ABSTRACT

This paper presents high-fidelity simulations of the Vortex-Induced Vibration (VIV) phenomena using a new computational model based on the high-order spectral difference (SD) method on unstructured grids. The SD method has shown promise in the past as a highly accurate, yet sufficiently fast method for solving unsteady viscous compressible flows. A Riemann solver is used to compute the inviscid fluxes at the cell interfaces, and the viscous fluxes are computed using an averaging mechanism based on the fluxes from the two cells that share the interface. A third-order Runge-Kutta scheme is used to advance time. In this viscous, compressible flow solver, the displacement of an elastically mounted bluff body has been coupled to the lift force created by vortex shedding. The solver is validated by correlating the lift and drag coefficients with previous published results of two cases: single rigid cylinder, and single elastically mounted cylinder. The rigid cylinder case validates the accuracy of the fluid solver. The elastically mounted case validates the fluid-structure coupling. We simulate the phenomenon of wake galloping with two cylinders after the solver has been validated using single cylinder VIV phenomenon.

NOMENCLATURE

\( A^* \) Normalized maximum cylinder displacement
\( c, \zeta \) Dimensional and normalized damping coefficient
\( C_L, C_D \) Lift and drag coefficients
\( d \) Cylinder diameter
\( dof \) Degrees of Freedom
\( f_N, f^*, F_N \) Cylinder natural frequency, frequency ratio, and normalized frequency
\( k \) Dimensional spring constant
\( m, m^* \) Dimensional and normalized cylinder mass
\( Re \) Reynolds number
\( SD \) Spectral difference
\( St \) Strouhal number
\( t, T \) Dimensional and normalized time
\( u, U^* \) Dimensional and normalized velocity
\( VIV \) Vortex-induced vibration
\( y, Y \) Dimensional and normalized cylinder displacement
\( \mu \) Fluid viscosity
\( \rho \) Fluid density

INTRODUCTION

When a structure or bluff body is subjected to a wind load, it is possible for it to undergo unintended vibrations due to aerodynamic instability phenomena. The main types of such phenomena are vortex-induced vibrations (VIV), galloping and fluttering, and wake galloping. VIV is the most classic type of aerodynamic instability phenomenon and is frequently used to generally describe the other types. VIV occurs when fluid flow over bluff bodies forms unsteady vortices on the trailing side of the body. As the vortices shed off of the body, oscillatory forces are applied to the body. These oscillatory forces cause small amplitude vibrations in a plane normal to the fluid flow. If the oscillatory forces are applied at a frequency near the structure’s natural fre-
As the airfoil lifts, the torsional inertia forces the airfoil to twist downward, causing a plunging motion \[2\].

Galloping and flutter are a special case of VIV where the derivative of the steady state lift coefficient is negative. Thus, as the structure oscillates in one direction, the lift forces generated by the body’s shape decrease and become negative to put it back. These types of phenomena are frequently called diverging and self-regulating since the structure’s lift coefficient causes increased divergence from its resting place and eventually cause its return. In the case of flutter, this motion is accompanied by a torsional twist in the body, causing a pitching and plunging motion, as shown in Fig. 1. As the body lifts and twists, the torsional stiffness forces the body to twist back, causing a plunging motion. This is the phenomenon witnessed during the Tacoma Narrows Bridge accident in 1940 \[1\].

In 2008, a research group from the University of Michigan proposed a new device called VIVACE \[3\] for harnessing renewable energy using VIV. Wake galloping occurs when multiple tandem bodies undergo VIV. The downstream bluff body is affected by the shedding wake from the upstream body. This can cause an increase in vibration amplitude in the downstream body. If the distance between the two bodies is too great (about 6 times the body diameter) or too small (about 2 times the body diameter) wake galloping may not occur \[4\].

If the body is not designed properly, these phenomena can have disastrous effects. Historically, VIV (and its derivative forms) have been intentionally designed out of systems with prejudice due to the extreme destructive power displayed during engineering failures involving VIV. This power has been seen in simple cases such as water flowing past riser tubes bringing oil up from the seabed, flow around heat exchanger tubes, and wind flowing across launch vehicles, and in more famous cases such as the structural failure of Ferrybridge power station cooling towers in England which was resulted from VIV of turbulent flow \[5\].

