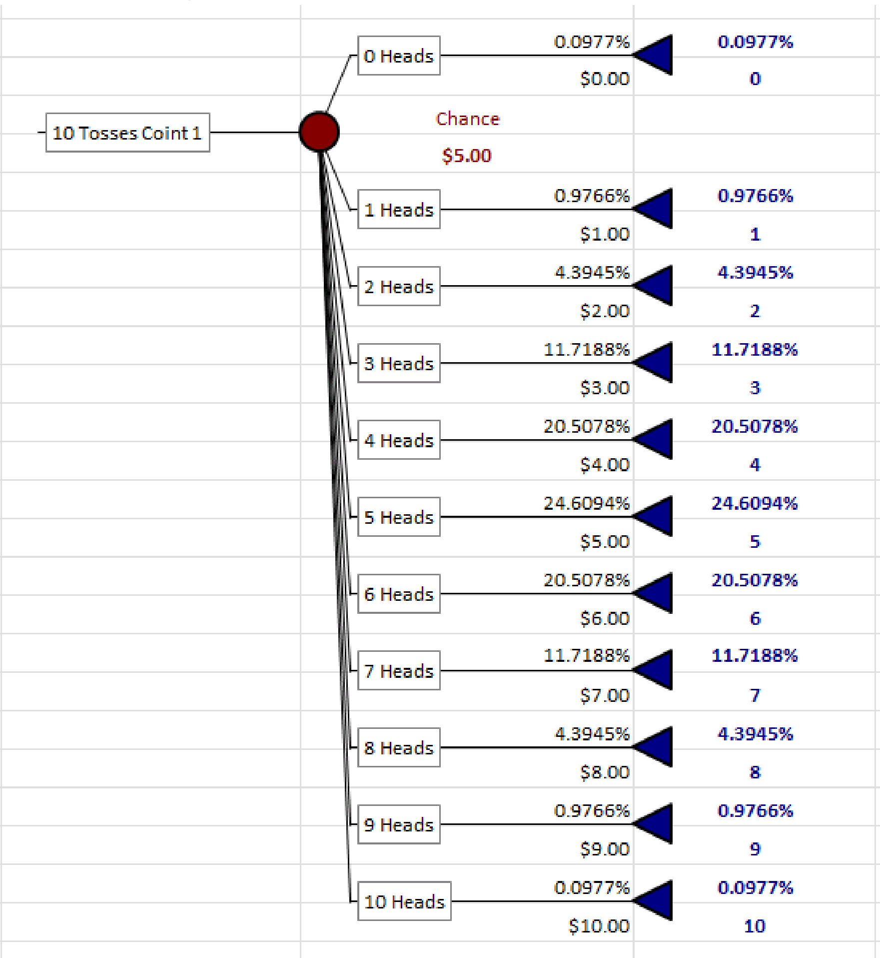
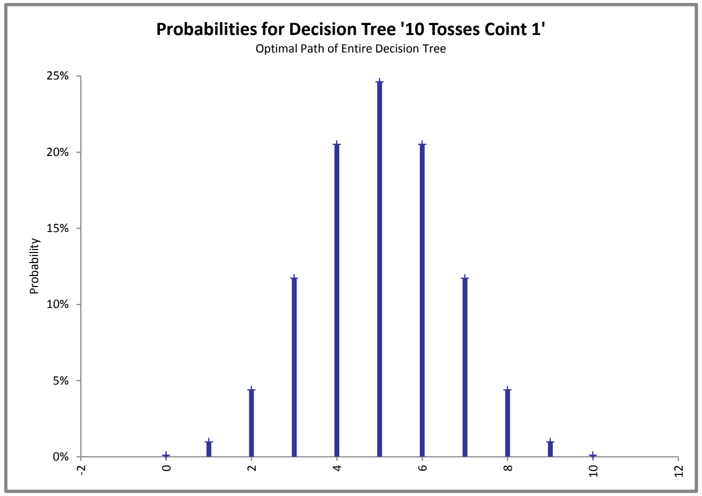
## Decision Analysis Software: Precision Tree

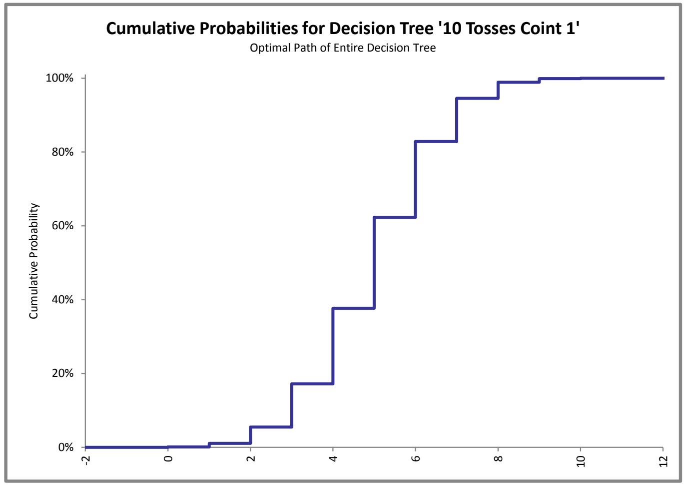
### Probability Node




### Risk Profile (RP) – Probability Mass Function (PMF)




### Cumulative Risk Profile (CRP) – Cumulative Distribution Function (CDF)



# OUTLINE

1. Coin Tosses
2. Decision Making under Uncertainty
3. Decision Trees or Influence Diagrams?
4. Elements of Decision Analysis

1. Imagine we have two coins:
   Coin 1 shows heads **50%** of the time
   Coin 2 shows heads **75%** of the time

Coin 1


Coin 2


2. When heads turns up, you win **a pot of money**. When tails turns up, you do not get anything.

You have to choose between Coin 1 and Coin 2

Which one would you choose? Coin 2

WHAT ASSUMPTION DID YOU MAKE?

You assumed that the pot of money you win is the same regardless of the coin you chose!

# AN INTRO TO DECISION ANALYSIS

1. Imagine we have two coins:

   Coin 1 shows heads 50% of the time
   Coin 2 shows heads 75% of the time

**Coin 1**

**Coin 2**

2. Each time heads turns up, you win **the same pot of money**. When tails turns up you do not get anything, regardless of the coin you throw.

   You have to choose between two alternatives
   Alternative 1: Throwing **ten times** with Coin 1
   Alternative 2: Throwing **five times** with Coin 2

## Which alternative would you choose?

Alternative 1 you expect to win 5 times and
Alternative 2 you expect to win 3.75 times

