Problem 1: (25 points)

Let $G = (V,E)$ be the following weighted undirected graph: $V = \{1, 2, 3, 4, 5, 6, 7\}$ and $E = \{(1, 4), (3, 4), (3, 6), (2, 6), (4, 2), (2, 5), (1, 6), (4, 6), (1, 5), (4, 7), (7, 5)\}$, with respective weights $8, 11, 3, 6, 9, 20, 12, 3, 12, 16, 10$.

a) Find a minimum spanning tree in $G$.

b) Designate node 1 as a source node. Find the distance between 1 and all the other nodes using the greedy algorithm. Show the value of the DIST array at every step.

Problem 2: (25 points)

Let $x[1:n]$ be a sorted array. Define $y[1:n]$ as follows: for all $i = 1, 2, \ldots, n$, $y[i]$ is the number of times the value of $x[i]$ repeats in the input array $x[1:n]$. For example, if $n = 4$ and $x[1:4]$ is $7, 7, 10, 12$, then $y[1] = y[2] = 2$ (because 7 occurs twice), $y[3] = 1$ (because 10 occurs once), and $y[4] = 1$.


b) Write a divide-and-conquer algorithm that takes as input an arbitrary sorted array $x[1:n]$, and returns as output the array $y[1:n]$. Analyze the time of your algorithm.

Problem 3: (25 points)

The most frequent value in an array $A[1:n]$ is the value that occurs the most in $A$. For example, if $A[1:7]$ is $1, 4, 2, 4, 2, 5, 4$, then the most frequent value in $A$ is 4.

a) Write an algorithm that takes as input an arbitrary unsorted array $A[1:n]$ and returns the most frequent value in $A$. (Hint: use sorting and your algorithm of Problem 2 above.)

b) Give the time complexity of your algorithm.

Problem 4: (25 points)

You have $n$ employees and $n$ jobs. It costs $c_{ij}$ dollars for job $i$ to be done by employee $j$ (the $c_{ij}$'s are given input). The job assignment problem is to assign exactly one job to each employee in such a way that the sum of the costs of the $n$ jobs is minimized.

a) Write a greedy algorithm for the job assignment problem.

b) What is the time complexity of this algorithm?

c) Show by a counter example that this greedy method does not always yield an optimal solution (Hint: You can find a counter example where $n=2$).