While engineering practice seeks to design out the VIV phenomena, there has been considerable research into understanding the intense non-linear, unsteady, multi-dimensional flow that causes VIV. The experiments conducted by Feng in 1963 are generally considered the first modern study of VIV. Feng introduces many of the concerns and questions about VIV which have still not been answered. Even though theoretical, experimental, and numerical research into VIV has been going on for decades, the fluid behavior and structural responses cannot be fully explained, only empirically predicted. One of the main problems is that VIV is extremely sensitive to the physical properties of the system. Making simple assumptions, such as only allowing vibrations in the plane normal to fluid flow, can cause wide divergence in results. Much of the research by Feng \[6\], Khalak and Williamson \[7\], Pan et al. \[8\], Zhao and Cheng \[9\], Zhao et al. \[10\], and Ji et al. \[11\] has addressed a single cylinder in a cross flow current undergoing one-degree of freedom (1dof) vibrations. In these studies, the cylinder is in some way restrained from vibrating parallel to fluid flow. However, VIV is inherently multi-dimensional and many situations in reality (such as suspension cables on bridges) do not display that constraint. Consequently, there have been 2-degree of freedom (2dof) studies by Mittal and Kumar \[12\], Jeon and Gharib \[13\], Jauvtis and Williamson \[14\], and Prasanth et al. \[15\] which address transverse and downstream vibrations of the body. These researchers notice specific cases where the phase between the transverse and downstream oscillations is offset such that the cylinder moves in a circular, half-circular, or figure-8 motions, as shown in Fig. 2.

Even though the physical phenomena in the 1dof and 2dof systems are not well understood, work has been done on study-
ing VIV wake galloping effects on multiple cylinders within one domain. This allows for anywhere between 0 to 4 dof to be investigated, although, in these cases the leading cylinder is usually fixed and the downstream cylinder attached elastically some distance behind the first (measured in intervals of cylinder diameter, d). The work done by Shimizu [16], Kim [1], Assi et al. [17], and Jung and Lee [4] suggest that wake galloping can be used a form of energy harvesting by converting the oscillations into electrical current, as seen in Fig. 3. Understanding and predicting the wake galloping response to the system’s properties is still quite difficult, yet Jung and Lee report a field test where the wake galloping device is mounted on a bridge to harvest wind energy and possibly power remote sensor devices that would otherwise require connection to the power grid.

In addition to the difficulty presented by the VIV sensitivity to physical properties and assumptions, a second problem is the wide range of variables to control. The above mentioned studies have observed extremely non-linear VIV effects to varying free stream velocities, cylinder sizes, spring and damper coefficients, and Reynolds numbers. To help gather and compare data, a set of non-dimensionalized variables are constructed in Eqn. 1-8 [15, 18].

\[
\begin{align*}
Y &= \frac{y}{d} \quad (1) \\
T &= \frac{tu}{d} \quad (2) \\
m^* &= \frac{4m}{\rho \pi d^2} \quad (3) \\
\zeta &= \frac{c}{\sqrt{2k/m}} \quad (4) \\
U^* &= \frac{u}{f_N d} \quad (5) \\
A^* &= \frac{\gamma_{\text{max}}}{d} \quad (6) \\
f^* &= \frac{f_{\text{shed}}}{f_N} \quad (7)
\end{align*}
\]

with,

\[
\begin{align*}
F_N &= \frac{f_N d}{u} = \frac{1}{U^*} \quad (8) \\
St &= \frac{(f_{\text{shed}} d)/u}{u} \quad (9) \\
f_N &= \frac{1}{2\pi} \sqrt{\frac{k}{m}} \quad (10) \\
C_L &= \frac{F_L}{\frac{1}{2} \rho u^2 d} \quad (11) \\
Re &= \frac{\rho u d}{\mu} \quad (12)
\end{align*}
\]

where \( y \) is the cross-flow displacement, \( t \) is the time, \( u \) is the free stream velocity, \( d \) is the cylinder diameter, \( m \) is the cylinder mass, \( \rho \) is the fluid density, \( c \) is the structural damping coefficient, \( k \) is the structural spring coefficient, \( f_N \) is the structural natural frequency, \( \gamma_{\text{max}} \) is the maximum amplitude of cross-flow vibration, \( f_{\text{shed}} \) is the vortex shedding frequency, \( St \) is the Strouhal number, \( C_L \) is the aerodynamic lift coefficient of the lift force \( F_L \), \( Re \) is the Reynolds number, and \( \mu \) is the fluid viscosity.