**CHOOSE ALTERNATIVE 1**

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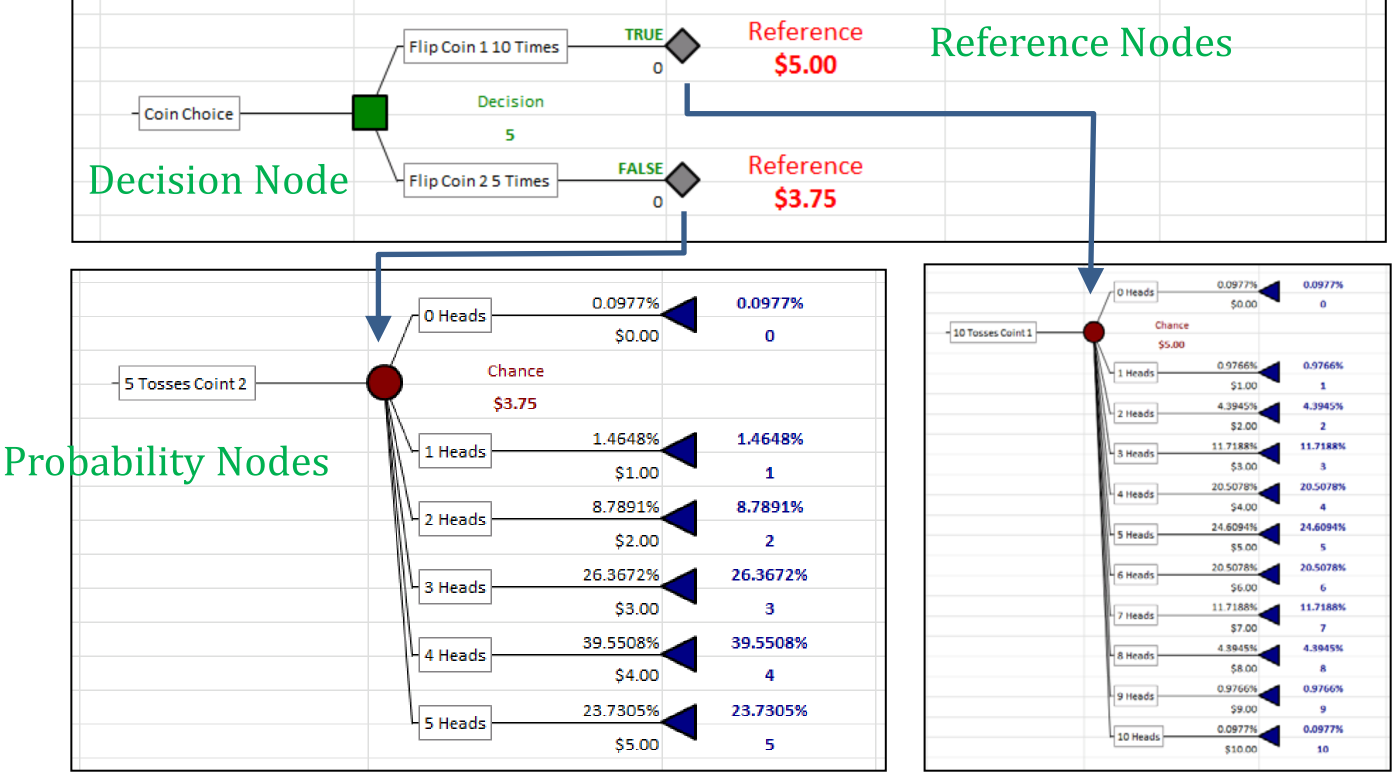
## A DECISION TREE: The Basic Risky Decision

| Alternative | Pr(Heads) | N Trial | Pay-Off per "Heads" | Average Pay-Off |
|---|---|---|---|---|
| Coin 1 | 0.5 | 10 | $1.00 | $5.00 |
| Coin 2 | 0.75 | 5 | $1.00 | $3.75 |




Our objective is to **maximize pay-off.** So **faced with uncertainty** of **pay-off outcomes** we choose the alternative with largest average pay-off..

# AN INTRO TO DECISION ANALYSIS

| Alternative | Pr(Heads) | N Trial | Pay-Off per "Heads" | Average Pay-Off |
|---|---|---|---|---|
| Coin 1 | 0.5 | 10 | $1.00 | $5.00 |
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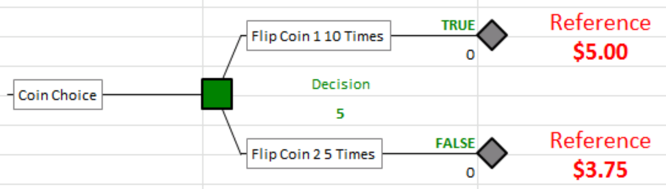



Cumulative Risk Profiles of both Alternatives

## Observe from CRP's on the Right

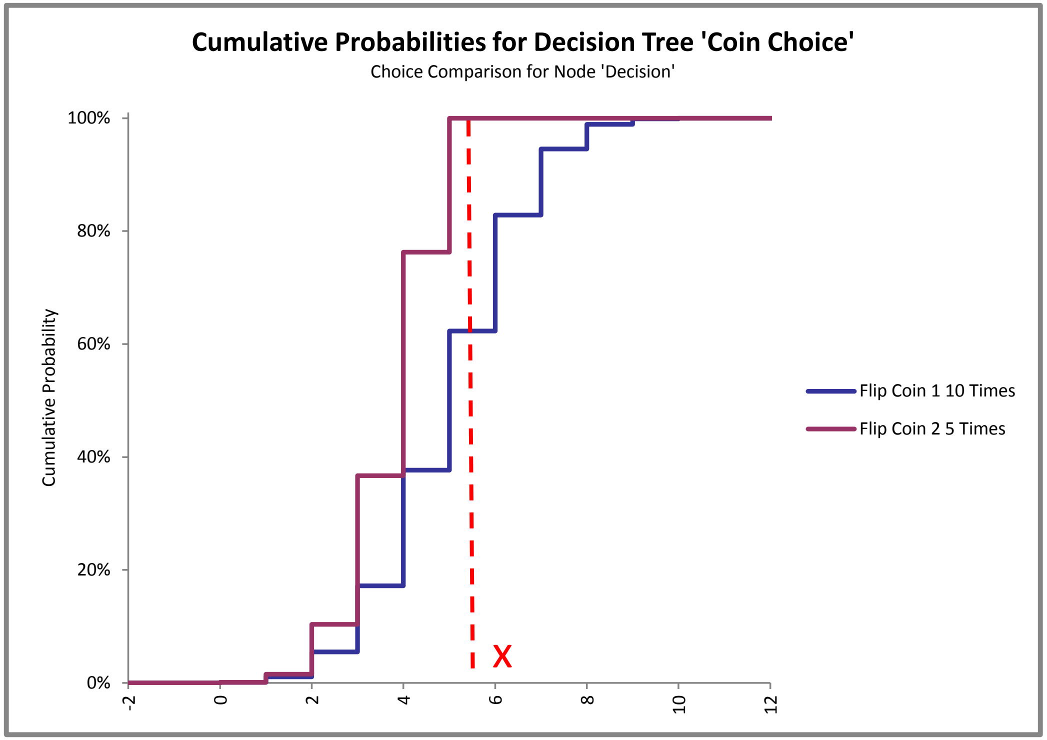
$$\Pr(X \le x \mid Coin\ 1) \le \Pr(X \le x \mid Coin\ 2)$$

$$\Updownarrow$$

$$\Pr(X > x \mid Coin\ 1) \ge \Pr(X > x \mid Coin\ 2)$$

1. Deterministic Dominance
2. Stochastic Dominance
3. Make Decision Based on Averages

Chances of an "Unlucky" Outcome Increase going from 1, 2 to 3



1. Imagine we have two coins:

   Coin 1 shows heads **50%** of the time

   Coin 2 shows heads **75%** of the time

Coin 1

Coin 2

2. Each time heads turns up **with Coin 1 you win $2**. Each time heads turns up **with Coin 2 you win $4**. When tails turns up you do not get anything.

