Ques.1: Consider the process of improving the performance of a program by optimizing some of the code in the program and running it in an enhanced (i.e., optimized) form. Suppose that optimized instructions run 20 times faster than sequential.

- Derive the speedup equation when \( x\% \) of the instructions are optimized.
- Determine the percentage of code that must be optimized to get a speedup of 2, 5, and 10 respectively.
- If 25% of the code cannot be optimized (due to the inherently sequential nature such as I/O etc.), then what is the maximum speedup you can achieve.

Ans: The speedup \( S \) is defined as
\[ S = \frac{T_{seq}}{T_{par}} \]
where \( T_{seq} \) is non-optimized sequential and \( T_{par} \) is optimized (parallel) time. For each sequential cycle, the optimized time is \((1/20 = 0.05)\). If each sequential instruction takes 1 cycle, the sequential time for a program with \( N \) instructions is \( T_{seq} = N \) cycles. The time for a program with \( x\% \) of its instructions optimized is
\[ (N \times 0.05 	imes x + N \times 1 \times (1-x)) = N(0.05x + (1-x)). \]
Therefore speedup
\[ S = \frac{N}{N(0.05x + (1-x))} = \frac{1}{1-0.95x}. \]
Use this equation to determine the values of \( x \) for \( S=2,5,10 \).

- For \( S=2 \), we get \( x = \frac{100}{95}(1 - \frac{1}{2}) = 52.6\% \)
- For \( S=5 \), we get \( x = \frac{100}{95}(1 - \frac{1}{5}) = 84.2\% \)
- For \( S=10 \), we get \( x = \frac{100}{95}(1 - \frac{1}{10}) = 94.7\% \)

If 25% of the code cannot be optimized, then maximum value of \( x = 0.75 \). Substitute in the speedup equation to get maximum speedup
\[ S = \frac{1}{1-0.95\times0.75} = 3.47. \]

(Practice Problem) Ques.3: This question requires you to generalize Amdahl’s law to the case when multiple enhancements are possible. Three enhancements with the following speedups are proposed for a new architecture:

- \( \text{Speedup}_1 = 30 \)
- \( \text{Speedup}_2 = 20 \)
- \( \text{Speedup}_3 = 15 \)

Only one enhancement is usable at a time (but multiple can be used over the entire application). If enhancements 1 and 2 are each usable for 25% of the time, what fraction of the time must enhancement 3 be used to achieve an overall speedup of 10 for the entire application?

Ans: Amdahl’s Law can be generalized to handle multiple enhancements. If only one enhancement can be used at a time during program execution, then
\[ \text{Speedup} = \frac{1}{[1 - \sum_i FE_i + \sum_i FE_i/SE_i]} \]
where \( FE_i \) is the fraction of time that enhancement \( i \) can be used and \( SE_i \) is the speedup of enhancement \( i \).
For a single enhancement the equation reduces to the familiar form of Amdahl’s Law. With three enhancements we have
\[ \text{Speedup} = \frac{1}{[1 - (FE_1 + FE_2 + FE_3) + (FE_1/SE_1) + (FE_2/SE_2) + (FE_3/SE_3)]} \]
Substituting in the known quantities gives
\[ \text{Speedup} = \frac{1}{[1 - (0.25 + 0.25 + 0.45) + (0.25/30) + (0.25/20) + (0.45/15)]} \]
Solving the above equation for \( FE_3 \) gives \( FE_3 = 0.45 \). Thus, the third enhancement must be usable 45% of the time.