Even with these normalized values, it is difficult to create a single chart, table, or medium to effectively communicate the cylinder response of each experiment. This is partly due to the nonlinear amplitude response. For example, a typical VIV amplitude response (A*) for a variety of normalized flow velocities (U*) can be seen in Fig. 4. There are clearly three nonlinear regimes, namely the initial branch, lower branch, and upper branch. However, this only holds true if the mass-damping coefficient (m*) is sufficiently small, in the order of 10^{-1} [18]. When m* increases in the order of 10^{2} the regimes shift, as shown in Fig. 5.

Although the non-linearity cannot necessarily be explained, it has been well established empirically. This allows numerical solutions to be compared with experimental results for validation of their methods. The present work investigates 1dof transverse vibrations in elastically mounted rigid cylinders using a massively parallel high-order spectral difference (SD) solver. The code is built for moving and deforming unstructured grids [19] which allows the solver to analyze a wide variety of bluff body shapes and configurations, including square cylinders, tandem cylinders, or even clusters of parallel cylinders. To validate the solver, benchmark cases using a single elastically mounted cylinder have been compared to results from previous literature.

**COMPUTATIONAL MODEL**

**Spectral Difference Method**

The SD method has shown promise in the past as a highly accurate, yet sufficiently fast method for solving unsteady viscous flows. More detailed derivation and analysis of the viscous,
Mass-Spring-Damper System

To allow the cylinder to vibrate, a mass-spring-damper system is modeled as shown in Fig. 6. The upstream cylinder is rigid. The downstream cylinder is free to translate vertically. A spring and damper connect the downstream cylinder to the rigid ground. In some cases of this study, only a single cylinder is used. In which case, the first cylinder is elastically mounted and the second cylinder is removed.

The equation to describe the elastically mounted cylinder can be written as:

\[ m\ddot{y} + c\dot{y} + ky = F_L \] (13)

Using the Newmark-\(\beta\) method, we can discretize Eqn. 13 in the time domain into the following second order approximations.

\[
\ddot{y}_{n+1} = \frac{(F_L - ky_n - c\dot{y}_n)}{m} \quad (14)
\]

\[
\dot{y}_{n+1} = \dot{y}_n + \Delta t/2(\ddot{y}_n + \ddot{y}_{n+1}) \quad (15)
\]

\[
y_{n+1} = y_n + \Delta t \dot{y}_n + (1/2 - \beta)\Delta t^2\ddot{y}_n + \beta\Delta t^2\ddot{y}_{n+1} \quad (16)
\]

where \(\beta\) is the Newmark variable, which is typically set to \(\beta = 1/4\), and \(n\) is the number of time steps \(\Delta t\). The Newmark-\(\beta\) method is stable under the condition that [21]:

\[
\frac{\beta \Delta t^2}{1 + \frac{1}{m} \Delta t^2 \beta} < 4.0
\] (17)

To easily compare results between different studies, it is useful to now non-dimensionalize the variables by normalizing them with the system parameters in Eqn. 1-8. Based on Eqn. 1 and 2 it can be seen that,

\[ \ddot{y} = u\ddot{Y} \] (18)

\[ \ddot{Y} = \frac{u^2}{d}\ddot{Y} \] (19)
Therefore, Eq. 13 can be rewritten,
\[ \ddot{Y} + \frac{c d}{m u} \dot{Y} + \frac{k d^2}{m u^2} Y = \frac{1}{2} \rho d^2 \left( \frac{1}{2} C_L \right) \] (20)

By substituting Eq. 5 and 8,
\[ \ddot{Y} + \frac{c}{m f_N} F_N \dot{Y} + \frac{k}{m f_N^2} F_N^2 Y = \frac{1}{2} \rho d^2 \left( \frac{1}{2} C_L \right) \] (21)

Lastly, by substituting Eq. 3, 4, and 7 and simplifying, the non-dimensional form of Eq. 13 is:
\[ \ddot{Y} + 4 \pi F_N \xi \dot{Y} + (2 \pi F_N)^2 Y = \frac{2 C_L}{\pi m} \] (22)

These non-dimensional variables can be used as a basis of comparison between various studies using different dimensional values.