   You have to choose between two ALTERNATIVES

   Alternative 1: Throwing **ten times** with Coin 1

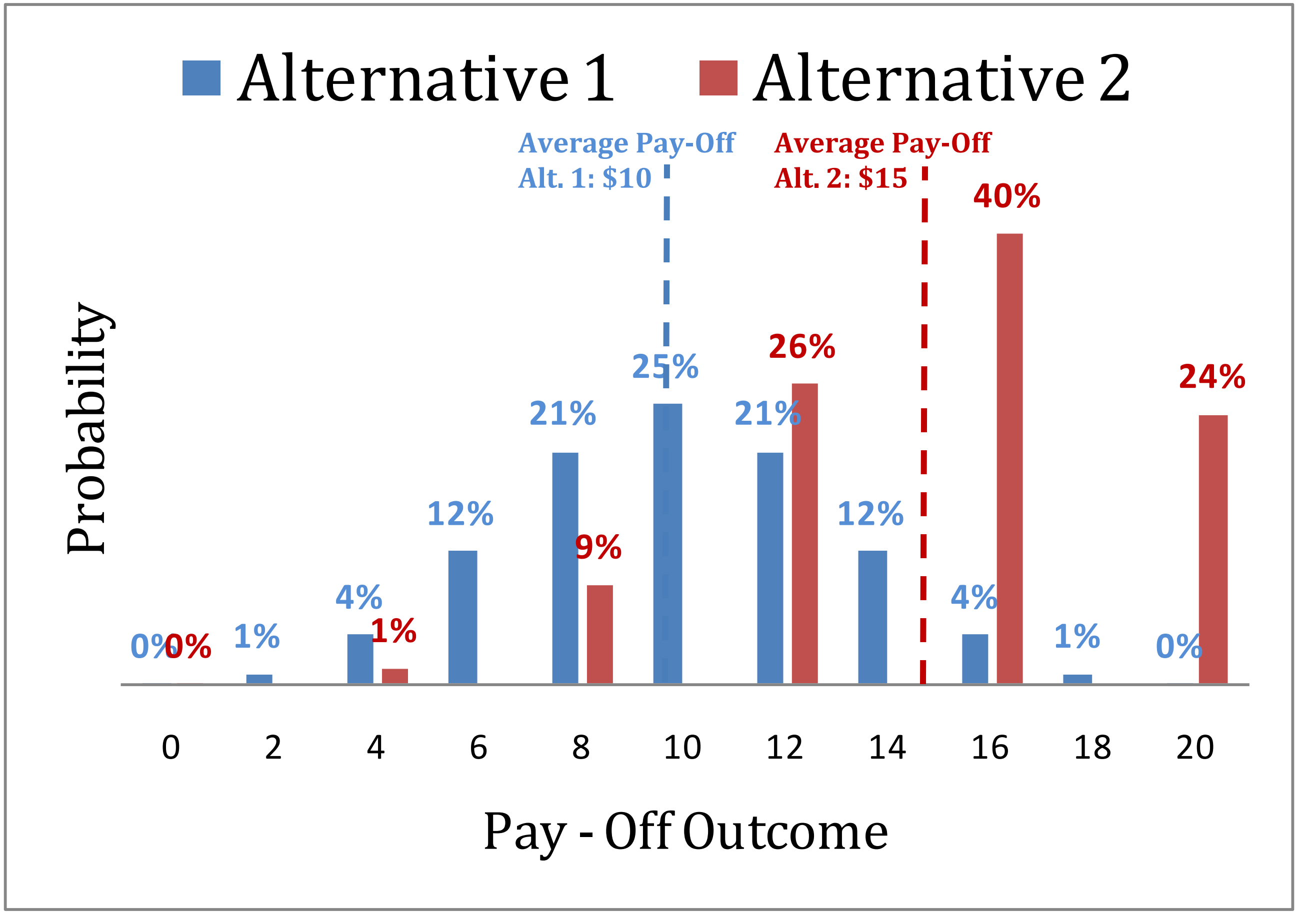
   Alternative 2: Throwing **five times** with Coin 2

# Which alternative would you choose?

Alternative 1 you average 5 * $2 = $10

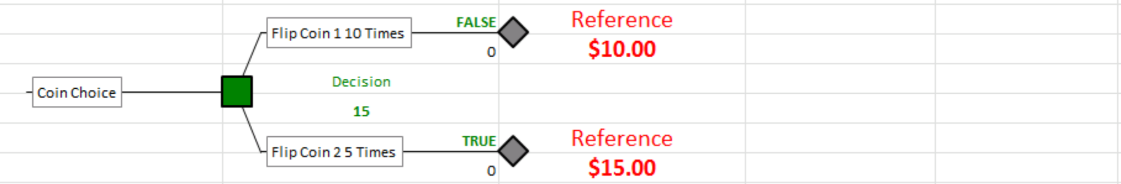
Alternative 2 you average 3.75 * $4 = $15

**CHOOSE ALTERNATIVE 2**




Our objective is to **maximize pay-off.** So **faced with uncertainty** of **pay-off outcomes** we choose the alternative with largest average pay-off.

| Alternative | Pr(Heads) | N Trial | Pay-Off per "Heads" | Average Pay-Off |
|---|---|---|---|---|
| Coin 1 | 0.5 | 10 | $2.00 | $10.00 |
| Coin 2 | 0.75 | 5 | $4.00 | $15.00 |




Please Note Optimal Choice and Stochastic Dominance "Switched"

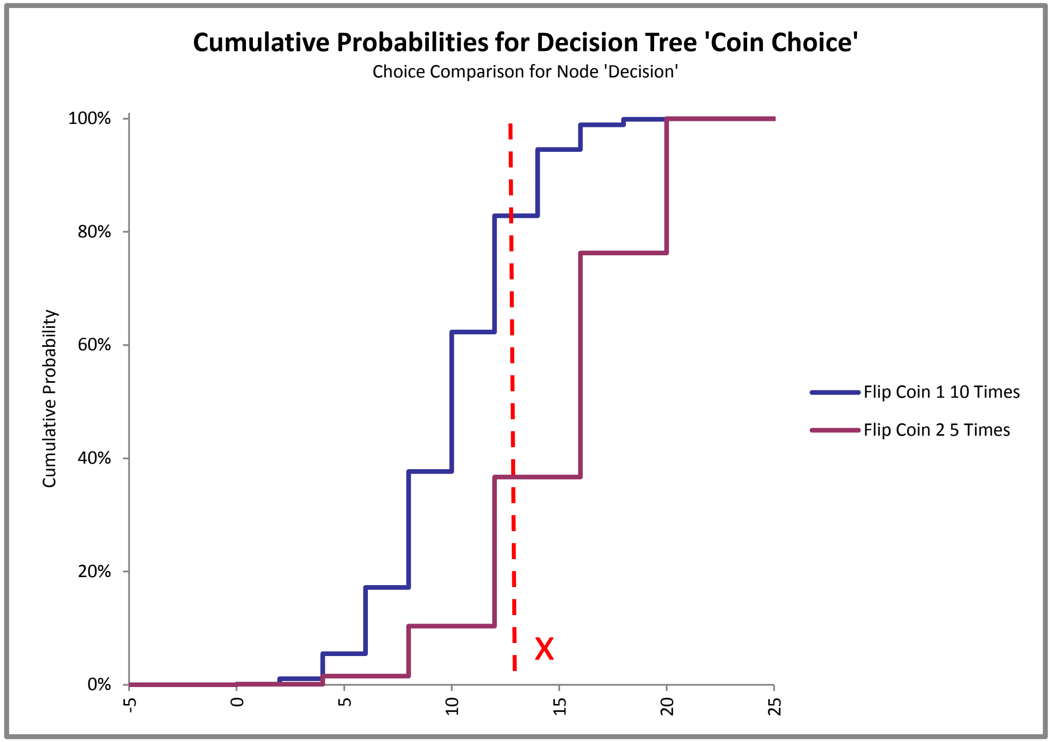
CRP' S of both Alternatives

Observe from CRP's on the Right

$$\Pr(X \leq x | Coin\ 2) \leq \Pr(X \leq x | Coin\ 1)$$
$$\Updownarrow$$
$$\Pr(X > x | Coin\ 2) \geq \Pr(X > x | Coin\ 1)$$

1. Deterministic Dominance
2. Stochastic Dominance
3. Make Decision Based on Averages

Chances of an "Unlucky" Outcome Increase going from 1, 2 to 3



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# Conclusion?

When choosing between **two alternatives** entailing a series of coin toss trials, the following comes into play:

1. The number of trials ***N*** in each alternative
2. The probability of success ***P*** per trial
3. The pay-off amount ***W*** per trial

AVERAGE PAY-OFF = N × P × W

**Is it required to know the absolute value of N, P and W to choose between these two alternatives?**

1. Imagine we have two coins:

   Coin 2 shows heads **1.5 times more** than Coin 1

2. When heads turns up with Coin 2 **you win 2 times the amount** when heads turns up with Coin 1.

   You have to choose between **Two Alternatives**

   Alternative 1: Throwing **2*N times** with Coin 1

   Alternative 2: Throwing **N times** with Coin 2

   P = % Heads turns up with Coin 1,

   W = $ amount you win with Coin 1.

$$\frac{\text{Average Pay – Off Alternative 2 :}}{\text{Average Pay – Off Alternative 1 :}} \quad \frac{\cancel{N} \times 1.5 \times \cancel{P} \times \cancel{2} \times \cancel{W}}{\cancel{2} \times \cancel{N} \times \cancel{P} \times \cancel{W}}$$

Average Pay-Off Alt. 2/Average Pay-Off Alt. 1 = 1.5

# Conclusion?

When choosing between **two alternatives** entailing a series of trials, we can even make a choice if just we know **the multiplier between the average pay-offs**. That is, even when the absolute pay-off values over the two alternatives are unknown/uncertain

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| Alternative | Pr(Heads) | N Trial | Pay-Off per "Heads" | Average Pay-Off |
|---|---|---|---|---|
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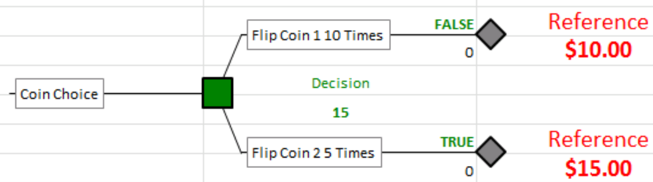
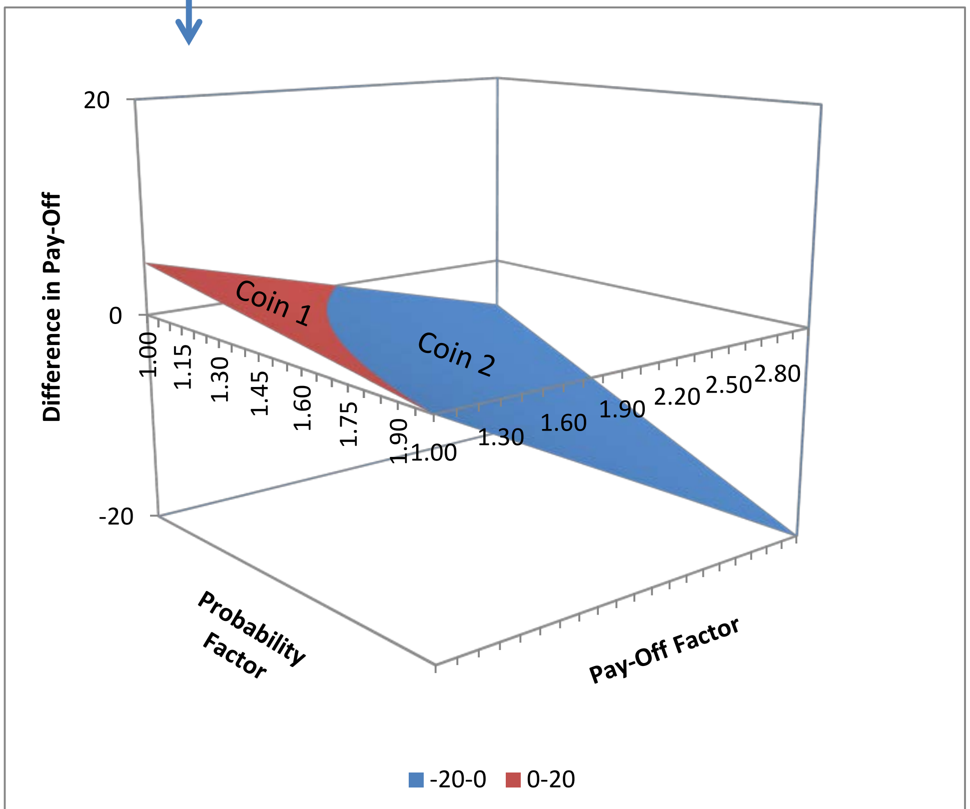
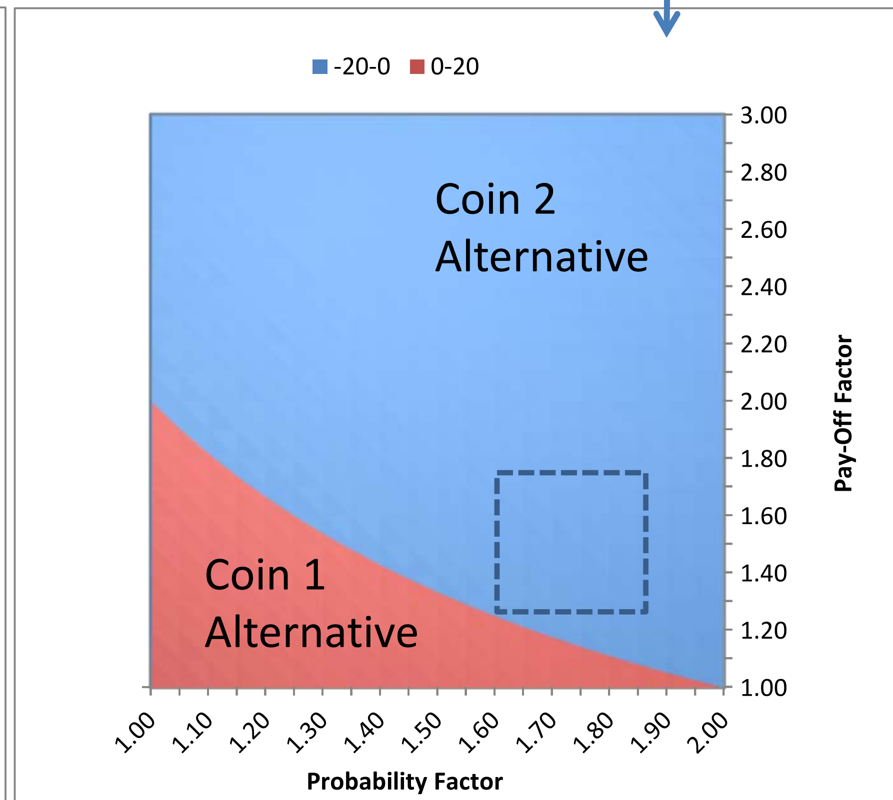