**Mesh Deformation and Domain Decomposition**

The domain for this system was chosen to be a large rectangle, 60d wide and 40d high as shown in Fig. 7. Thus the blockage, \( B \), is \( 1/40 = 2.5\% \). The upstream cylinder is centered at \((0,0)\) and the downstream at \((4d,0)\). The domain for one cylinder validation is identically shaped but without the downstream cylinder. The inlet (left) and outlet (right) boundaries are constrained using Dirichlet conditions of flow speed and pressure. The top and bottom boundaries have symmetric constraints applied to minimize the boundary interference. A close-up of the mesh around the cylinders can be seen in Fig. 8. The single cylinder mesh uses 19,181 elements and the tandem cylinder mesh uses 23,873 elements. These meshes were selected since they are sufficiently fine to have converged on accurate solutions.

After the cylinder response was approximated using the Newmark-\( \beta \) method, each vertex of the mesh is deformed at each time step by some displacement \( y_v \), where:

\[ y_v = (1 - C_{deform}) y_{cyl} \] (23)

\[ C_{deform} = \begin{cases} 
0 & r < 0 \\
10r^3 - 15r^4 + 6r^5 & 0 \leq r \leq 1 \\
1 & r > 1
\end{cases} \] (24)

\[ r = \frac{r_v - r_{min}}{r_{max}} \] (25)

where \( r_{min} \) is the distance of 0.6d and \( r_v \) is the distance of the vertex to the cylinder center. \( r_{max} \) is a prescribed outer limit of mesh deformation. \( y_{cyl} \) is the y displacement found using the Newmark-\( \beta \) method. Thus the physical mesh is deformed based on a blended analytical function such that each node’s deformation is based on its proximity to the deforming cylinder. Vertices with \( r_{min} \) of the cylinder deform with the cylinder, and vertices outside of \( r_{max} \) do not deform at all. These blended functions are also used in [19, 22].

To allow for higher order solutions and finer meshes, the domain has been decomposed into separate domains, each to be solved on a separate processor. This essentially splits the problem into a group of parallel problems which can be solved si-
FIGURE 9. Domain decomposition onto 4 processors. Each shade represents the mesh quadrant solved by a different processor. The processor interface is the boundary layer between each processors portion of the domain

multaneously on different processors within a cluster of servers. A decomposed mesh using 4 processors can be seen in Fig. 9. Since the fluid flows across the domain, and the solutions for the cells on one side of a processor interface depend on the solutions of the cells across the interface, some cross-processor communication and extra memory space is required for two portions of the domain interact. Message Passing Interface (MPI) is used to communicate values between the processors.

VALIDATION

In the following section two validation cases are tested to ensure that the solver produces accurate results. The first case is a single rigid cylinder in a flow field. The second case is a single elastically mounted cylinder in the same field.

Single Rigid Cylinder

This test case uses a single cylinder in a flow field with $Re = 100$. The freestream Mach number is set to 0.2. This “rigid” single cylinder is actually an elastically mounted cylinder allowed to vibrate in the vertical direction connected by a very stiff spring and damper ($U^* = 0.063$ and $\zeta = 50$). This effectively reduces the maximum $Y$ to the order of $10^{-6}$. Figure 10 shows lift and drag coefficients for single “rigid” cylinder. Table 1 reports a comparison between the present compressible viscous flow at Mach number 0.2 to other numerical and experimental studies for incompressible viscous flow.

The reported lift and drag coefficients are very closely matched with the previously published results. The rms lift coefficient, $C_L^r$, is slightly higher than those reported by Liang et al. [20] and Sharman et al. [23] yet below that reported by Kang [25]. The mean drag coefficient, $C_D$, is slightly higher than those reported by other authors, however the rms drag, $C_D^r$, matches that reported by Sharman et al. [23]. It is possible that the slight compressibility of the present fluid may cause the slight increase in mean drag coefficient, but the effect is minimal. Mittal and Tezduyar [26] report a mean drag coefficient around 1.4 for the same fluid flow conditions. Overall, this validation proves that this solver correctly predicts VIV when the cylinder is fixed.