| | |
|---|---|
| Prob. Factor | 1.5 |
| Pay-Off Factor | 2 |

## 2D – Strategy Region Diagram



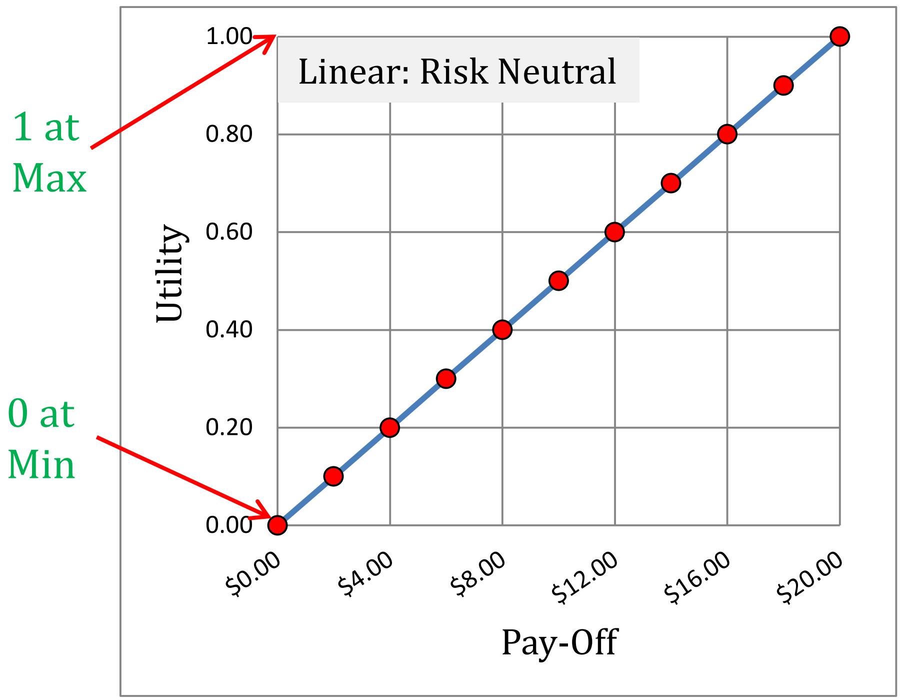

## 2D – Strategy Region Diagram



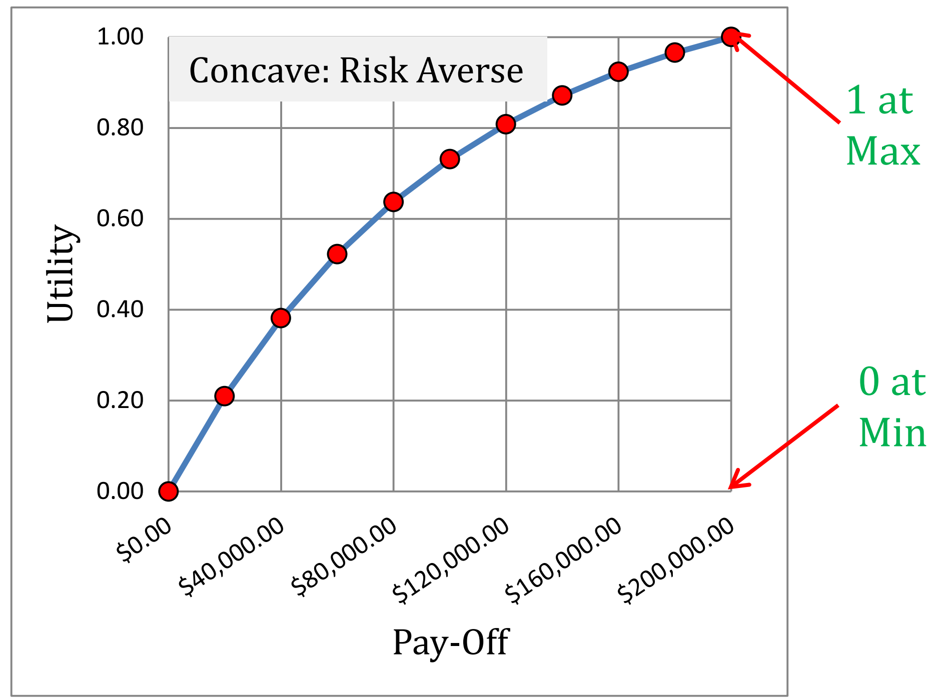
# Conclusion?

When choosing between **two alternatives** entailing a series of trials, we can make a choice if we know **the sign of the difference between the average pay-offs**, even when **only ranges** are available for the pay-off probability factors using **a strategy region diagram.**

# What if your Value for Money depends on the amount you win per Coin Toss?




**Scenario 1:**
Winning $2 with "Heads" Coin 1




**Scenario 2:**
Winning $20,000 with "Heads" Coin 1

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# What if your Value for Money Changes depends on your wealth?

- **Linear Utility Function** implies the Decision Maker (DM) is **Risk Neutral**. A DM is **Risk Neutral** if he/she is **indifferent** between a bet with an expected pay-off and **a sure amount** equal to the expected pay-off.

- **Concave Utility Function** implies a Decision Maker (DM) is **Risk Averse**. A DM is **Risk Averse** if he/she is **willing to accept less money** for a bet with a certain expected pay-off **than the expected pay-off for sure**.

- **Convex Utility Function** implies a Decision Maker (DM) is **Risk Seeking**. A DM is **Risk Seeking** if he/she is **willing to pay more money** for a bet with a certain expected pay-off **than the expected pay-off for sure**.

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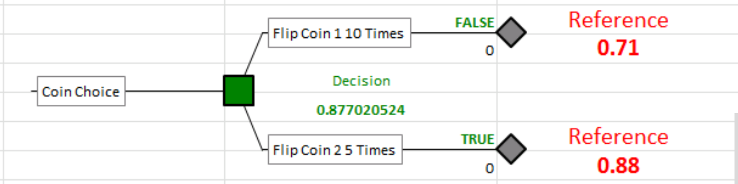
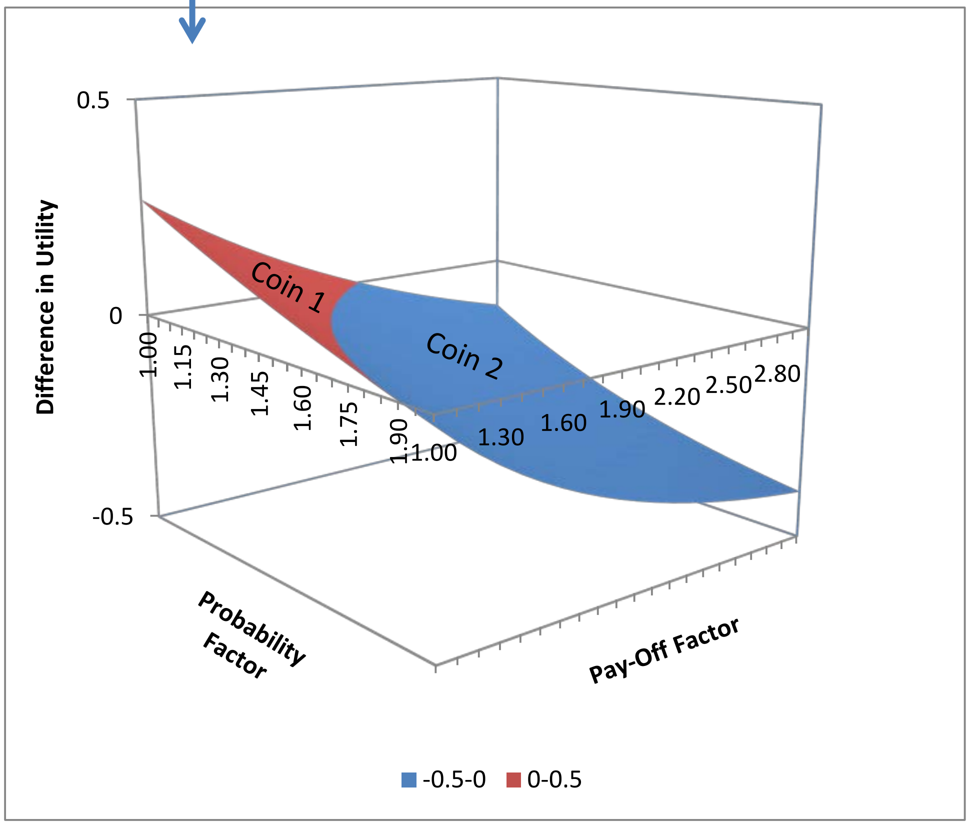
| Alternative | Pr(Heads) | N Trial | Pay-Off per "Heads" | Average Pay-Off |
|---|---|---|---|---|
| Coin 1 | 0.5 | 10 | $20,000.00 | $100,000.00 |
| Coin 2 | 0.75 | 5 | $40,000.00 | $150,000.00 |

| | |
|---|---|
| Prob. Factor | 1.5 |
| Pay-Off Factor | 2 |




Now Max. Exp. Utility

## 2D – Strategy Region Diagram

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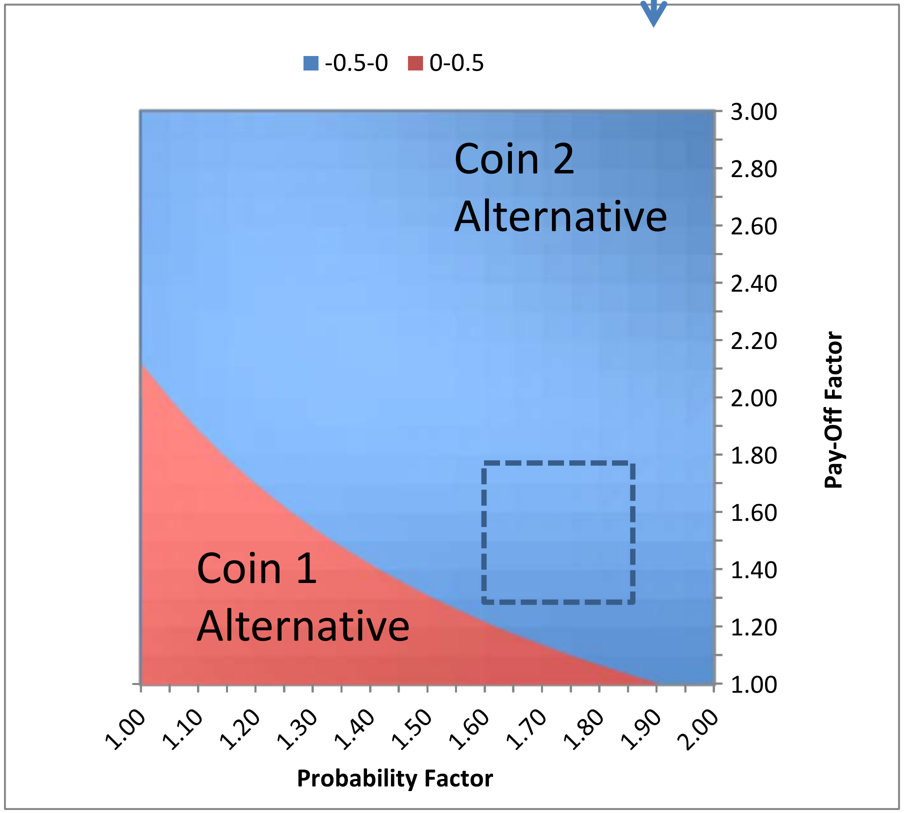

# AN INTRO TO DECISION ANALYSIS

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| Alternative | Pr(Heads) | N Trial | Pay-Off per "Heads" | Average Pay-Off |
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| Coin 1 | 0.5 | 10 | $20,000.00 | $100,000.00 |
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| | |
|---|---|
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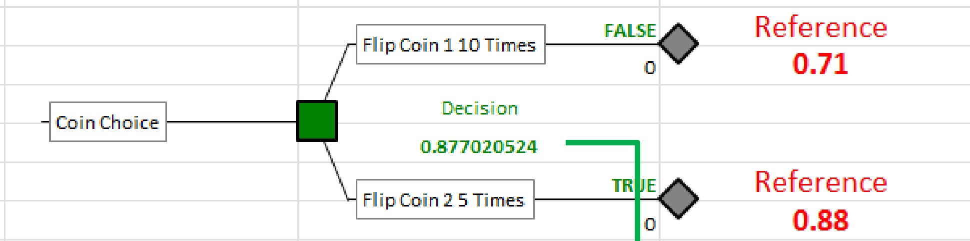
| Pay-Off Factor | 2 |



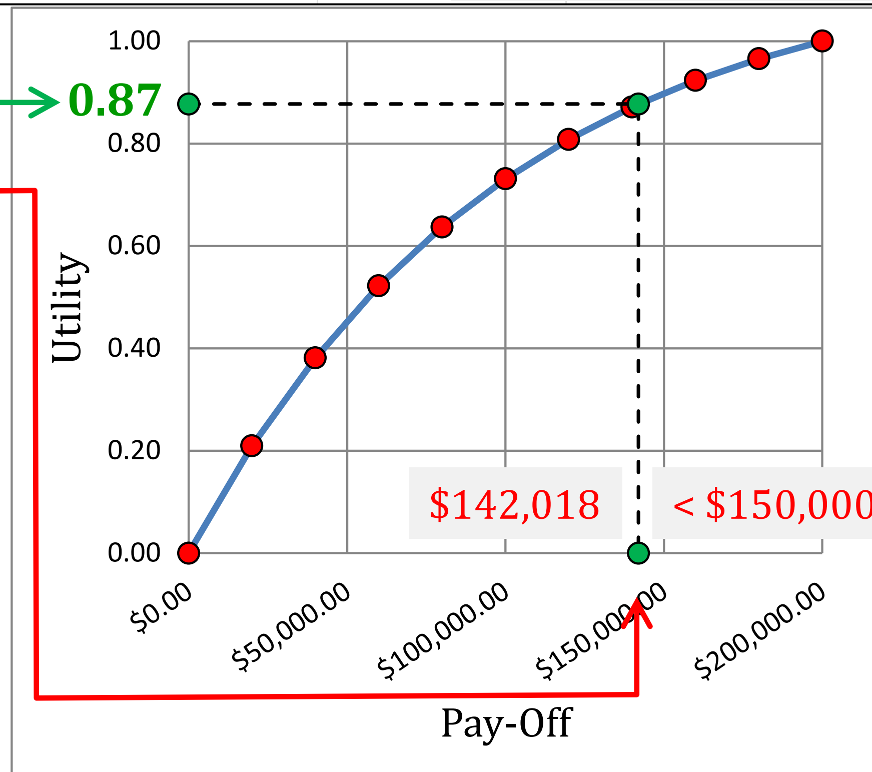

Now Max. Exp. Utility

For how much money are you willing to sell this decision?

$142,018

Called **Certainty Equivalent (CE)**

Provides for an
**Operational Interpretation**
of
**the Utility Concept**.