Single Elastically Mounted Cylinder

This test case uses a single elastically mounted cylinder with $\zeta = 0.0$, $m^* = 10$. A variety of flow conditions have been used, but in all cases $U^* = 0.06Re$. This condition effectively couples the spring coefficient to the flow velocity. Doing so allows the response amplitude, $A^*$, to be plotted for a variety of Re, as shown in Fig. 11 which compares the present amplitudes to those predicted by Prasanth et al. [15, 27]. Similarly, Fig. 12 compares

FIGURE 10. Lift and drag coefficients for the single rigid cylinder in a flow field with Reynolds number 100

TABLE 1. Comparison of present results with other results for flow over a single rigid cylinder at Reynolds number 100

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Re no.</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Nodes</td>
<td>19393</td>
<td>33400</td>
<td>14441</td>
<td>13696</td>
<td>62127</td>
</tr>
<tr>
<td>Blockage</td>
<td>0.025</td>
<td>0.0312</td>
<td>0.02</td>
<td>0.047</td>
<td>–</td>
</tr>
<tr>
<td>$C_L^r$</td>
<td>0.239</td>
<td>0.232</td>
<td>0.23</td>
<td>–</td>
<td>0.32</td>
</tr>
<tr>
<td>$C_D^r$</td>
<td>1.385</td>
<td>1.365</td>
<td>1.33</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>$C_D$</td>
<td>0.006</td>
<td>0.0086</td>
<td>0.0064</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>St no.</td>
<td>0.166</td>
<td>0.164</td>
<td>0.164</td>
<td>0.165</td>
<td>0.165</td>
</tr>
</tbody>
</table>
the present lift coefficient amplitude to those predicted by Prasanth et al. The jump in vibration amplitude and lift coefficient amplitude marks the synchronization, or “lock-in” region, where the frequency ratio (Eqn. 7) approaches unity. It is this region which contains the highest energy transfer from the fluid to the structure.

**FIGURE 11.** The maximum normalized vibration amplitude for a single elastically mounted cylinder

The amplitude of vibration and lift coefficients match very well with those reported by Prasanth et al. [15, 27]. A plot of the Y-displacement against time is shown in Fig. 13. In addition, these values are similar to those reported by Khalak and Williamson [18]. This validation proves that this solver correctly and accurately couples the fluid and structure responses together. This fluid-structure interaction can now be used to investigate more complicated cases, such as tandem cylinder wake galloping.

**FIGURE 13.** Nondimensional displacement of the elastically mounted cylinder over nondimensional time

**TANDEM CYLINDER WAKE GALLOPING**

Figure 14 shows the vortex shedding of two synchronized tandem cylinders at $Re = 200$, with the contour of $\omega_d/u_\infty$ ranging from -4 to 4. The first cylinder is rigid and the second is elastically mounted with $U^* = 6.6$, $m^* = 10.0$, and $\zeta = 0.0$. The mesh has 8,374 cells such that there are 30 cells around the periphery of each cylinder. The simulation was performed using a third-order SD method. The cylinder walls are dealt by a high-order curved representation. The center-to-center distance between the cylinders is 4d. Figure 15 shows the non-dimensional displacement, $Y$, of the second cylinder over non-dimensional time.

Note that the amplitude of vibration for this tandem cylinder wake galloping case is nearly twice that of the single elastically mounted cylinder case shown in Fig. 11. The addition of the upstream cylinder relative to the elastically mounted cylinder has dramatically increased the vibrational amplitude. That is, more energy is converted from the fluid into the kinetic energy of the cylinder. In a system like that proposed by Jung and Lee [4] in Fig. 3 the electricity generation is partially dependent on the amplitude of vibration.

**CONCLUSION**

A massively parallel, high-order spectral difference solver is extended for simulating Vortex-Induced Vibrations in bluff bodies based on a robust and accurate scheme for moving and de-
wake galloping case is nearly twice that of the single elastically mounted cylinder case. The addition of the upstream cylinder relative to the elastically mounted cylinder has dramatically increased the vibrational amplitude. That is, more energy is converted from the fluid into the kinetic energy of the cylinder. Thus more energy could be extracted from the vibrating cylinder and converted into electricity. Further investigation of the wake galloping response to a variety of flow and structural conditions is already underway.

ACKNOWLEDGMENT

Chunlei Liang and Andrew DeJong would like to acknowledge a research grant from ONR with award no. N000141210500. They are also grateful to the intramural seed funds awarded by the George Washington University.

REFERENCES