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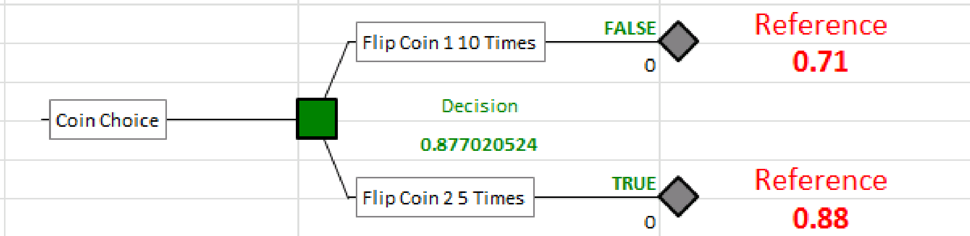



| | |
|---|---|
| Prob. Factor | 1.5 |
| Pay-Off Factor | 2 |

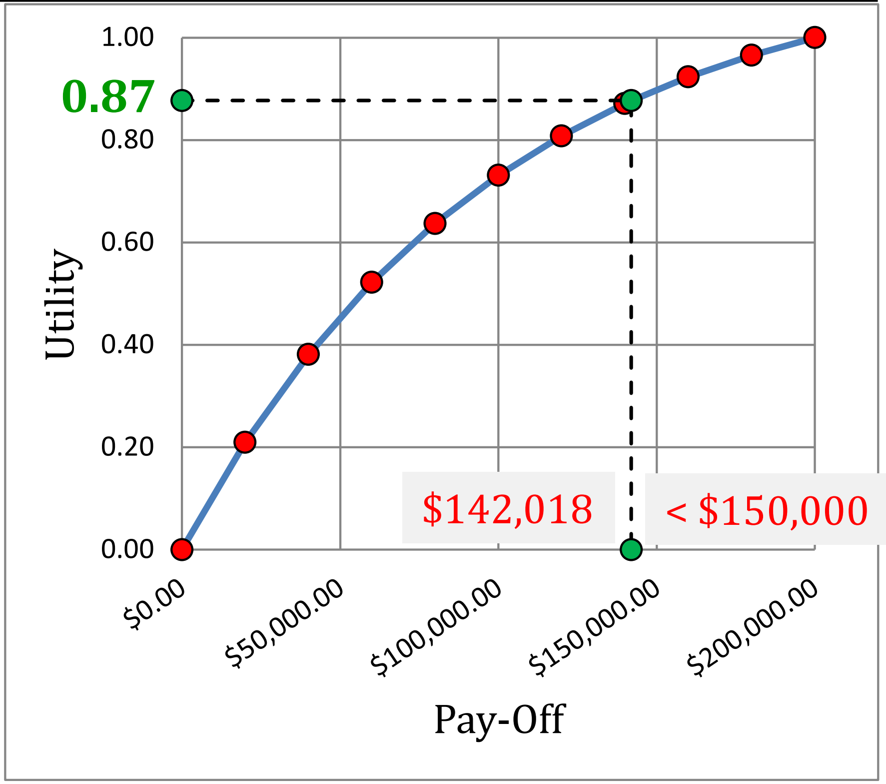
Now Max. Exp. Utility

How much money are you willing to give up to not play?

$150,000 - $142,018 =

$7,982

Called Risk Premium



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1. Coin Tosses
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3. Decision Trees or Influence Diagrams?
4. Elements of Decision Analysis

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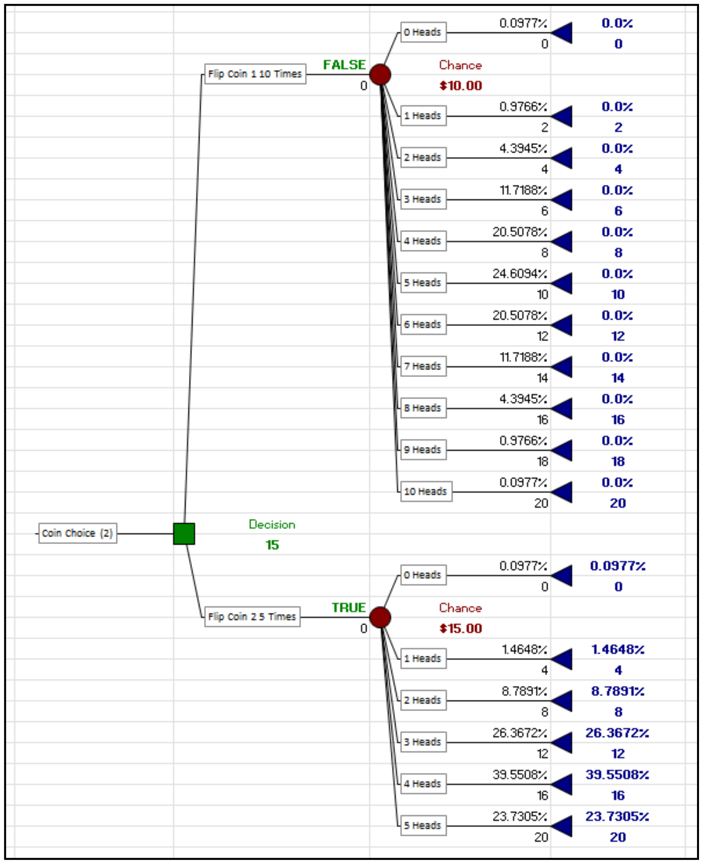
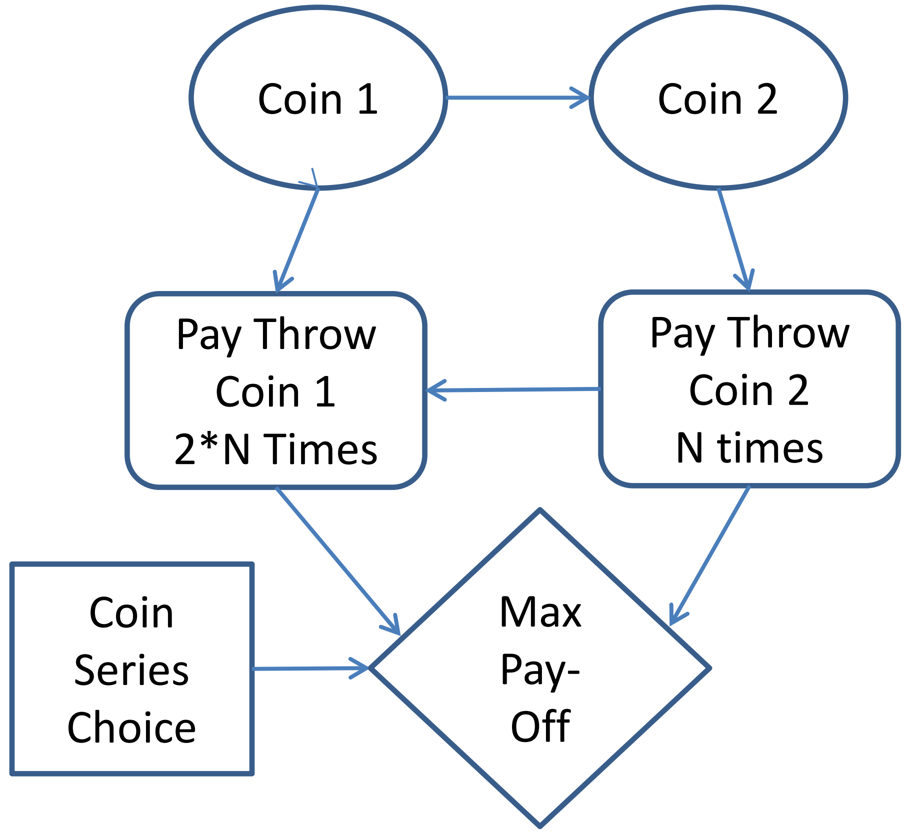
# Decision Trees or Influence Diagrams?




Lot of Detail, but becomes unwieldy




Lack of Detail, Higher level View and makes Dependence explicit

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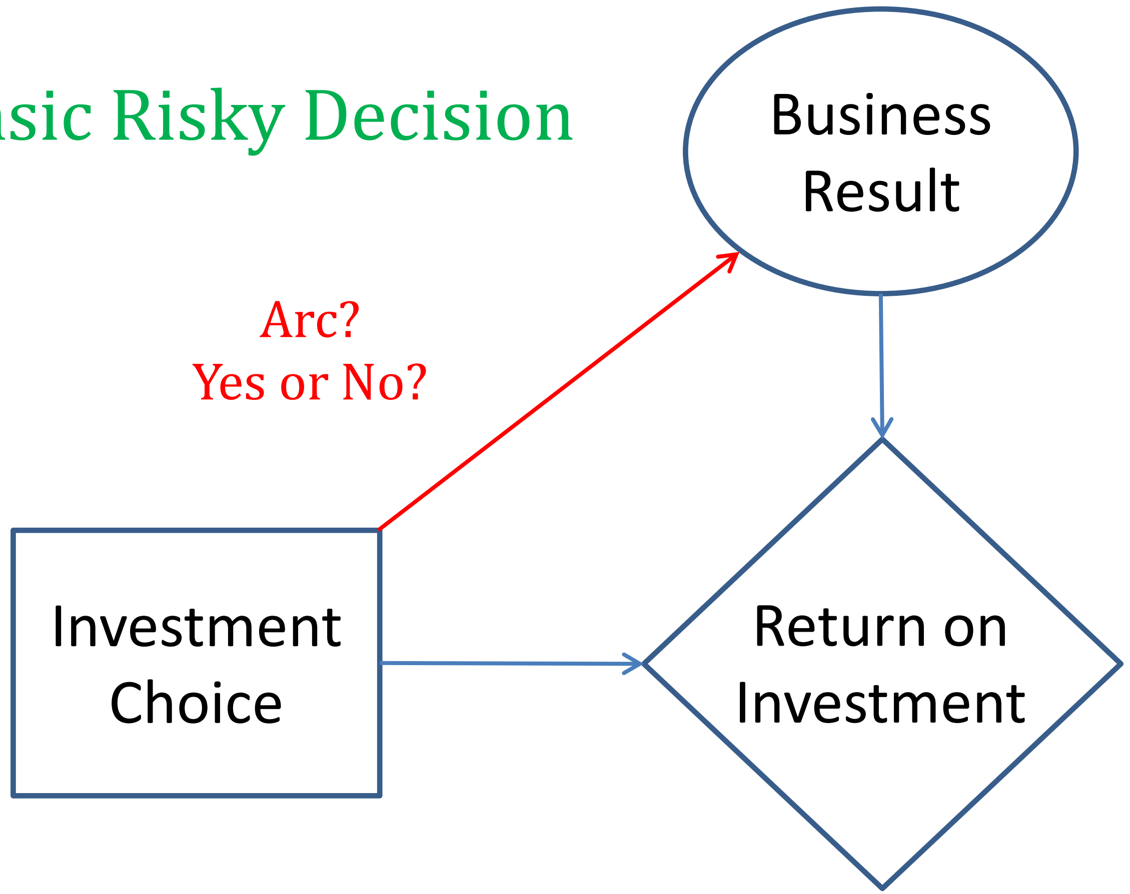
# Some Basic Influence Diagram Examples

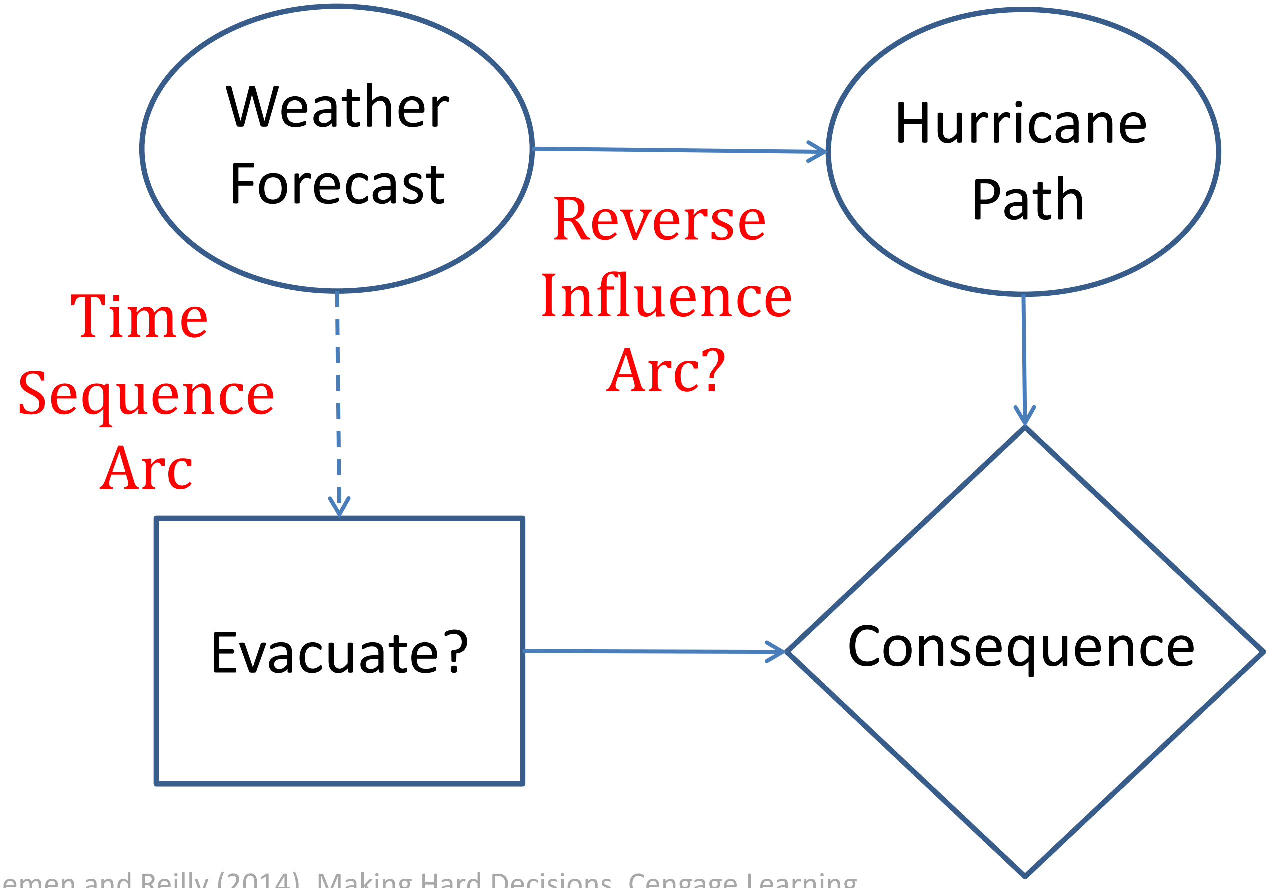
## Basic Risky Decision




Source: Clemen and Reilly (2014), Making Hard Decisions, Cengage Learning

# Some Basic Influence Diagram Examples

## Imperfect Information

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# Elements of Decision Analysis (DA)

- **Multiple Decisions:** The immediate one and possibly more. Decisions are sequential in time. The DP is called dynamic.
- **Multiple Uncertainties:** Each uncertainty node requires a probability model. Multiple uncertainty nodes may be statistically dependent.
- **Multiple or Single Objectives:** In case of multiple conflicting objective the trade-off between objectives needs to be modelled.
- **Multiple values:** Evaluation of achievements of each individual objective requires description of a utility function for each one (linear, concave, convex?)

DA's are Complex!

# Skill Set/Techniques for Decision Analysis (DA)

- **Decision Tree/Influence Diagrams:** To structure and visualize DP's, identify its elements and prescribe the method towards evaluation.

- **Expert Judgement (EJ) Elicitation:** To describe/specify probability models of "one-off" uncertainty nodes and to combine expert judgements.

- **Statistical Inference:** In DA the inference is typically Bayesian in nature. Is used when uncertainties reveal themselves over time to refine/update probability models or combine available data with Expert Judgement.

- **Utility Theory:** To describe "The Decision Maker's" risk attitude/ appetite for the evaluation of a single objective and to formalize trade-off between multiple objectives.

## Thus, a DA is Normative in Nature !